CGU HS Committee on River Ice Processes and the Environment
22nd Workshop on the Hydraulics of Ice-Covered Rivers
Canmore, Alberta, Canada, July 9-12, 2023.

Using logistic regression to identify the key hydrologic controls of ice-jam flooding near the Peace-Athabasca Delta: assessment of uncertainty

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The Peace-Athabasca Delta (PAD) in northern Alberta is one of the world's largest inland freshwater deltas, home to many species of fish, mammals, and birds. In the past five decades, the PAD has experienced prolonged dry periods in-between rare floods, accompanied by reduction in the area covered by lakes and ponds that provide habitat for aquatic life. In the Peace sector of the PAD, this likely resulted from reduced frequency of spring flooding caused by major ice jams that form in the lower Peace River. There is debate in the literature regarding the factors that promote or inhibit the formation of such ice jams, deriving from physical process studies, paleolimnological studies and – recently – statistical analysis founded in logistic regression. The logistic regression attempts to quantify ice-jam flood (IJF) probability given the values of assumed explanatory variables but involves considerable uncertainty. Herein, different sources of uncertainty are examined and their effects on statistical inferences evaluated. It is shown that epistemic uncertainty can be addressed by selecting direct explanatory variables, such as breakup flow and ice cover thickness, rather than more convenient proxies that rely on winter precipitation and degree-days of frost. Structural uncertainty, which pertains to the unknown mathematical relationship between IJF probability and the selected explanatory variables leads to different probability predictions for different assumed relationships, but does not seem to modify assessments of statistical significance. The uncertainty associated with the relatively small sample size (number of years of record) may be complicated by known physical constraints on IJF occurrence. Overall, the logistic regression corroborates physical understanding that points to breakup flow and freezeup level as primary controls of IJF occurrence.

1. Introduction

The Peace–Athabasca Delta (PAD) in northern Alberta (Figure 1), is one of the world's largest inland freshwater deltas and homeland for the Indigenous Peoples of the region. It has been designated a Ramsar wetland of international importance and is largely located within the Wood Buffalo National Park (WBNP), itself being a UNESCO World Heritage Site (WBNP 2019). During the past five decades, this complex and dynamic region has, in-between rare overland floods, experienced prolonged dry periods and considerable reduction in the area covered by lakes and ponds that provide habitat for aquatic life [Peace-Athabasca Delta Project Group 1972, 1973; Prowse et al. 1996, 2002; Peters 2003; Peters et al. 2006; Ward et al. 2018, 2020, 2021]. The drying trend coincides with the regulation of Peace River, which began with construction (1968), reservoir-filling (1968–1971) and operation (1972 onwards) of the W.A.C. Bennett hydroelectric dam in British Columbia, located some 1200 km upstream of the PAD (Figure 1). There is debate in the scientific literature regarding the possible effect of regulation on the drying of the PAD [see Beltaos (2023a) for relevant bibliography].

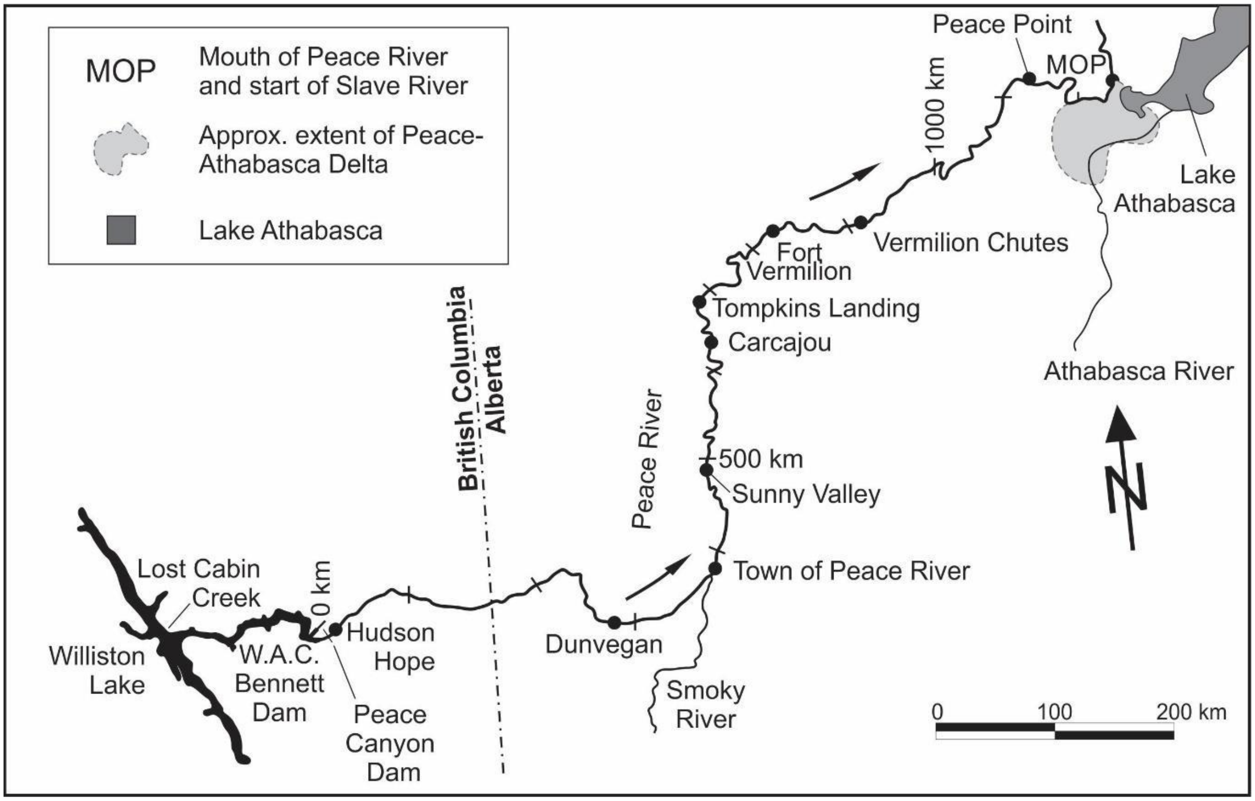

Fig. 1. Plan view of Peace River and Peace-Athabasca Delta (showing only the northern portion of Athabasca River). From Jasek and Pryse-Phillips (2015), with changes.

Concern over the long-term health and sustenance of PAD ecosystems is underscored by climate change and future construction of more dams [Jasek and Pryse-Phillips, 2015]. As a result of a UNESCO Reactive Monitoring Mission report [UNESCO 2017], prompted by a petition from Indigenous Peoples, Canadian federal and provincial authorities commissioned a strategic assessment [IEC 2018] of WBNP. This assessment culminated in development of the WBNP

Action Plan [WBNP 2019], which incorporated Indigenous knowledge, to address several recommendations towards preserving the ecological integrity of this important World Heritage Site. The success of the Action Plan depends on sound understanding of the processes and hydroclimatic variables that control the supply of water, sediment, and nutrients to the PAD basins. Ongoing concerns prompted a second Monitoring Mission in 2022, which may result in further recommendations, possibly including addition of the Wood Buffalo National Park to UNESCO's List of World Heritage in Danger. (UNESCO team is investigating the deterioration of Wood Buffalo National Park - Alberta Native News / accessed on 27 April 2023).

The recharge of the delta depends in part on the formation of extensive spring ice jams that can trigger overland inundation. Ice jamming in the lower Peace River is the primary mechanism that can replenish the high-elevation - or "perched" - lakes and ponds of the Peace Sector of the PAD (Prowse and Conly 1998). The Historical and Traditional Knowledge record of ice-jam floods (IJFs) includes events of varying magnitude, labelled 1, 2, 3 (Peterson and Courtorielle 1992; Peterson 1995) or small, moderate and large, respectively (Timoney 2009). It is the large, or magnitude 3, variety that can generate the extensive overland flooding that is necessary to replenish perched basins. In what follows, the term IJF will be used to denote large events. During the 20th and 21st centuries, IJFs have been reported for the years 1900, 1904, 1920, 1932, 1933, 1934, 1942, 1943, 1948 (may have been of magnitude 2, and is not usually included in the IJF series), 1958, 1963, 1965, 1972, 1974, 1996, 1997 and 2014.

Early and ongoing research (Prowse et al. 1993; Prowse and Conly 1998; Beltaos et al. 2006; Beltaos 2019) indicates that, in addition to climate, regulation has contributed to the post-1968 reduction of IJF frequency via enhanced fall/winter flows and higher Peace River freezeup levels. A high freezeup level (HF) enhances the resistance of the winter ice cover to mobilization (Beltaos 2008, 2023b), and thence can reduce the rate of progression of the Peace River breakup front in years when the initially thermal breakup becomes dynamic. This change typically occurs near Sunny Valley (Fig. 1), following arrival of ice runs from the Smoky River, a major tributary that joins the Peace slightly upstream of the town of Peace River (Jasek 2019a, 2019b; Emmer et al. 2021). A slow advance of the breakup front, which is characterized by relatively frequent and prolonged jamming along the way, delivers inconsequential volumes of ice rubble to the delta reach of Peace River; additionally, the ice cover is by that time degraded by thermal effects and its capacity to initiate and retain ice jams is diminished. The result is minor, if any jamming (e.g. 2020 breakup event; Beltaos and Carter 2021).

Contrary to this understanding, the role of HF in IJF occurrence appeared to be secondary in the study by Lamontagne et al. (2021), who applied logistic regression to identify controlling variables and calculate IJF probability. This research avenue is uncommon in the study of river ice processes and therefore merits careful examination and assessment. The objective of this paper is to explore the uncertainties that were identified by Lamontagne et al. (2021), assess their implications for the selection of controlling variables, and delineate what can, or cannot, be inferred with confidence from the logistic regression.

2. Background Information

Logistic regression on binary outcomes, such as flood/no flood, during the spring breakup of any given year, attempts to quantify the probability of flood occurrence (P), given a set of known or hypothesized explanatory variables. It assigns respective values of 1 and 0 to flood and non-flood years and assumes that the probability of flood occurrence is expressed as:

$$P\{IJF/(x_1, x_2, x_3, \ldots)\} = \frac{1}{1+\exp(-\boldsymbol{X})} \qquad [1]$$

in which x_1, x_2, x_3 are the explanatory variables and

$$\boldsymbol{X} = "logit" = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + \cdots \qquad [2]$$

with b_0, b_1, b_2, b_3 being coefficients, of which the numerical values are furnished by the regression. The form of Eq. 1 ensures that P always varies between 0 and 1 (as it should), regardless of the value of X, while Eq. 2 assumes that the effects of the explanatory variables are linear and additive. Together, Eqs. 1 and 2 imply that $X = \ln\{P/(1-P)\}$ = natural logarithm of the "odds ratio" and is called the "logit" in statistical literature. To those who are accustomed to physics-based research, it would appear that considerable serendipity is needed to come across linear and additive logits (see also Section 4). This is a key point because the numerical values of b_0, b_1, b_2, … and their associated p-values depend on the structure of Eq. 2.

A primary consideration in logistic regression is the sample size, which in the present case, is the number of years of record. A common, though not universally accepted, empirical requirement is that the sample should contain at least 10 events per variable (van Smeden et al. 2019). For applications to IJFs in the lower Peace River (frequency ~0.1), use of two variables would require sample sizes of at least 2*10/0.1 = 200 years (300 years in the case of 3 variables, 400 years for 4 variables, and so on). By contrast, the available hydrometric data span a period of < 60 years.

Lamontagne et al. (2021) noted that the bias introduced by relatively short records is successfully reduced by maximizing a penalized likelihood function (Firth 1993), and applied Firth logistic regression to known IJFs using potentially relevant explanatory variables. They adopted the total winter (Nov. to Apr.) precipitation, WP, at Grande Prairie (supplemented, as needed, with data from nearby Beaverlodge) as a proxy for breakup flow, which is the primary factor that drives dynamic breakup events; and the accumulated degree-days of frost at Fort Vermilion (DDF) as a proxy for ice thickness, which is a resisting factor that promotes ice jamming. The freezeup level, HF, a resistance factor, was also considered initially, but eventually discarded by Lamontagne et al. (2021) as not being statistically significant (p-value > 0.05). [HF is defined as the highest 7-day running average water level during ice-cover conditions in late fall and early winter; herein, as in the Lamontagne et al. (2021) study, HF is assessed at Peace Point (Fig. 1), using historical WSC (Water Survey of Canada) data from the nearby hydrometric gauge]. Other factors, such as November flows and rapidity of spring melt were also considered but did not seem to improve upon the ultimately adopted logistic model, which was solely based on the aforementioned proxy variables WP and DDF. This model predicts increasing IJF probability with increasing breakup flow potential, as reflected in the winter precipitation and with increasing resistance to breakup, as reflected in the coldness of the winter. In the present context, the word "model" signifies an assumed version of Eq. 2, following statistical determination of the equation's coefficients.

Lamontagne et al. (2021) noted that none of the examined models had strong predictive power and the "best we can do is speak to what is more or less likely". This feature is visually illustrated in Fig. 2: probability estimates resulting from the adopted model for each one of the years of the examined record produced a wide overlap range (~ 0.2 to 0.7), containing 5 of the 7 flood events of the instrumental record and 6 non-flood events.

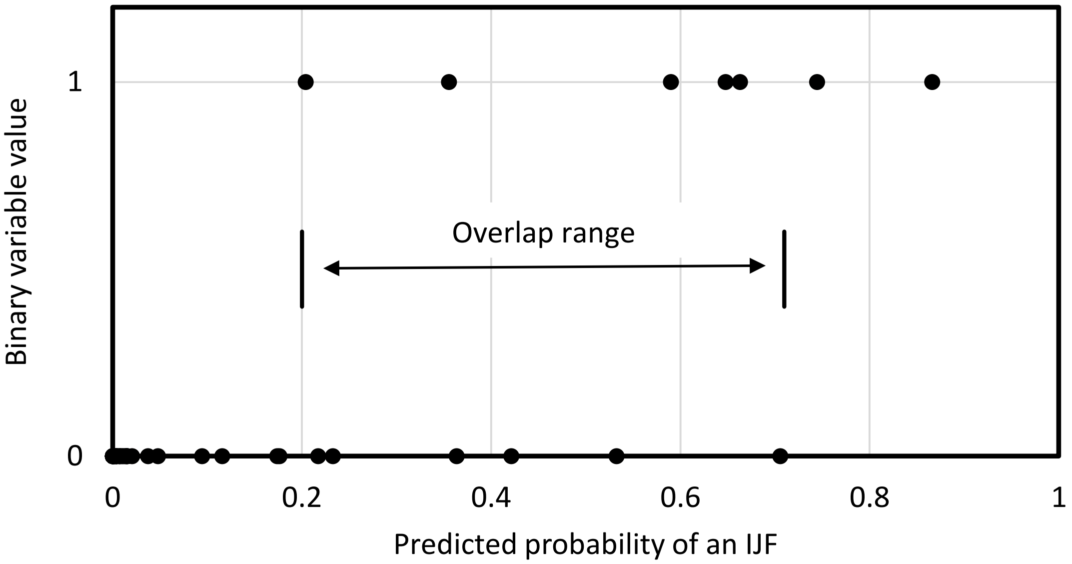

Fig. 2. IJF occurrence (binary variable = 1) or non-occurrence (binary variable = 0) versus predicted probability using the Lamontagne et al. (2021) regression model. Note large overlap range that includes 5 IJFs and 6 non-floods.

With reference to "discarded" variables, Lamontagne et al. (2021) cautioned that:
"It is important to note that just because a factor is not statistically significant ... does not necessarily mean that the factor is unimportant to generating large ice jam floods. It could be that the factor's relationship to flood generation is different than assumed by the models we tested (structural uncertainty), or that the sample size is too small to precisely estimate the effect of the factor on ice jam floods (parametric uncertainty). It could also be that the factors used as proxies for the physical drivers are not good (epistemic uncertainty)". (Lamontagne et al. 2021).

This statement is particularly relevant to the dismissal of HF as an explanatory variable, contrary to what physical understanding and experience indicate for the Peace and other rivers (Prowse and Conly 1998; Beltaos 2008, 2023b). For instance, simple counting of IJF and non-flood occurrences within successive, one-metre wide, ranges of HF indicates empirical probabilities that rapidly decrease as HF increases (Beltaos and Peters 2021). The discrepancy between empirical evidence and logistic regression is explored in the following sections by assessing how uncertainty influences the degree of confidence that can be attached to statistical inferences regarding IJF occurrence in the lower Peace River.

3. Epistemic Uncertainty

In addition to freezeup levels, daily mean breakup flow and sporadic ice thickness data are available for the studied period of record (1962 to 2020, minus the reservoir-filling years 1968-1971). Such data derive from WSC hydrometric records for the gauge located near Peace Point (Fig. 1). To characterize breakup flow using a single variable, one may simply select the maximum spring flow during the period when the gauge height is influenced by ice, which is flagged by the symbol B in the WSC records. To capture aspects of both, the magnitude and duration of high flows, one may calculate the peak value of the running average flow over a few to several days, during the B-period. Herein, the peak 7-day average (Q7) is used; resulting conclusions are similar when using maximum breakup flow instead of Q7.

As noted in Beltaos (2021) and illustrated in Figs 3 and 4, neither WP nor DDF are robust proxies for breakup flow and ice thickness, respectively. A common feature in both figures is the large scatter, which means that any given value of WP or of DDF corresponds to a wide range of breakup flow or of ice thickness, respectively. In the case of WP, the scatter in Fig. 3 likely results from the interannual variability of pre-breakup rates of melt and from reliance on a single meteorological station to represent the snowpack of the entire basin.

With reference to Fig. 4, the coldness of the winter, which is quantified by the accumulated degree-days of frost, or DDF, is occasionally considered a surrogate for river ice thickness. This assumption has merit if one is comparing multi-year average thickness values at different river sites: on average, we can reasonably expect to find thicker ice at river sites where the winters are generally colder than at sites where the winters are milder. However, this assumption breaks down when we compare ice thickness values from different years at the same site (as in Fig. 4). Comparable thickness data, albeit less voluminous, from the records of the Fort Vermilion gauge exhibit a similar pattern, when plotted against local values of DDF (Beltaos and Carter 2021). This kind of scatter has been noticed by others in the past and explained in terms of snowfall conditions: "*…the largest variations from year to year at a given site are associated with the thickness of the snow on the ice, and secondarily by variations in the coldness of the winter period of thickening*" (Ashton, 2011). This quote is reproduced from a seminal paper written by the late G.D. Ashton, one of the world's foremost experts in river and lake ice growth.

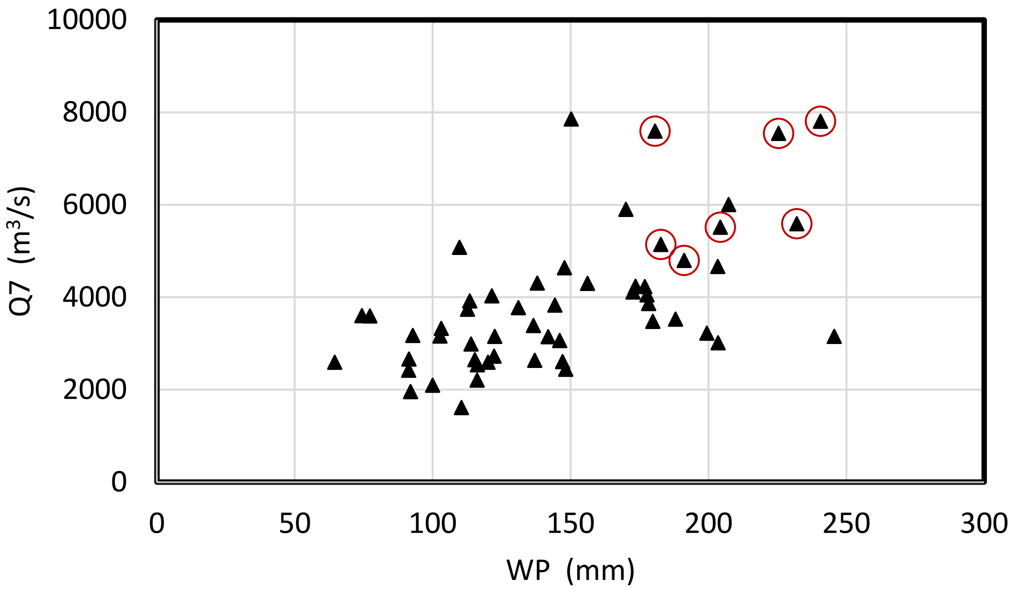

Fig. 3. Peak 7-day running average flow (Q7) at Peace Point during the breakup period versus Grande Prairie winter precipitation (WP), as determined by Lamontagne et al. (2021). The red circles mark IJF years. Data range: 1962-2020, excluding the reservoir-filling years 1968-1971. Flow data source: WSC archives for the Peace Point gauge. https://wateroffice.ec.gc.ca/

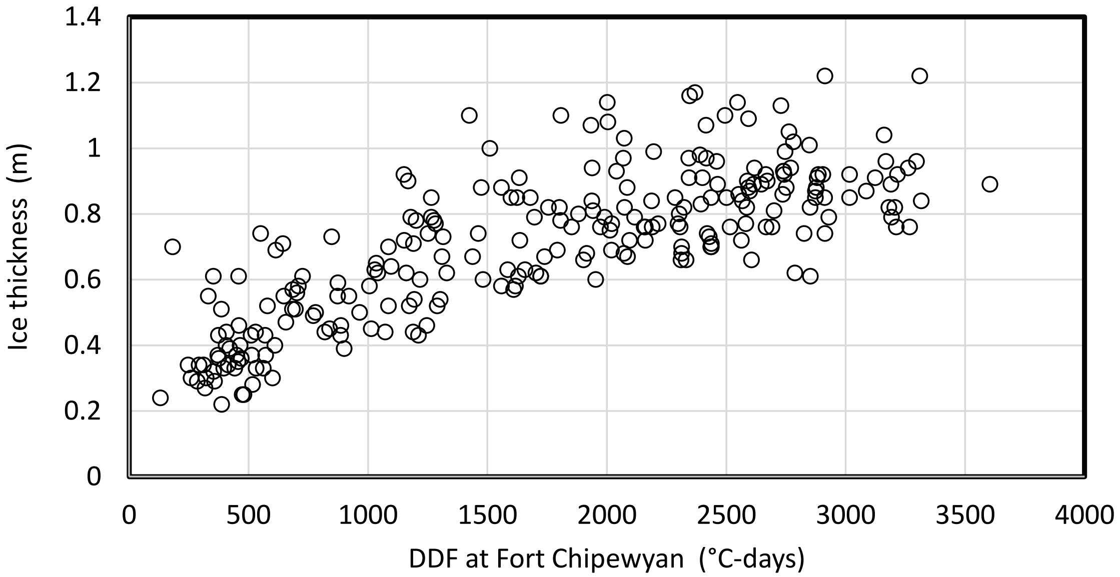

Fig. 4. Cross-sectional average ice thickness near the WSC Peace Point gauge, plotted versus accumulated degree-days of frost at Fort Chipewyan starting on October 1 of the year preceding the breakup year (Fort Chipewyan is located on Lake Athabasca, ~90 km SE of Peace Point). Adapted from Beltaos and Carter (2021). Data range: 1962-2021. Thickness data source: WSC archives for the Peace Point gauge

An additional contributing factor to the epistemic uncertainty is the fact that WP and DDF are correlated to each other (colder winters generally produce more snow in the studied area), which contradicts one of the assumptions made in logistic regression. Lamontagne et al. (2021) described the degree of correlation as “moderate” (Pearson’s $r \approx 0.60$) and pointed out that some of the information in the record is therefore redundant. Correlated variables could hinder correct interpretation of the statistical results. For example, the apparent positive influence of the DDF on the chances of IJF occurrence could largely reflect the positive effect of a deep snowpack.

Logistic regressions on Q7 alone, HF alone, and end-of-winter ice thickness (h_{io}) alone indicate statistical significance for Q7, near-significance for HF, and lack of significance for h_{io} (Table 1). Simultaneous regressions of (Q7, HF) and (Q7, HF. h_{io}) indicate significance for Q7 and HF but lack of significance for h_{io}. Regardless of the assumed model, the regressions indicate that Q7 and HF have positive and negative effects on the chances of an IJF, respectively. However, the model of h_{io} alone points to a negative effect ($b_1 < 0$), while the model that combines h_{io} with Q7 and HF indicates a positive effect ($b_3 > 0$).

Table 1. Coefficients and p-values associated with logistic regression on different combinations of explanatory variables.

Model variables	Model coefficients (and p-values)			
	b_0	b_1	b_2	b_3
Q7 (m^3/s)	-6.905 (0.00035)	0.0011 (0.0025)	NA	NA
HF (m)	169.95 (0.067)	-0.804 (0.064)	NA	NA
h_{io} (m)	-0.557 (0.844)	-1.398 (0.664)	NA	NA
S5 (°C-days)	1.680 (0.299)	-0.026 (0.045)	NA	NA
Q7, HF	321.83 (0.037)	0.0015 (0.0065)	-1.550 (0.034)	NA
Q7, HF, h_{io}	360.50 (0.049)	0.0019 (0.014)	-1.805 (0.044)	14.752 (0.110)

Notes: annual values of the explanatory variables derive from WSC records for the Peace River gauge at Peace Point; reservoir filling years, 1968-1971 are excluded; the year 2006 is excluded when the regression involves Q7 (insufficient data); Q7 and HF may have been underestimated in 2018 and 2020, respectively; their p-values decrease slightly if either one is increased.

A fourth, potentially relevant, variable is the degree-days of thaw (S5) above a base temperature of -5°C (Bilello 1980), accumulated up to the day when breakup is initiated at Peace Point. It reflects the degree of thermal decay that the ice cover is subjected to, prior to being mobilized. When regressed alone, S5 appears to be significant, though the associated constant is not (Table 1). It becomes insignificant when it is regressed along with other variables (p-value ~ 0.4), possibly because it correlates with both Q7 and with HF (Pearson's r ≈ -0.32, 0.30, respectively). In view of the lack of clear significance for h_{io} and S5, the remainder of this paper will focus on the primary variables Q7 and HF, which are practically uncorrelated (Pearson's r ≈ 0.07).

To visualize how IJF probability varies as a function of breakup flow and freezeup level, the (Q7, HF) model in Table 1 is selected and the indicated model coefficients entered in Eqs. 1 and 2. The results are illustrated in Fig. 5: for any given breakup flow, the probability P(IJF) decreases as HF increases; for any given HF, P(IJF) increases as Q7 increases.

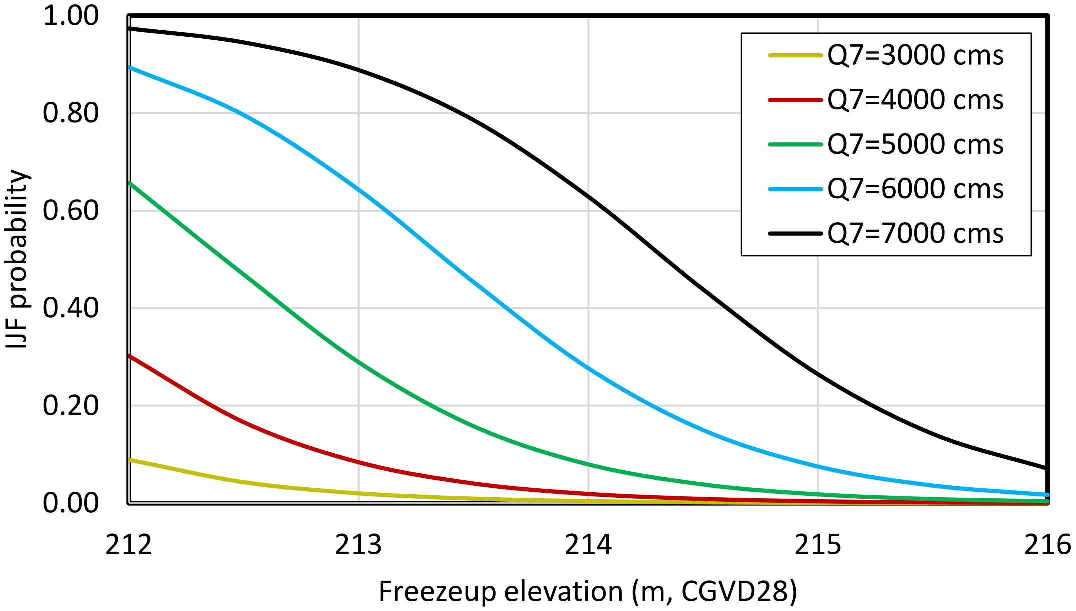

Fig. 5. Variation of the probability of an ice-jam flood (P_{IJF}) with Peace Point freezeup level and breakup flow, as calculated by logistic regression on Q7 and HF. Logistic regression was performed using the Firth bias-reduced algorithm (Wessa 2016).

The preceding results remove the epistemic uncertainty associated with the proxy variables WP and DDF but two other types of uncertainty remain. Therefore, there is no guarantee that the probabilities indicated in Fig. 5 are accurate; their main value is in qualitatively illustrating the effects of Q7 and HF on the chances of ice-jam flooding in any one year.

4. Structural Uncertainty

As intimated in Section 2, the linearity assumed in the logit formulation (Eq. 2) is most likely erroneous because complex physical processes are seldom quantified in terms of simple linear sums of their controlling factors. Different powers and exponents, products, or logarithms of the respective variables are also possible. The "true" logit function is unknown, but helpful insights can be gleaned by varying the formulation of Eq. 2 and repeating the regression. Noting that, at Peace Point, the resistance to ice cover mobilization depends directly on the difference 216-HF (Beltaos 2023b), the following expressions were tried:

$$\boldsymbol{X} = b_0 + b_1 Q7 + b_2 DF \quad [3]$$

$$\boldsymbol{X} = b_0 + b_1 \sqrt{Q7} + b_2 (DF)^3 \quad [4]$$

$$\boldsymbol{X} = b_0 + b_1 \ln{(Q7)} + b_2\, ln(DF) \quad [5]$$

in which DF = 216-HF. The elevation 216 m (CGVD28) is slightly higher than the highest known HF of 215.9 m, so that DF is always positive. Equation 3 is the conventional linear expression for the logit, while Eq. 4 sums arbitrarily selected powers of Q7 and DF; Eq. 5 effectively involves the product $(Q7)^{b1}(DF)^{b2}$. Table 2 summarizes the results of logistic regressions with these models and shows that all p-values are lower, or slightly higher, than the conventional significance limit of 0.05. The same applies to various other models that were also tried, each involving different functions of Q7 and DF. It is concluded that these two variables are indeed statistically significant controls of IJF occurrence.

Table 2. Exploring structural uncertainty for models that utilize breakup flow and freezeup level as the explanatory variables.

Model variables	Model coefficients (and p-values)		
	b_0	b_1	b_2
Q7, DF	-12.39 (0.0048)	0.0014 (0.0058)	1.53 (0.039)
$(Q7)^{0.5}$, $(DF)^3$	-15.26 (0.0017)	0.185 (0.0032)	0.037 (0.045)
ln(Q7), ln(DF)	-69.84 (0.0086)	7.64 (0.0096)	3.94 (0.066)

Predictions generated by the 3 models of Table 2 are compared in Fig. 6 in terms of respective isolines for P(IJF) = 0.50. As can be expected from Eqs. 1 and 2, the isoline of the linear model (second row of Table 2) is straight. This is not the case for the nonlinear models, which generate concave or convex curves, depending on the assumed formulation. Isolines for other values of P(IJF) have similar shapes as the corresponding lines of Fig. 6; for the simple linear model, the isolines are parallel to each other. Five of the seven flood events that occurred during the period 1962 to 2020 plot above the 0.5 isoline.

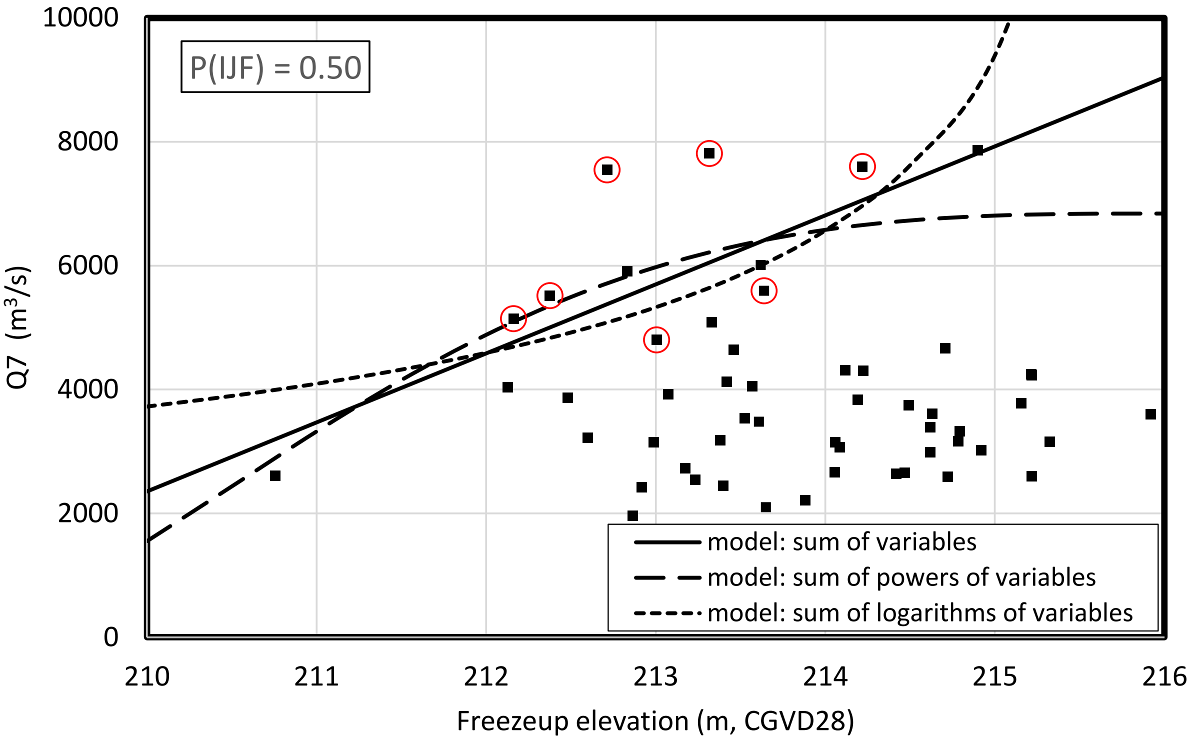

Fig. 6. Isolines for P(IJF) = 0.50, as generated by the three models of Table 2. The data points represent observed annual values of Q7 and HF at Peace Point for the period 1962 to 2020, excluding the reservoir-filling years 1968-1971; the red circles mark IJFs.

The predictive power of the Table 2 models has been visually assessed using graphs similar to that of Fig. 2. Best performance is exhibited by the logarithmic equation, which points to a product, rather than a sum, of the explanatory variables. As shown in Fig. 7, the overlap range now only involves two non-flood points. The rightmost of these points applies to 1967, a year for which the historical record mentions a flood, but without details as to the cause (Timoney 2009). Numerical modelling with HECRAS (Beltaos 2019) has indicated that a 1967 flood would require presence of an ice jam in the lower Peace River. If an IJFdid occur in 1967, the overlap range would be considerably reduced and contain a single non-flood occurrence. [This potential reduction of the overlap range does not apply to Fig. 2, where the rightmost non-flood data point derives from the year 1962].

It is concluded that structural uncertainty does not influence assessments of the statistical significance of the explanatory variables. However, it can lead to different predictions of IJF probability, depending on the structure of the assumed model. Of course, these findings apply to the present case study and may or may not be of general validity.

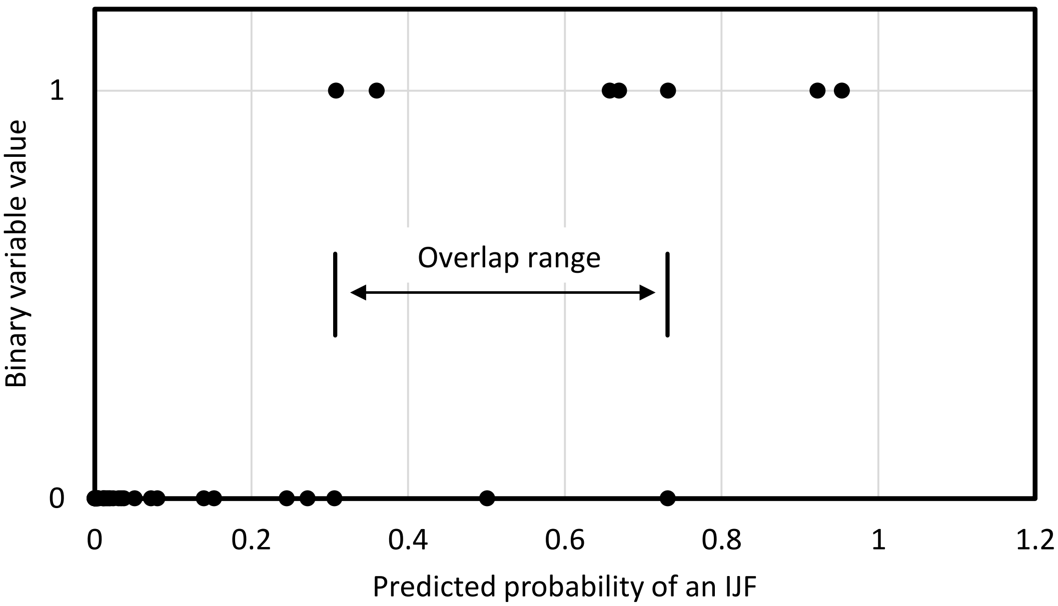

Fig. 7. IJF occurrence (binary variable = 1) or non-occurrence (binary variable = 0) versus predicted probability using the third model of Table 2, which essentially involves the product of powers of the explanatory variables.

5. Other Sources of Uncertainty

Recalling the quotation from Lamontagne et al. (2021) in Section 2, a third type of uncertainty derives from samples that are too small for logistic regression (parametric uncertainty). This issue is addressed by means of the Firth (1993) bias-reducing approach. Nevertheless, the events-per-variable requirement suggests that each additional variable can generate further uncertainty, potentially straining the limits of the Firth approach. Therefore, it is advisable to keep the number of explanatory variables to a minimum when performing logistical regression on small samples.

Independently of statistical theory, it has been determined by physics-based numerical modelling that a necessary, albeit not sufficient, condition for an IJF is that the river flow at Peace Point should exceed 4000 m^3/s (Beltaos 2003). Consequently, P(IJF) is nil if Q7 is less than 4000 m^3/s. However, all Q7-related regressions that have been performed so far predict nonzero probabilities for all values of Q7. This is illustrated in Fig. 5, which also indicates that associated prediction errors will not be pronounced when HF exceeds ~213 m. This condition applies to most years of the record (Fig. 6), which primarily reflects regulated conditions and typically higher freezeup levels than under natural conditions.

6. Discussion

The preceding analysis has highlighted some of the advantages and limitations of applying logistic regression to identify the factors that control the occurrence of IJFs in the lower Peace River and compute IJF probability as a function of these factors. The main advantage is that statistical significance or non-significance of anyone factor does not depend on the type of equation used to represent the logit, once weak proxies are replaced by direct variables. The main disadvantage is the lack of confidence in predicted probabilities, which change with logit structure and are demonstrably in error when the breakup flow is less than a physics-derived threshold. Therefore, predicted IJF probabilities, regardless of logit structure, are only helpful as qualitative indications. Their use in quantitative mode to, for example, generate climate-related projections of future IJF frequencies can lead to unrealistic results.

Recalling Fig. 6, one may note that data points representing IJFs congregate in the "northwestern" region of the graph. This feature also applies to a plot of the maximum breakup flow versus HF (Beltaos 2019). If one were to empirically draw a line in Fig. 6, demarcating the region where IJFs occur, and taking into account the 4000 m^3/s constraint, this line would most resemble the concave isoline. This again suggests that a product, rather than a sum, of functions of the explanatory variables DF (= 216-HF) and Q7 is more appropriate for the logit.

7. Summary and Conclusions

Single-station winter precipitation and degree-days of frost accumulated over the period November to April are shown to be weak proxies for breakup flow and ice thickness, respectively. Use of the latter variables in logistic regression demonstrates the statistical significance of the freezeup level and its negative effect on the probability of ice-jam flooding near the PAD during the spring breakup of Peace River. This effect fully aligns with empirical evidence from various rivers and with physical understanding of the breakup process in Peace River. It is further pointed out that the simple linear sum of variables that is conventionally assumed to represent the structure of the logit is very likely an erroneous quantification of complex physical processes. Trial regressions with different structural formulations of the logit indicate that the assessment of significance of key explanatory variables does not change across different models. However, the calculated probability of an IJF does vary from one model to the next and its primary value is therefore of qualitative nature. Comparison of probability isolines to actual occurrences on the breakup flow – freezeup level plane suggests that the correct, albeit unknown, logit likely involves a product, rather than a sum, of functions of these two variables.

Acknowledgments

This study was conducted as part of Environment and Climate Change Canada's contribution to the Wood Buffalo National Park World Heritage Site Action Plan. The writer thanks J. Lamontagne, M. Jasek and J.D. Smith for helpful discussion and Grande Prairie winter precipitation data. All logistic regressions performed in the course of this study utilized the free software provided by P. Wessa (Wessa 2023).

References

Ashton, G.D. 2011. River and lake ice thickening, thinning, and snow ice formation. Cold Reg. Sci. Technol. 68, 3–19.

Beltaos, S. 2003. Numerical modelling of ice-jam flooding on the Peace-Athabasca Delta. Hydrol. Processes, 17: 3685–3702. doi:10.1002/hyp.1355.

Beltaos, S. 2008 (editor). River ice breakup. Water Resources Publications, Highlands Ranch, Co., USA.

Beltaos, S. 2019. Ice-jam flood regime of the Peace-Athabasca Delta: Update in light of the 2014 event. Cold Reg. Sci. Technol. , 165, 102791. https://doi.org/10.1016/j.coldregions.2019.102791

Beltaos S. 2021. Assessing the Frequency of Floods in Ice-Covered Rivers under a Changing Climate: Review of Methodology. *Geosciences*. 2021; 11(12):514. https://doi.org/10.3390/geosciences11120514

Beltaos, S. 2023a. "The Drying Peace–Athabasca Delta, Canada: Review and Synthesis of Cryo-Hydrologic Controls and Projections to Future Climatic Conditions" *Sustainability* 15, no. 3: 2103. https://doi.org/10.3390/su15032103

Beltaos, S. 2023b. Resistance of river ice covers to mobilization and implications for breakup progression in Peace River, Canada. Hydrological Processes. 37(3):e14850. wileyonlinelibrary.com/journal/hyp 1 of 11 https://doi.org/10.1002/hyp.14850

Beltaos, S. and Carter, T. 2021. Minor 2020 ice jamming in lower Peace River despite extreme breakup flows: assessment of hydroclimatic controls. CGU HS Committee on River Ice Processes and the Environment 21st Workshop on the Hydraulics of Ice Covered Rivers, Saskatoon, Saskatchewan, Canada, August 29-September 1, 2021.

Beltaos, S. and Peters, D.L. 2021. Naturalizing the freezeup regimes of regulated rivers and exploring implications to spring ice-jam flooding. Hydrol. Process. 35, e14321

Beltaos, S., Prowse, T. D., and Carter, T. 2006. Ice regime of the lower Peace River and ice-jam flooding of the Peace-Athabasca Delta. Hydrological Processes, 20(19), 4009–4029.

Bilello, M. A. (1980). Maximum thickness and subsequent decay of lake, river and fast sea ice in Canada and Alaska. U.S. Army, Cold Regions Research and Engineering Laboratory, Report 80-86, Hanover, New Hampshire, USA, 160.

Emmer, S., Kovachis Watson, N., and Nafziger, J. (2021). A Tale of Two Basins: The 2020 river ice breakup in northern Alberta, Part II: The Peace River. CGU HS Committee on River Ice Processes and the Environment 21st Workshop on the Hydraulics of Ice Covered Rivers, Saskatoon, Saskatchewan, and online, August 29 to September 1, 2021.

Firth, D., 1993. Bias Reduction of Maximum Likelihood Estimates. Biometrika 80 (1), 27–38. https://doi.org/10.2307/2336755.

Independent Environmental Consultants (IEC). 2018. Strategic environmental assessment of potential cumulative impacts of all developments on the World Heritage Values of Wood Buffalo National Park. Volume 1: final SEA report, Markham, Ontario, Canada. 247 pp.

Jasek, M., and Pryse-Phillips, A. (2015). Influence of the proposed site C hydroelectric project on the ice regime of the Peace River. Canadian Journal of Civil Engineering, 42(9), 645–655.

Jasek, M. (2019a) ‘An emerging picture of Peace River break-up types that influence ice jam flooding of the Peace-Athabasca Delta part 1: the 2018 Peace River break-up’, in Proceedings of the 20th Workshop on the Hydraulics of Ice Covered Rivers. 20th Workshop on the Hydraulics of Ice Covered Rivers, Ottawa, ON, Canada, 31 pp.

Jasek, M. (2019b) ‘An emerging picture of Peace River break-up types that influence ice jam flooding of the Peace-Athabasca Delta part 2: Insights from the comparison of the 2014 and 2018 break-ups’, in Proceedings of the 20th Workshop on the Hydraulics of Ice Covered Rivers. 20th Workshop on the Hydraulics of Ice Covered Rivers, Ottawa, ON, Canada, 24 p.

Lamontagne, J.R.; Jasek, M.; Smith, J.D. Coupling physical understanding and statistical modeling to estimate ice jam flood frequency in the northern Peace-Athabasca Delta under climate change. Cold Reg. Sci. Technol. 2021, 192, 103383.

Peace-Athabasca Delta Project Group (1972) The Peace-Athabasca Delta, A Canadian Resource.Summary Report, Canada, Alberta, Saskatchewan, 144 pp.

Peace-Athabasca Delta Project Group (1973) Peace-Athabasca Delta Project, Technical Report. Canada, Alberta, Saskatchewan, 176 pp.

Peters, D.L. 2003. Controls on the Persistence of Water in Perched Basins of the Peace-Athabasca Delta, Northern Canada. Ph.D. thesis, Trent University, Peterborough, Ont., Canada.

Peters, D.L., Prowse, T.D., Pietroniro, A., and Leconte, R. 2006. Flood Hydrology of the Peace-Athabasca Delta, Northern Canada. Hydrol. Processes, 20: 4073–4096. doi:10.1002/hyp.6420.

Peterson, M. 1995. Peace–Athabasca Flood History Study. Peace–Athabasca Delta Technical Studies, Task F.1 - Flood History. Peace–Athabasca Delta Technical Studies, Communications: Fort Chipewyan, Canada. 32pp.

Peterson, M. and Courtorielle, W. 1992. Flood history study, Peace/Athabasca Delta, Wood Buffalo National Park. Data Report 92- 14/WB, Fort Chipewyan, Canada, 62pp.

Prowse, T.D. and Conly, F.M. 1998. Effects of climatic variability and flow regulation on ice-jam flooding of a northern delta. Hydrol. Processes, 12: 1589–1610.

Prowse, T. D., Aitken, B., Demuth, M.N. and Peterson, M. 1996. Strategies for restoring spring flooding to a drying northern delta. Regulated rivers: research and management, 12: 237-250.

Prowse T.D., Peters, D., Beltaos, S., Pietroniro, A., Romolo, L., Toyra, J. and Leconte, R. 2002. Restoring ice-jam floodwater to a drying delta ecosystem. Water International 27(1): 58–69.

Prowse, T.D., Demuth, M., and Peterson, M. 1993. Environmental aspects of river ice jamming on the Peace-Athabasca Delta. In Proceedings of the 7th Workshop on the Hydraulics of Ice Covered Rivers (1993), Saskatoon, Saskatchewan: NHRI Symposium Series No. 12. pp. 333–337.

Timoney, K.P. 2009. Three centuries of change in the Peace–Athabasca Delta, Canada. Clim. Change, 93: 485–515. doi:10.1007/s10584-008-9536-4.

UNESCO. 2017. Reactive Monitoring Mission (RMM) to Wood Buffalo National Park, Canada. Mission Report March 2017. UNESCO World Heritage Centre - WHC International Union for Conservation of Nature (IUCN). Available from http://whc.unesco.org/en/documents/156893.

van Smeden, M., Moons, KGM, de Groot, JAH, Collins, GS, Altman, DG, Eijkemans, MJC and Reitsma, JB. 2019. Sample size for binary logistic prediction models: Beyond events per variable criteria. Statistical Methods in Medical Research, Vol. 28(8) 2455–2474.

Ward, E.M. and Gorelick, S.M. 2018 Drying drives decline in muskrat population in the Peace-Athabasca Delta, Canada. Environ. Res. Lett. 13 124026

Ward, E.M., Wysong, K. and Gorelick, S.M. 2020. Drying landscape and interannual herbivory-driven habitat degradation control semiaquatic mammal population dynamics. Ecohydrology 13(1): e2169 https://doi.org/10.1002/eco.2169

Ward, E.M., Solari. K.A., Varudkar, A., Gorelick, S.M., and Hadly, E.A. 2021. Muskrats as a bellwether of a drying delta. Nature Commun Biol. 2021 4(1):750. Muskrats as a bellwether of a drying delta | Communications Biology (nature.com)

WBNP. 2019. Development of a multi-jurisdiction Action Plan to protect the World Heritage Values of Wood Buffalo National Park. Fort Smith, N.T., Canada ISBN: R64-546/2019E-

PDF. Available from https://www.pc.gc.ca/en/pnnp/nt/woodbuffalo/info/action [accessed April 2020].

Wessa P., (2016), Bias Reduced Logistic Regression (v1.0.6) in Free Statistics Software (v1.2.1), Office for Research Development and Education, URL http://www.wessa.net/rwasp_logisticregression.wasp/

Wessa, P. (2023), Free Statistics Software, Office for Research Development and Education, version 1.2.1, URL https://www.wessa.net/