Calibration of Ice Growth Models for Bare and Snow Covered Conditions: A Summary of Experimental Data from a Small Prairie Pond

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Ice thickness is an important consideration in cold regions engineering. In many cases ice thickness measurements are unavailable or incomplete. Difficulties are encountered when attempting to reconstruct historical ice thickness records at specific locations because often only regional meteorological data is available. Furthermore, the characteristics of the snow cover affect the growth rates of the ice cover as much, or more than, the meteorological conditions. Records do not provide information on the actual snow depths, densities, and thermal conductivities; thereby requiring approximations on the basis of observations elsewhere.

This paper describes field measurements of ice growth and snow conditions under controlled conditions on a small prairie-like pond. The objective of these measurements is to provide a basis for choosing salient heat transfer parameters to model ice growth. Simulations of measured ice thicknesses under both the ambient snow pack and in an area where the snow was cleared on a daily basis provide an assessment of the thermal effects of snow covers. Ice thicknesses for both conditions could be simulated reasonably well with an ice albedo of 0.8, a thermal conductivity of snow between 0.08 and 0.12 W/m -°C, a convective heat transfer coefficient of 15 W/m^2-°C, and a snow density of 250 kg/m^3.

1. Introduction

Ice thickness is an important consideration in cold regions engineering from a number of perspectives. Calculation of ice forces on riparian structures requires estimates of late-winter ice thicknesses. Forecasting bearing capacities of ice covers on lakes and rivers requires an understanding of the growth rates of ice covers during the winter period. The theoretical aspects of ice growth are reasonably well understood, but require the quantification of the heat flux at the ice or snow surface and the depth of the snow cover and its corresponding thermal properties.

Difficulties are encountered when attempting to reconstruct historical ice thickness records because often only regional meteorological data is available. This precludes the definition of site-specific wind speeds, vapour pressures, and cloud covers that are required in the comprehensive calculation of heat flux. Furthermore, the characteristics of the snow cover affect the growth rates of the ice cover as much or more than the meteorological conditions. While on one hand, snow reduces ice growth rates by insulates the ice, it also can contribute to the formation of thicker ice by causing snow ice to form. Usually, the effects of the snow cover can only be estimated from regional snowfall records. These records do not provide information on site specific snow depths, densities, and thermal conductivities; thereby requiring approximations on the basis of observations elsewhere.

This paper describes provides estimates of heat transfer coefficients over ice and snow on the basis of readily available regional values of meteorologic characteristics, evaluates a model that calculates the growth of ice, and implicitly determines the thermal properties of a prairie snow cover. Measurement of ice thicknesses and snow depths are provided for two winters: 1992-93 and 1993-94. Simulations of the growth of the ice cover under both the ambient snow pack and in an area where the snow was cleared on a daily basis provide an assessment of the thermal effects of the snow cover.

2. Ice Cover Growth

An ice cover ultimately forms on a lake or reservoir when the surface temperature is supercooled for a sufficiently long time for a stable ice sheet to form. In shallow lakes, even light winds are sufficient to prevent thermal stratification and the entire water body cools more or less at the same rate. If the well mixed condition is maintained by constant wind, the entire water column cools to 0°C, frazil forms at the surface and an ice cover forms. However, if winds are intermittent, surface ice usually forms during periods of low or no wind when the surface supercools even while the bulk water temperature is above freezing. The higher the temperature of the water body, the longer is the required duration of no wind conditions to establish supercooling at the water surface. Andres (1995) describes the near-surface water temperature gradients and the relationship between the bulk water temperature and the duration of calm periods required for the formation of an ice cover.

Once an ice cover has formed, the growth of the ice cover is a function of the heat flux at the ice or snow surface, the depth of the snow cover and its corresponding thermal properties, and the thickness of the ice itself. The rate of growth of the ice is determined by the difference between the heat flux to the atmosphere and the heat being transferred to the ice from the water column. In shallow fast flowing rivers, the latter term is not usually an issue because the water temperature in a river is usually very close to zero when the surface ice forms. However, in very deep flowing rivers and certainly in lakes and reservoirs this may not be the case.

Four main mechanisms (Andres, 1984, 1993) contribute to the total heat flux, $\boldsymbol{Q_t}$ at the snow or ice surface. They are solar radiation, long wave radiation, evaporation, and conduction. The total heat flux can be divided into two main components: the solar radiation component, $\boldsymbol{Q_s}$ and the temperature-related heat transfer component, $\boldsymbol{Q_a}$ which is the sum of the long wave radiation, evaporation, and conduction. The solar radiation component is given by

$$Q_s = (1 - a)Q_i \qquad \textbf{[1]}$$

where a is the albedo, $\boldsymbol{Q_i}$ is the incident shortwave radiation, and $\boldsymbol{Q_s}$ is the net shortwave radiation absorbed at the surface of the water body.

The temperature dependent components of the heat flux can be combined and linearized in the form

$$\boldsymbol{Q_a = H_a(T_a - T)} \qquad \textbf{[2]}$$

where $\boldsymbol{T_a}$ and $\boldsymbol{T}$ are air and water temperatures, respectively, and $\boldsymbol{H_a}$ is a heat transfer coefficient which accounts for wind, barometric pressure, relative humidity, and cloud conditions. This linear approximation is acceptable as a first approximation because many of the above noted effects cannot be accounted for, even for the most rigorous analysis. The value for $\boldsymbol{H_a}$ for a particular location can be determined by calibration. Adding equations **[1]** and **[2]** gives the net heat flux at the air/snow/ice interface.

$$Q_t = (1 - a)Q_i + H_a(T_a - T) \qquad \textbf{[3]}$$

The ice growth can be calculated by equation **[4]** (Ashton, 1986), in which $\boldsymbol{h_i}$ and $\boldsymbol{h_s}$ is the thickness of the ice and the snow, respectively and $\boldsymbol{k_i}$ and $\boldsymbol{k_s}$ is the thermal conductivity of the ice and the snow, respectively. The latent heat of fusion **L** is 333,000 J/kg and the density of ice $\boldsymbol{r_i}$ is 920 kg/m^3. The other terms have been defined previously. This equation accounts for heat loss from the ice cover to the atmosphere by convection, heat introduced to the ice cover from the water by conduction, and heat introduced by solar radiation.

$$r_i L \frac{dh_i}{dt} = \left[\frac{T_a}{\frac{h_i}{k_i} + \frac{h_s}{k_s} + \frac{1}{H_{ai}}}\right] - Q_{wi} - Q_s \qquad \textbf{[4]}$$

The heat conducted to the ice cover from the water $\boldsymbol{Q_{wi}}$ depends on the water temperature, the thermal conductivity of the water column, and any inflow of heat from the bottom of the water column. This term can be significant for the case with flowing warm water under the ice cover where the convective processes dominate (Calkins, 1984). In a lake or pond, however, the dominant heat transfer process between the water and the ice is conduction and in most shallow lakes the quantity of heat delivered to the ice cover by this process is small. Andres (1995) found that excluding this term from the heat calculations in a shallow pond resulted in only a 2% increase in the calculated ice thickness.

3. Field Measurements

The Millwoods Pond is a storm retention lake located on the southern edge of Edmonton, Alberta at a latitude of about 53.4° north (Figure 1). The surface area of the pond is about 10,000 m^2 and the pond has a maximum depth of about 2 m in winter. The drainage area of the pond is about 1.0 km^2 of which about 20% is impermeable pavement or roof surface. During the winter and even in the early spring there is limited inflow of water. Thus the level is relatively constant and there are no energy fluxes associated with inflow or outflow of water. Figures 2 and 3 show the snow-covered pond during the winter with the snow removed from one small area to measure the growth of ice without the insulating effects of the snow cover.

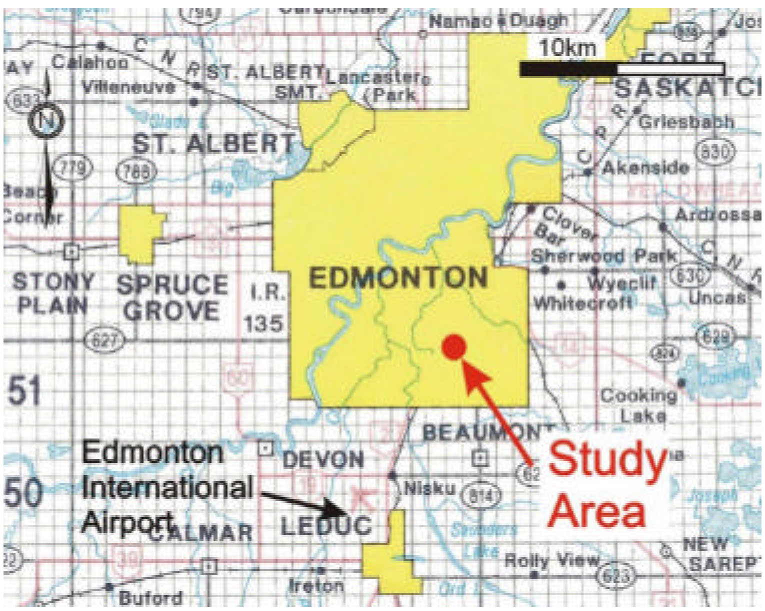

Figure 1 Study Area

Data related to ice growth and deterioration were collected in the winters of 1992-1993 and 1993-1994. Snow depths and densities, ice thicknesses and densities, and water

Figure 2 Snow covered pond with snow removed from small area

Figure 3 Close up view of cleared area

temperatures and depths were measured. Snow depths were monitored approximately once per week over the winters of 1992-1993 and 1993-1994. In 1992, significant amounts of snow did not develop until mid-December and the maximum snow depth on the ice did not exceed 0.2 m any time during the entire winter. In 1993, a limited amount of snow developed in early November, with a substantial snow fall in late December that persisted into early spring. The maximum snow depth was about 0.30 m in the middle of January, while on the average the snow pack had a thickness of about 0.25 m. The average late-winter snow density was about 250 kg/m^3 in both 1993 and 1994.

Ice thickness measurements were obtained daily in the early part of the winter, and about once per week in late winter. During both winters, an area of the ice cover was kept free of snow so that the thermal insulating effects of the snow could be quantified and the effects of the snow cover on the growth of the snow ice could be evaluated. A coarse net was placed on the ice underside early in the winter to provide an internal reference so that thickness changes on both the top and bottom of the ice could be determined. The maximum ice thickness under the snow was about 0.60 m for both years, however in 1993 the growth of snow ice due to greater snow depth produced 0.1 to 0.2 m of ice.

As expected, the maximum thickness of the snow free ice was greater than that of the snow covered ice. In 1992-1993 the cleared ice grew to a thickness that was about 20% greater than the snow covered ice. In 1993-1994 the cleared ice grew to a thickness that was about 60% greater than the snow covered ice. This differential is related to the difference in snow accumulations for the two years and the fact that the coldest part of the 1992-1993 winter occurred when there was very little snow anywhere on the pond. The measured ice thickness and snow depth are shown in Figures 4 and 5 for 1992-93 and 1993-94, respectively.

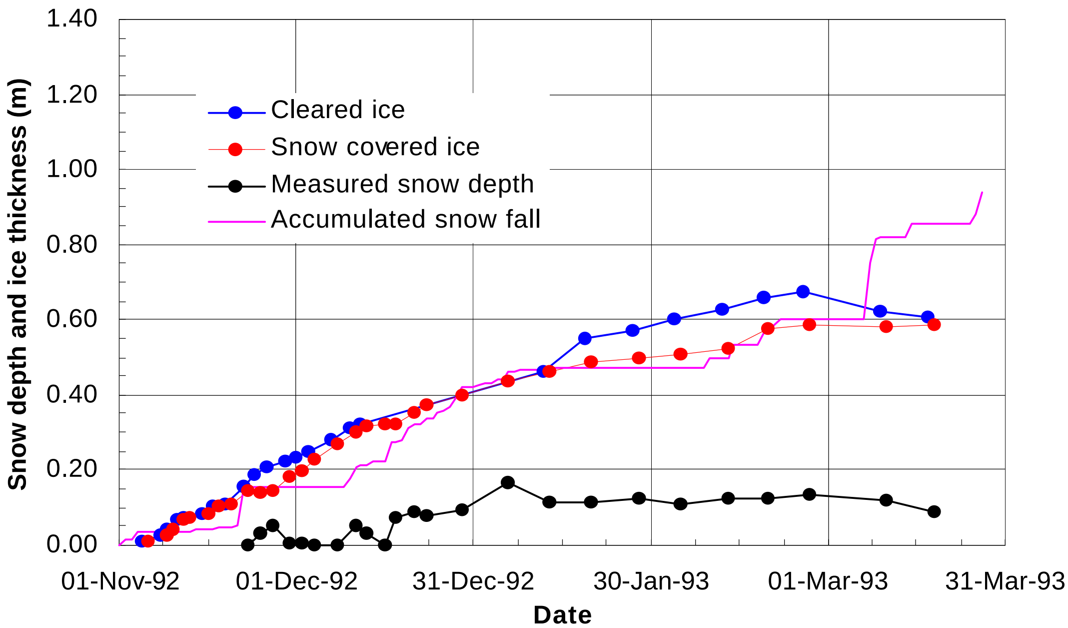

Figure 4 Snowfall, snow depth, and ice thicknesses, 1992-93

Regional values of air temperature and precipitation were measured at the Edmonton International Airport, about 10 km south of the pond site. Solar radiation was measured at Stony Plain, about 30 km west of the pond site. In 1992-1993, cold conditions persisted between mid December and late January, with a cold spell in mid February and mid March. In 1993-1994, it was relatively mild throughout November and December and then remained cold throughout January and February.

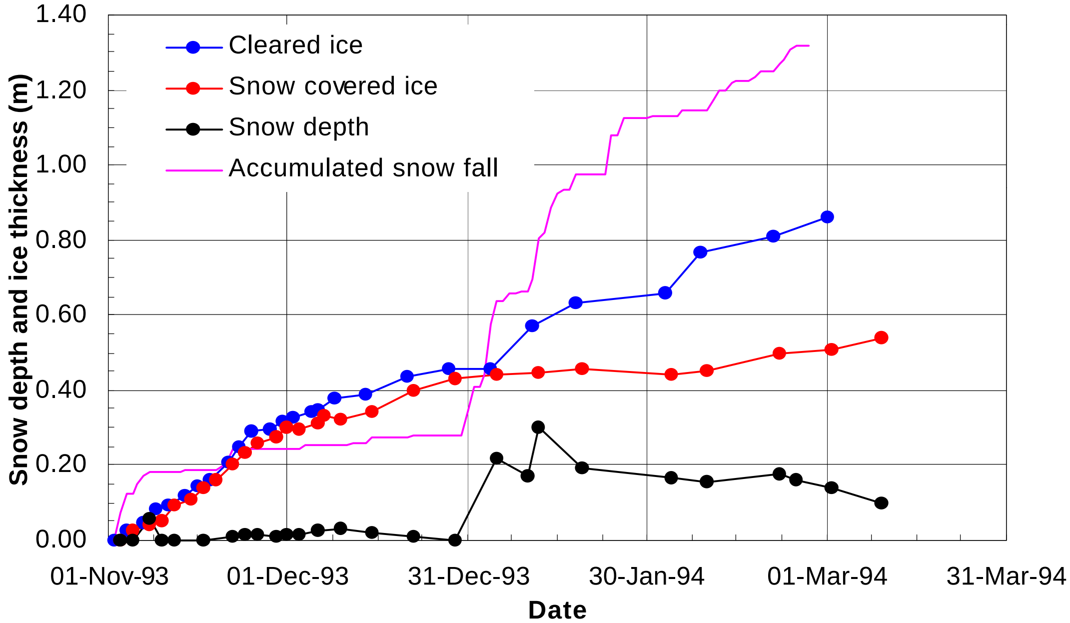

Figure 5 Snowfall, snow depth, and ice thicknesses, 1993-94

4. Simulations and Parameter Evaluations

Ice Growth with no Snow Cover

The simplest situation for ice growth is the case with no snow cover. In both years it was evident that the measured ice thickness could be simulated using an albedo for ice of 0.80 and a heat transfer coefficient of 15 W/m^2-°C, while assuming the textbook value of the thermal conductivity for ice of 2.2 W/m -°C (Figures 6 and 7).

It is interesting that the model performed poorly in the early part of the winter when the ice cover was thin and the air temperature hovered around freezing. This may be explained by the propensity for convective cooling to be more efficient than convective warming. In other words, the minimum daily temperature, should it be below freezing, should be weighted higher than the maximum temperature if it is above freezing. This may overcome the generally reduced efficiency of the convective warming because of the inherent greater stability of the colder air at the ice/air interface, relative to the warm air above. This argument is borne out by Van Der Vinne's (1995) estimates of the heat transfer coefficient during the melting period being in the order of 8 to 10 W/m^2-°C. That is, convective cooling is about twice as efficient as convective warming for the same difference in air temperature.

Figure 8 illustrates the sensitivity of the calculated ice thicknesses to changes in the convective heat transfer coefficient. It is evident that there is only about a 0.1 m variation in the calculated ice thickness for values of the heat transfer coefficient in the range between 5 and 20 W/m^2-°C. The variation in the albedo of the ice surface, however, has a large effect on the calculated ice thickness (Figure 9). Assuming an albedo of 0.5 causes an under prediction in the ice thickness of about 0.2 m or 20% of the actual thickness at the end of the winter. Assuming all the solar radiation is reflected causes the ice thickness to be over estimated by about 0.2 m or 20%.

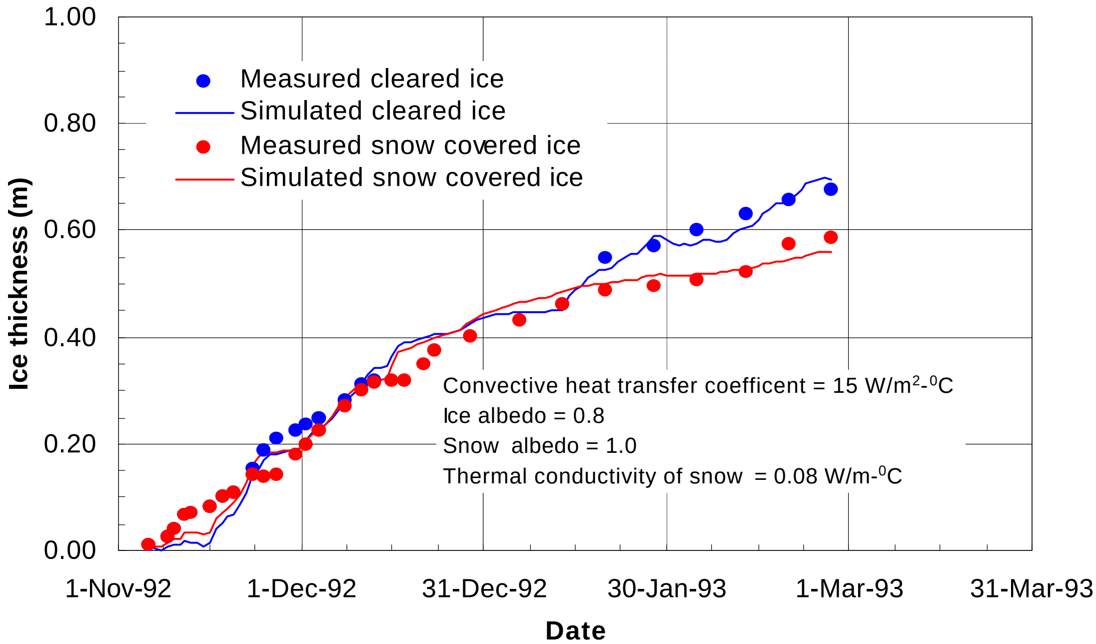

Figure 6 Measured and simulated ice thicknesses, 1992-93

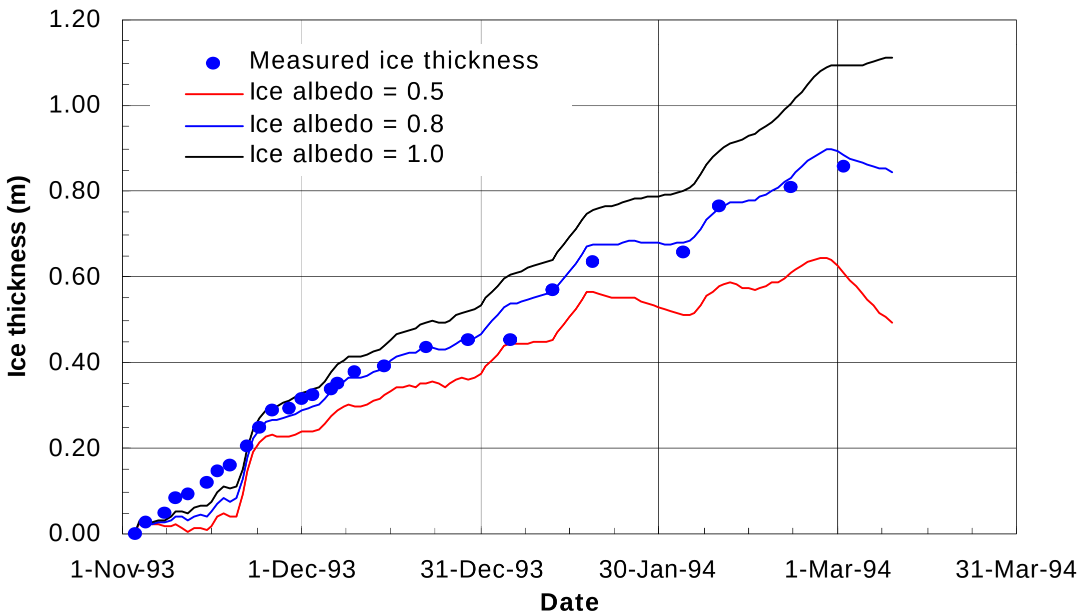

Figure 7 Measured and simulated ice thicknesses, 1993-94

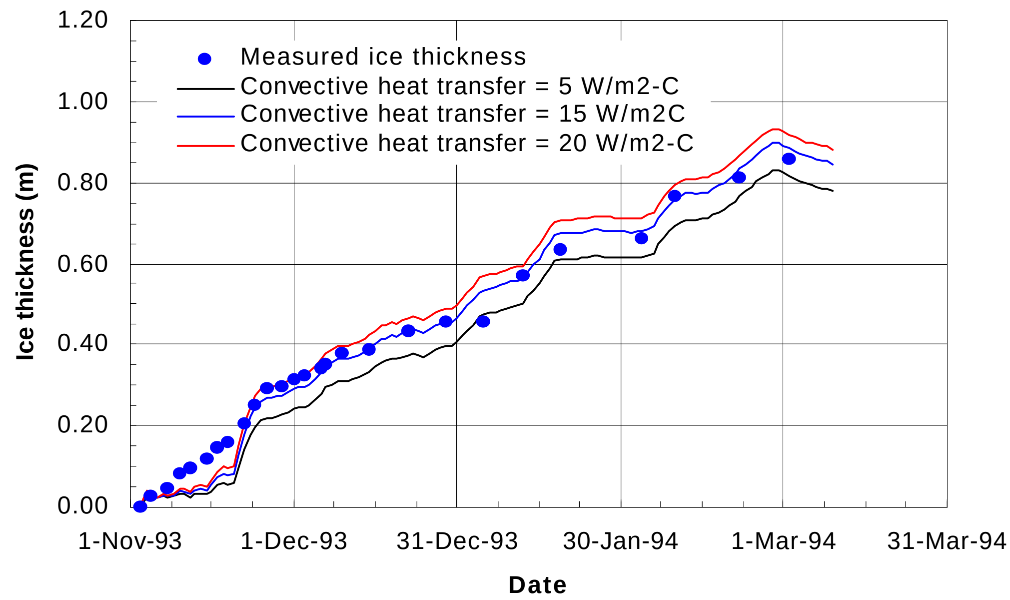

Figure 8 Sensitivity of calculated ice thickness to changes in the convective heat transfer coefficient

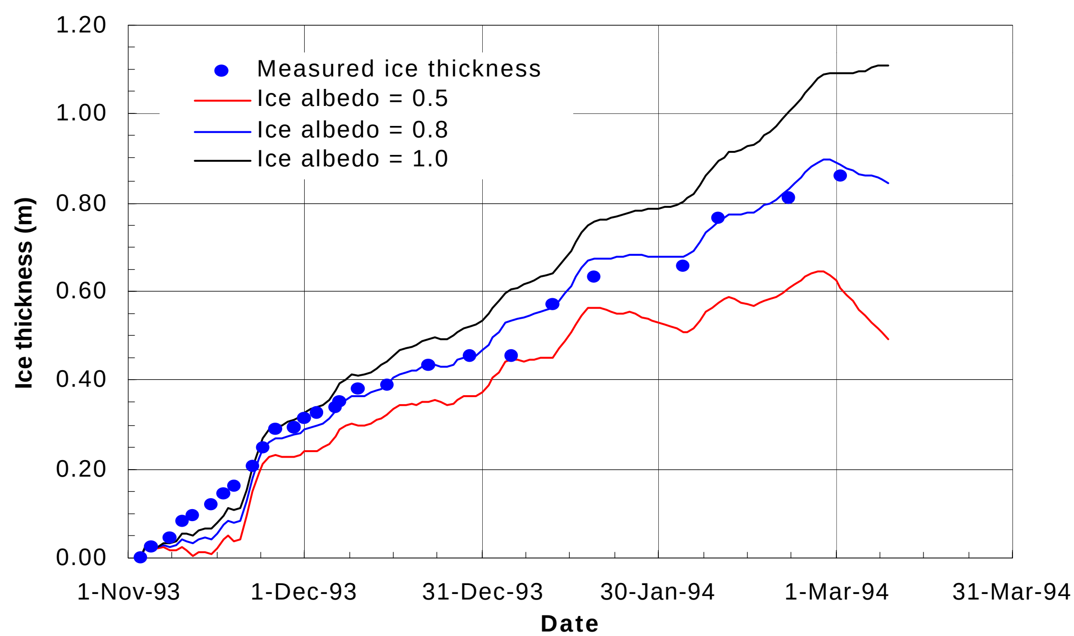

Figure 9 Sensitivity of calculated ice thickness to changes in the albedo of the ice

Ice Growth Under a Snow Cover

The ice growth under a snow cover can be simulated if the depth of the snow cover, its density, and its thermal conductivity is known. One of the biggest difficulties is to determine the actual snow depth on the ice surface on the basis of the accumulated winter snow fall. Figures 4 and 5 show that the actual snow depth is substantially less than the accumulated snow fall if one was to assume that the density of the snowpack is the same as the density of the snow fall. Figure 10 shows that the measured snowpack can be reasonably simulated from the snow fall by assuming an in situ density of 250 kg/m^3.

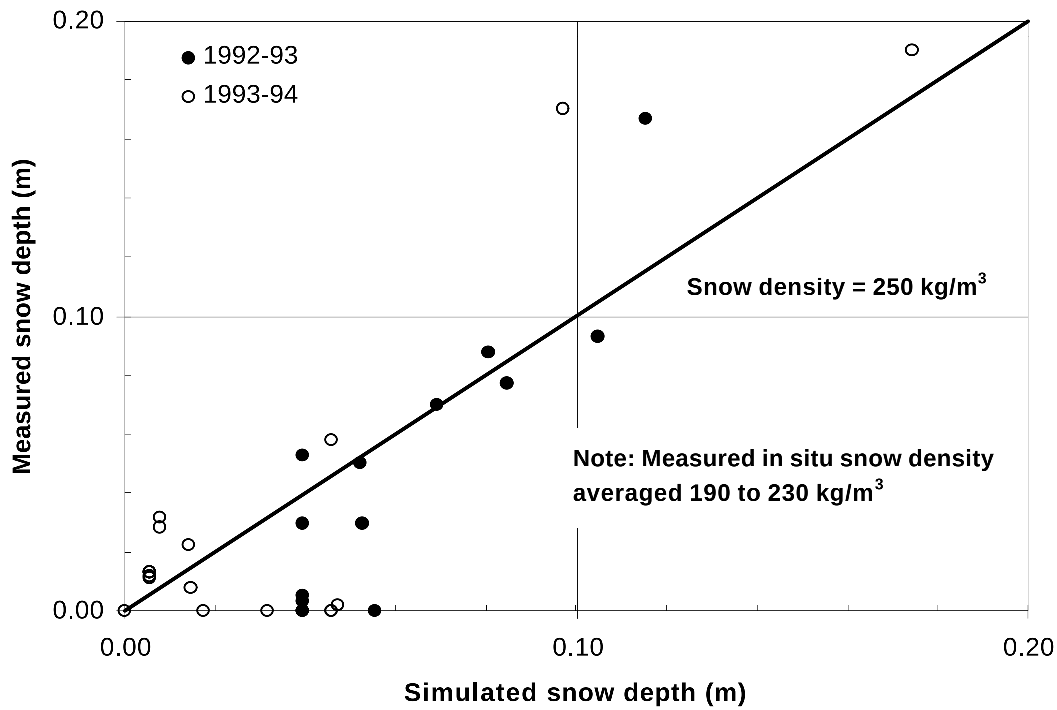

Figure 10 Modelled snowpack density

The thermal conductivity of the snow is a function of its density and can change, depending on the state of the snowpack, as determined from its thermal history. In most instances, the temporal and vertical changes in the thermal conductivity of the snow are difficult to calculate or simulate, and therefore one usually adopts a representative value of the thermal conductivity.

The 1992-1993 observations provided the best case to calibrate the model because the snow thickness was not great enough to cause the growth of snow ice. Figure 6 illustrates the best fit curve using an albedo of 1.0 (all the solar radiation is absorbed by the snow and is used to change the snow structure), a convective heat transfer coefficient of 15 W/m^2-°C as defined for the bare ice condition, and a snow thermal conductivity of 0.08 W/m-°C. Attempts were made to verify the calibrated value of the thermal conductivity for the 1993-1994 data. It is apparent that a value of 0.12 W/m-°C is more appropriate for that year.

However, there are mitigating circumstances. In 1993-1994 there was a substantial amount of snow ice that developed over the course of the winter and the additional thickness could only be simulated by assuming a larger value of the thermal conductivity of the snow.

Table 1 summarizes the meteorological and the modelling coefficients used to simulate the measured ice thicknesses and snow depths.

Table 1 Summary of salient meteorological characteristics and model parameters

Characteristic	1992-93	1993-94
Temperature range (°C)	-34.6 to 5.0	-35.1 to 2.6
Total snowfall (cm)	94	132
Maximum snow depth (m)	0.14	0.30
Effective density of snow pack (kg/m^3)	250	250
Maximum cleared ice thickness (m)	0.68	0.81
Maximum ice thickness under snow (m)	0.59	0.64
Convective heat transfer coefficient (W/m^2-°C)	15	15
Albedo of ice surface	0.8	0.8
Thermal conductivity of snow pack (W/m-°C)	0.080	.12

Conclusions

This paper has summarized the ice thicknesses and snow depths as measured on the Millwoods Pond in 1992-1993 and 1993-1994, along with the regional meteorological characteristics. For practical computations of ice growth rates in Western Canada, it is reasonable to assume an ice albedo of 0.8 if there is no snow on the ice and a snow albedo of 1.0 once snow has accumulated on the ice cover. The effective thickness of the snow pack can be estimated from the snowfall by assuming a snow pack density of 250 kg/m^3 The thermal conductivity of snow is in the range of 0.08 to 0.12 W/m-°C and a convective heat transfer coefficient of 15 W/m^2-°C is most appropriate to estimate the temperature-related heat flux at both the ice and snow surface. Accounting for the heat flux from the water to the ice sheet does not significantly affect the calculated ice thickness.

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