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In-Situ Deflection Measurements of Ice Bridges: Results and Implications on Design

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A number of techniques are available to design ice bridges. Mechanistic approaches require estimates of the modulus of elasticity and/or the strength of the ice sheet. Empirical approaches provide only general rules of thumb without consideration of ice conditions and load characteristics. Regardless of what approach is used, without site specific information from which the ice characteristics can be determined, it is difficult to both optimize the construction process and identify the risk that users would assume.

This paper summarizes the results of a number of deflection measurements that have been undertaken in Northern Alberta. From these measurements, bulk ice properties such as modulus of elasticity and strength are deduced and generalizations are made that can apply to the design of ice bridges. On the basis of the collected data, the efficacy of empirical design approaches are evaluated with respect to ice mechanics theories, and suggestions are made to improve the current design process.

1. Introduction

When rivers and lakes freeze-over in cold regions, the floating ice covers can be utilized for a number of transportation and construction related purposes. Ice bridges are used as replacement of summer ferries during winter season, as part of winter road systems, and to provide temporary access to remote sites. Ice covers also provide construction and maintenance platforms for bridges, pipelines, and other water related structures.

In some cases the natural ice can be utilized without modification but in most cases flooding must be undertaken to attain a required thickness. This flooding is used both to accelerate the growth of the ice and to generate an ice thickness greater than the surrounding natural ice. Flooding can be time-consuming and costly if the ice bridge is long or requires excessive thickness; therefore it is important to design the ice bridge as economically as possible while still providing an appropriate degree of safety for the end users.

It is difficult to both optimize the construction process and identify the risk that users would assume without site specific information from which ice characteristics can be determined. This paper summarizes the results of a number of deflection measurements that have been undertaken in Northern Alberta. Bulk ice properties such as modulus of elasticity and strength are deduced from these measurements. On the basis of the collected data, the efficacy of empirical design approaches are evaluated with respect to ice mechanics theories, and suggestions are made to improve the current design process.

2. Background

Ice bridge design can be either empirical or mechanistic. Empirical design methods are useful if the design is straight-forward and time of construction is not an issue. Mechanistic design methods are needed when unusual geometry is required or when construction time is limited.

Empirical techniques typically use a simple formula which relates bearing capacity to ice thickness. For example, Gold (1971) used break-through data to determine the appropriate coefficient in the theoretical relationship between the bearing capacity of ice and the square of the ice thickness. A variation of this formula is used in some jurisdictions where only one-half the thickness of flood ice is added to the natural ice thickness when calculating the bearing capacity. The advantage of these empirical formulas is their ease of use and simple data requirements. The disadvantage is that it is difficult to account for variations in ice properties so the methods tend to have large factors of safety.

Mechanistic techniques use equations based on the theoretical analysis of the behavior of a plate on an elastic foundation. The advantage of this mechanistic approach is that theoretical analysis allows more accurate calculation, thus less ice thickness may be needed so the ice bridge can be used more quickly. This approach is also useful for determining the bearing capacity for incomplete ice covers, or where slots or trenches are required in ice platforms. The disadvantage is that mechanistic methods require knowledge of ice properties such as modulus of elasticity and tensile strength as well as ice thickness.

Field testing of modulus of elasticity has been done by both sonic wave testing and compression test deflections. Sonic wave testing requires specialized equipment to measure the speed of sound waves through ice samples, but the technique only measures the instantaneous modulus, whereas an effective modulus is required for analysis. Compression testing of individual ice cores for the modulus of elasticity is difficult as well because horizontal cores are required to obtain the crystal orientation that represents the appropriate fracture plane. This typically requires that a large block of ice is cut and removed from the ice. As well, the testing equipment must be much stiffer than the ice which reduces the portability of the equipment. Furthermore, if samples must be transported to the lab, melting or freezing may occur which may change the ice properties.

Specific tests for tensile strength include flexural beam tests, ice core tests, and borehole jack techniques. One method of flexural beam testing is carried out by cutting a slot around three sides of a beam and then measuring the load required to fail the beam. These tests are time consuming, provide highly variable results and, worst of all, they are destructive tests so they cannot be performed on the ice bridge itself. Strength testing requires horizontal cores from the ice sheet which are difficult to obtain. As well, cores are much easier to prepare and test in compression so actual tensile strengths are estimated from the measured compressive strength. Borehole jack technology is much easier but requires specialized equipment. A specially constructed jack is lowered into a borehole in the ice and expanded until the walls of the borehole fail. This technique can only provide an index of strength due to the complex stress distribution generated so an alternative test is still required to provide an absolute strength value.

Most of the above test procedures are difficult to perform and provide highly variable results. All these complicated procedures can be replaced by a simple non-destructive deflection test, the results of which can be used to obtain both a bulk in-situ measurement of modulus of elasticity and tensile strength of the actual ice being used for construction. The use of this test requires some knowledge of the theoretical behavior of a floating ice sheet as well as empirical data which define the load and deflection required to generate failure in the ice sheet.

2. Theoretical Background

Two components of the behavior of a floating ice sheet are required to develop a deflection based ice testing procedure. The first component is a strain energy criterion for failure used by Beltaos (1978) to analyze creep failure. The second component is the elastic deflection of the ice sheet.

A strain energy failure criterion was used by Beltaos to analyze creep failure test results but it is also applicable for rapid failures. Beltaos found that work required to fail an ice sheet could be defined by the following equation

$$W_f = P_f w_f = 300\, h^{5/2} \qquad [1]$$

where h is the ice thickness in meters and W_f is the work done to the ice sheet in kilonewtons per meter at the point of failure. It should be noted that Beltaos defined failure as the onset of tertiary creep. This is somewhat less than the point of actual failure, but for the purposes of design, this definition works very well. The work done to the ice sheet is defined by the load, P_f times the maximum deflection, w_f, therefore Equation [1] also defines a relationship between load and deflection for a given ice thickness. The coefficient in Equation [1] was obtained from a best-fit of all of Beltaos' creep failure data. The failure data was not differentiated by load radius or load distribution so the equation does not account for differences in load configuration.

The elastic deflection of an ice sheet under a load also provides a relationship between load, P and maximum deflection, w_{max} for a given ice thickness. This relationship is as follows

$$P = 8\,\rho_w\,g\,l^2\,w_{max} \qquad [2]$$

where ρ_w is the density of water and g is the acceleration due to gravity. The characteristic length, l of the ice sheet defined by

$$l = [(Eh^3)/(12\,\gamma_w\,(1-\nu^2))]^{1/4} \qquad [3]$$

where E is the modulus of elasticity of the ice, ν is Poisson's ratio, and γ_w is the specific weight of water. Equation [3] can be re-arranged and substituted into Equation [2] so that the modulus of elasticity is expressed as function of the measured load and deflection.

Equations [1] and [2] can also be combined to express the failure load as a function of modulus of elasticity and ice thickness

$$P_f = 49\,[(E\,\gamma_w)/(12(1-\nu^2))]^{1/4}\,h^2 \qquad [4]$$

Once the modulus of elasticity is determined from the deflection test results using Equations [2] and [3], Equation [4] can be used to determine the failure load for a given ice thickness. This equation can be used for single loads on uniform ice covers but is not sufficient in its present form to analyze situations with multiple loads or non-uniform ice thickness. Furthermore, Equation [4] does not provide an indication of the stresses within an ice cover. Thus, there is no mechanism to evaluate the risk of failure within the context of a stress based safety factor for the loading condition.

Tensile stress analysis must be used to analyze multiple loads or non-uniform ice thickness. When a load is applied to the ice surface the maximum tensile stress in the ice cover occurs at the bottom of the ice cover under the load. The tensile stress diminishes as the distance from the load increases. Plate theory can be used to determine this stress distribution for an individual load. The failure stress can be estimated as the maximum tensile stress generated by the failure load defined by Equation [4]. This failure stress varies somewhat with the radius of the test load because Equation [1] is not sensitive to load radius while the maximum stress obtained from plate theory is.

Stresses from individual loads can be added together to obtain the cumulative stresses for multiple loads. This can be done with a spread-sheet type analysis for simple loading geometries or with a finite element model of the ice sheet for more complex ice geometries such as slots or partial ice covers. The dimensionless stress in the ice cover varies with distance from the load, $\boldsymbol{r}$ as follows

$$\sigma \boldsymbol{h}^2/\boldsymbol{P} = \boldsymbol{f}(\,\boldsymbol{r}/\boldsymbol{l}\,) \qquad [5]$$

A curve defining this function is shown in Figure 1. Values far from the load were obtained from thin plate theory while those near the load were determined by thick plate theory for a typical $\boldsymbol{h}/\boldsymbol{l}$ ratio of 0.09. The upper limit of the dimensionless stress occurs at the radius of the load, $\boldsymbol{b}$. The maximum value of the dimensionless stress is about 2.6 for a very small load radius. The dimensionless stress drops off quickly with distance from the center of the load to a value of 0.06 at $\boldsymbol{r}/\boldsymbol{l}$ = 2.0.

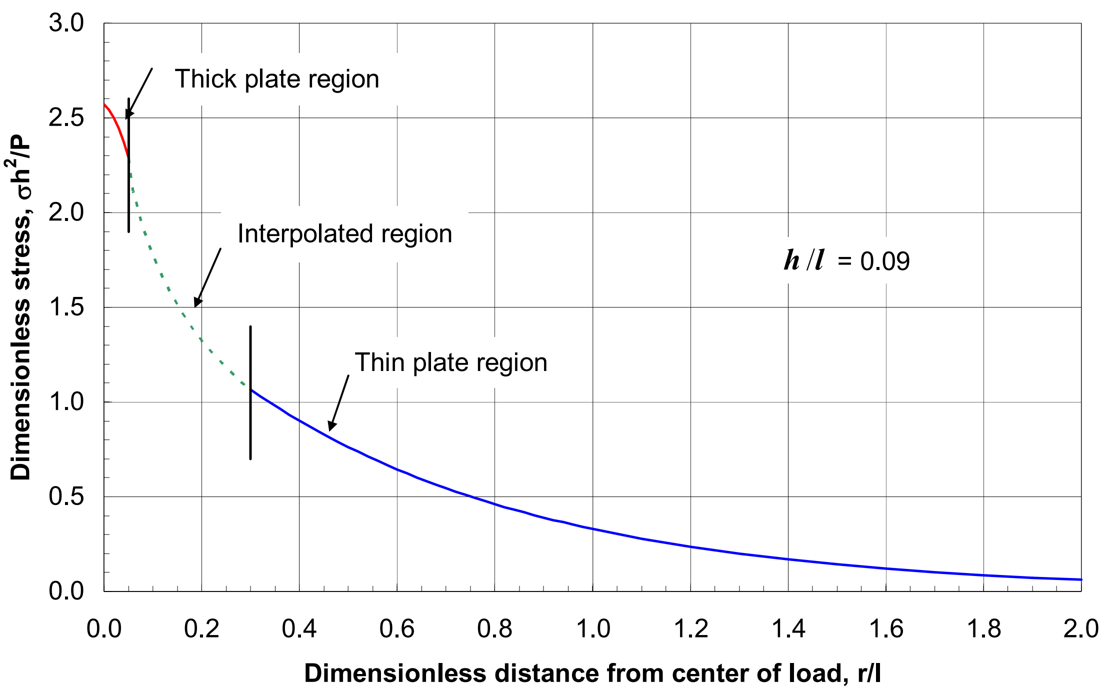

Figure 1 Variation of dimensionless stress with dimensionless distance from center of load.

Table 1 Summary of deflection test results

Waterbody	Ice Thickness (m)	Load (kg)	Load Radius (m)	Maximum Deflection (mm)	Modulus of Elasticity (GPa)	Tensile Strength (kPa)
N. Saskatchewan R.	0.75	15,070	2.05	23	1.7	2140
N. Saskatchewan R.	0.94	22,270	2.30	25	1.6	2037
Alemeda Res.	0.75	15,070	2.05	23	1.7	2140
Athabasca R.	0.86	16,718	2.40	14	3.7	2640
Athabasca R.	0.81	16,718	2.40	17	3.0	2395
Athabasca R.	0.81	23,976	2.40	38	1.3	1799
Athabasca R.	0.89	23,976	2.40	25	2.2	2318
Des Rochers R.	0.78	3,397	1.50	4	2.5	2699
Athabasca R.	0.93	29,291	3.70	30	2.0	1767
Athabasca R.	1.00	23,340	2.20	24	1.6	2243
Athabasca R.	0.24	805	0.80	6	2.2	2318
Athabasca R.	0.67	3,134	2.10	10	0.5	1257
Athabasca R.	0.90	21,115	2.40	36	0.8	1481
Athabasca R.	0.92	20,659	2.40	25	1.5	1913
Athabasca R.	0.55	3,134	2.10	9	1.2	1577
Athabasca R.	0.80	20,652	2.40	34	1.2	1665
Athabasca R.	0.91	20,652	2.40	34	0.8	1511
Average					1.7	1994
Standard Deviation					0.82	419

3. Deflection Measurements

The results of 17 ice deflection tests are summarized in Table 1. All of the tests were performed on natural ice which was thickened to some extent by surface flooding. The ice thickness for the tests ranged from a minimum of 0.24 m to a maximum of 1.00 m; however, due to the load bearing requirements of the ice bridges being tested, the typical ice thickness for testing tended to be between 0.8 and 0.9 m. The ice thicknesses reported in Table 1 are the average ice thicknesses in the vicinity of the load.

Deflection tests require a rod and level, a tape measure, and an appropriate test load. First, positions and elevations of a number of points on the ice are established. Next, the load is placed in the center of the points and then the elevation of each of the points is re-surveyed. Points nearest the load are measured first to reduce the effects of creep on these measurements. The geometry and position of the load are also measured. After the load is removed, ice thicknesses in the area of the load are measured. Creep measurements can also be obtained with this same testing apparatus simply by leaving the load in place for a period of time and measuring the deflections at a number of time intervals.

The load selected for a given test is based on the measured ice thickness and on the available load. The load selected is usually as conservative as possible from a safety point of view but large enough to generate a measurable deflection. The test loads ranged from a minimum of 805 kg to a maximum of about 29,300 kg.

The radius of each of the loads was determined so that stresses in the bottom of the ice cover could be estimated by Equation [5]. For example, the loads produced by tandem trucks are rectangular but the equivalent radius was determined from the measured axle lengths and spacing so that a single measure of load distribution could be adopted. The load radius of a tandem truck is typically between 2.05 and 2.40 m depending on the length of the vehicle.

Maximum deflections ranged from a minimum of 4 mm to a maximum of 38 mm but typically they were between 20 mm and 30 mm. This target range provided enough deflection to be accurately measured while still providing a reasonable factor of safety for untested ice. The error in the deflection reading may be as high as 1 mm.

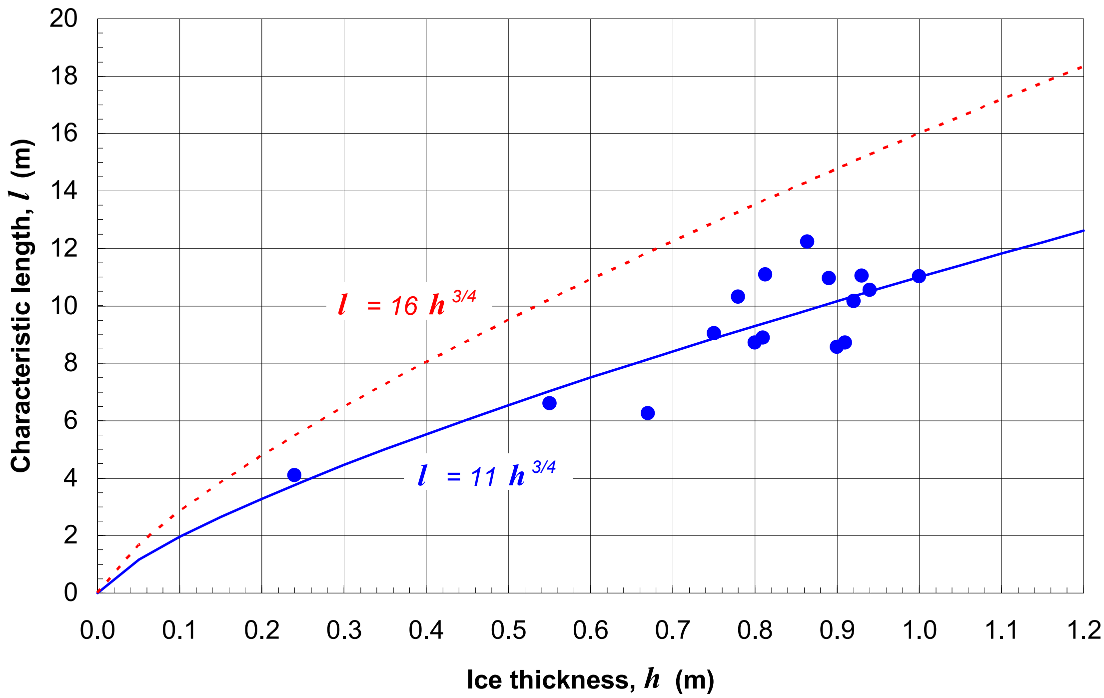

Figure 2 Variation of characteristic length with ice thickness.

The characteristic length of the ice sheets was determined from the measured load and maximum deflection using Equation [2]. The characteristic length provides a scale for the size of the deflection bowl around the load and is a function of the ice thickness and the modulus of elasticity. The deflection bowl extends approximately three times the characteristic length from the center of the load. The characteristic length ranged from a minimum of 4.1 m to a maximum of 12.2 m in the tests. The relationship between characteristic length and ice thickness is shown in Figure 2. The best-fit line through this data is described by

$$\boldsymbol{l} = 11\, \boldsymbol{h}^{3/4} \qquad [6]$$

The coefficient in this equation is somewhat lower than the value of 16 suggested by Gold (1971).

The total deflection bowl around the load was measured as a check of the elastic modulus calculation, by providing an independent measure of the characteristic length. It also provided an indication of the homogeneity of the ice in each of the tests. Discontinuities in the deflection curves can indicate the presence of cracks in the ice which may affect the test results and the bearing capacity of the ice sheet. A typical set of deflection measurements for an ice platform test are shown in Figure 3. Note that the deflection curve to the east of the load has much lower deflections. A crack was observed in the surface of the ice on this side of the load so it is likely that the crack has affected the stress distribution in this test.

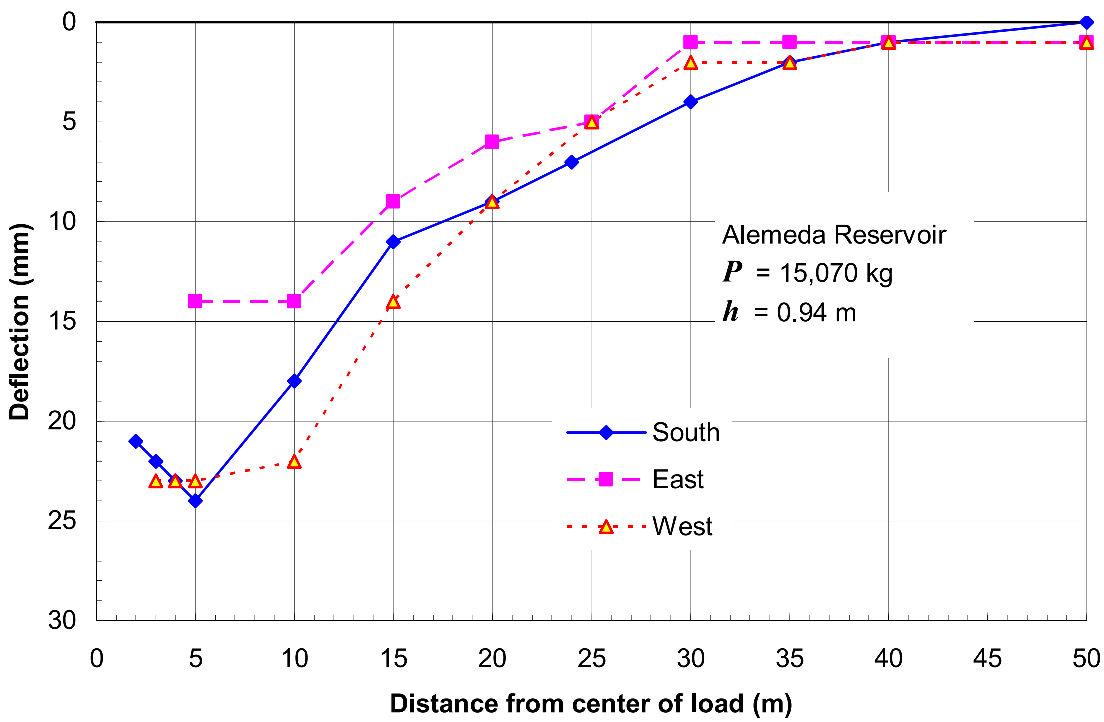

Figure 3 Variation of measured deflections with distance from center of load.

4. Measured Ice Characteristics

The modulus of elasticity of the ice for each of the tests was determined from the characteristic length using Equation [3]. The modulus of elasticity ranged from a minimum of 0.55 GPa to a maximum of 3.7 GPa in the tests. The average modulus of elasticity from the deflection tests was 1.7 GPa.

Values of modulus of elasticity in the literature range from the instantaneous modulus value of about 9.0 GPa for monocrystalline ice to effective modulus values in the range of 3.5 to 6.0 GPa for polycrystalline ice (Ashton, 1986). The results of the deflection tests indicate that the bulk value of effective modulus for large areas of ice is only about 1.7 GPa. This is substantially lower than the reported values and probably reflects the non-homogeneity that occurs in large scale ice samples.

The tensile strength of the ice for each of the tests was determined from the maximum tensile stress produced by the failure load calculated from Equation [4]. The tensile strength ranged from a minimum of 1260 kPa to a maximum of 2700 kPa in the tests. The average tensile strength was found to be about 2000 kPa.

The literature suggests that values of the tensile strength of ice vary with the grain size of the ice. A typical range of tensile strength for frazil type river ice with a grain size of 1-2 mm is between 1800 and 2500 kPa. Lake ice tends to have larger grain sizes of about 5-10 mm and thus a lower range of tensile strength of between 800 and 1200 kPa.
The average tensile strength of 2000 kPa obtained from the above analysis is more consistent with river ice; however, the tensile strengths were derived from Beltaos' creep failure loads which were obtained from tests on lake ice. The ice thickness in many of the lake ice tests was small so the grain sizes were likely smaller than 5 mm. Therefore the strengths obtained from Beltaos' study are expected to be higher than typical lake ice. The effects of variations in grain size and load radius on failures loads were not quantified by Beltaos' study. Further failure tests over a range of ice types and load radii are required to determine the effects of these parameters on the failure load and, therefore, the tensile strength.

5. Design Considerations

The deflection-based analysis enables the use a lower factors of safely because the ice characteristics have been quantified explicitly. For example, the load obtained for Gold's formula with a safe coefficient of 35,000 kg/m^2 typically has a factor of safety on the tensile strength derived from deflection measurements of about 6.0; whereas, a factor of safety of 3.0 or 3.5 on the tensile strength is typically used with the deflection-based analysis, depending on the type of use of the ice sheet. For example, a factor of safety of 3.0 may be used with a one-time load but 3.5 may be used when the load is applied repeatedly. These factors of safety of 3.0 or 3.5 are required to allow for variability and uncertainty in the evaluation of the tensile strength as well as to allow for undetected cracks in the ice.

The thinner ice requirements resulting from the application of this deflection-based analysis allows for much quicker utilization of ice bridges and reduces the cost of flooding. In some cases it allows a project to proceed even through the climate conditions are marginal.

The deflection-based analysis also enables the analysis of more complex loading scenarios such as the multiple loads produced by a tractor-trailer vehicle. Although these vehicles typically are the heaviest loads that an ice bridge might sustain, they are long and narrow so the load is distributed along the ice bridge. Analyzing the vehicle as a series of interacting loads will result in lower maximum stresses in the ice relative to a single load of the same weight, and thus, less ice thickness is needed to support the maximum loads.

Another complex loading scenario occurs when a backhoe or crane is required to work next to an open trench or slot in the ice. In these situations, a finite element model can be used to determine the maximum stresses in the ice cover for a particular geometry. A design ice thickness is then selected so that the maximum stress is less than the allowable stress obtained from the results of a deflection test.

This deflection based design technique has been used successfully to design numerous ice bridges and ice platforms using thinner ice covers that those suggested by standard empirical techniques. This has resulted in reduced flooding costs as well as increased durations of use. No failures have occurred when these techniques have been applied; however, additional ice failure data analyzed in this manner would provide even more confidence in this approach.

6. References

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