Experimental and numerical studies on the motion of river ice floes driven by a dispersive wave train

Takaaki Abe¹, Yasuhiro Yoshikawa² and Akashi Itoh³

¹Civil Engineering Research Institute for Cold Regions, 3-1-34, Hiragishi-Ijo, Toyohira-ku, Sapporo, Hokkaido, JAPAN 062-8602
tabe.ceri@gmail.com

²Kitami Institute of Technology, 165 Koen-cho, Kitami, Hokkaido, JAPAN, 090-8507
yoshi@mail.kitami-it.ac.jp

³Civil Engineering Research Institute for Cold Regions, 3-1-34, Hiragishi-Ijo, Toyohira-ku, Sapporo, Hokkaido, JAPAN 062-8602
ito-a2tu@ceri.go.jp

This research detailed the interaction between the dispersive wave train and ice floe in tsunami-induced wave propagation up rivers. First we conducted hydraulic experiments to investigate the effects of ice floe length ($L_i$) and thickness ($h_i$) using field observation data obtained when river ice breakup, transportation and scattering were caused by the 2011 Tohoku Pacific-Coast Earthquake tsunami. We measured transportation velocities of ice floes using an image analysis technique, namely, PTV and examined some aspects of the drifted motion of the ice floes. Second, particle based numerical simulation was conducted. The method we use is the Moving Particle Semi-implicit (MPS) method, which has been originally proposed for calculation of incompressible viscous fluid flows with free surfaces. To reproduce the motion of ice floes driven by a dispersive wave train, a simple model to find an approximated solution of the motion for a solid body in the fluid is used. Direct comparisons of experimental photos and their corresponding simulation snapshots are made in terms of the reproduced waveforms and the ice floe displacement. Then a quantitative comparison is made between the time series velocity predicted in the MPS method and the experimental results from PTV analysis. Velocity waveforms of calculation results are similar in shape to those from experiments. The importance of significant effect of dispersive wave trains on ice floe transportation is elucidated.
1. Introduction

The tsunami triggered by the 2011 Off the Pacific Coast of Tohoku Earthquake not only caused enormous damage to the coastal areas of the Thoku Region, but it also caused run-up in rivers nationwide (Abe et al. 2012b). River tsunamis observed in class-1 rivers in Hokkaido revealed issues peculiar to cold regions, such as the movement of large amounts of ice floe transport, ice jams and intrusion of ice floes into sluice gates. Photo 1 shows a video of an undular bore in the Shin-Kushiro River. In the first wave of the tsunami, just after 4:00 pm on March 11, the first peak of the wave was in a breaking form, and the second and subsequent peaks were in the form of continuous dispersive train. Late at night at the same observation point, there was not the breaking form shown in Photo 1; instead, undular bore continued for about two minutes, longer than the first peak of the wave. Observation of the maximum wave at Kushiro Port right before the bore was observed, at midnight, suggests that the bore was sufficiently developed by the large-scale tsunami in the process of run-up in the Shin-Kushiro River.

In cold snowy regions such as Hokkaido, ice forms in rivers from December to April. An experimental study in our previous report (Abe et al., 2011) revealed the close resemblance between the time-series change in transport velocity of ice and the waveform taken by wave gauges. It was revealed that ice floe transport generated in a river channel after the development of an undular bore like that described above dramatically increases the ice floe velocity. In the above-mentioned study, however, single-form models of ice floe were used, and trends with models of different lengths or thicknesses of ice were not examined.

The aim of this study is to clarify how the dimensions of the ice affect ice floe transport and how an undular bore interacts with an ice floe. To achieve this, hydraulic experiments were carried out using ice floes of different lengths or thicknesses. Moreover, using a particle method, a vertical two-dimensional numerical simulation was conducted on a river tsunami with ice in order to address the relation between the internal flow field of an undular bore and ice floe transport.

2. Study Methods

The experimental flume for river tsunamis (Figure 1) owned by the Civil Engineering Research Institute for Cold Regions was used for the hydraulic experiments. This flume is equipped with a wave paddle at the downstream end and a pump for flow generation, enabling the run-up of a river tsunami to be simulated in a non-uniform flow field. Utilized for measuring longitudinal waveform change, capacitance wave gauges (KENEK Corp.) were installed at the five positions shown in the profile view. These gauges were set 5 cm from the right-bank sidewall of the flume to avoid disturbance to the ice floe run-up. The five gauges in this paper are designated Ch.1, Ch.2, Ch.3, Ch.4 and Ch.5.

Despite the fact that the Tokachi River is a large-scale Class-1 river in Hokkaido, river ice easily forms on it. The experimental conditions were set in reference to the conditions of the river channel of the Tokachi River. The model scale was set at 1/30, and the bed gradient and the water depth \((D_m)\) on the upstream side of the wave paddle were set at 1/250 and at 0.80m, respectively, in order to satisfy Froude's law of similarity. The inflow was set at either 5.0 L/s or
2.5 L/s. For comparison, a test was conducted with no inflow. Length \((L_i)\) and thickness \((h_i)\) of ice models with square shapes in plain view were set in combinations of three lengths (6, 9 and 12 cm) and three thicknesses (1, 2 and 4 cm) in accordance with the results of dimensional measurement of drifting ice floes generated in the Tokachi River by the tsunami run-up of the 2011 Off the Pacific Coast of Tohoku Earthquake (Abe et al., 2012a).

Regarding the tsunami of the Off the Pacific Coast of Tohoku Earthquake, other phenomena characterizing rivers in cold regions were observed, such as the transport of vast ice floes that almost covered the width of the river (Matsukawa et al., 2012) and tsunami propagation under frazil ice (Yoshikawa et al., 2013). This paper focuses on the phenomenon of drifting ice floes, which formed after the breakup of river ice, transported by an undular bore.

To prevent flowing down of the ice floes by the channel flow, wire rods were installed vertically on the downstream side of the floe (Figure 2). While minimizing the impact on tsunami run-up, this system allows the observation of ice floe transport only in the upstream direction. To investigate the changes in longitudinal waveform with run-up in the flume and the ice floe transportation process, the locations (=\(d\)) of the floe models were set as 5.0 m, 10.0 m, and 15.0 m from the downstream end.

Digital cameras were installed above the flume to capture the process of ice floe transport. The resolution was set at 1920 x 1080 with a frame rate of 30 FPS, and the shooting range was set at around 5 m so as to record ice floe transport from commencement to termination. Control points were set on the sidewalls of the flume, starting from the base point \((x = 0 \text{ m})\) at the downstream end, allowing the positional relation between the tsunami and the ice floe to be read from images.

Particle tracking velocimetry (PTV) analysis on sequential images taken by the digital cameras provided the ice floe transport velocity \((\mathbf{U}_i = (U_{ix}, U_{iy}))\). Using Dipp-Flow Ver.2.00 (DITECT Corp.) for the image analysis, distortion correction of the lens and spatiotemporal correction were made.

To directly analyze the interaction between tsunami run-up and the drifting ice floes, a particle method that allows easy simultaneous analysis of violent motions of free water surface and on transport of drifting object was applied. The Moving Particle Semi-implicit (MPS) method, a particle method proposed by Koshizuka et al. (1996), was adopted for basic calculation. The basic equations are a continuity equation \(\left(\frac{\partial \rho}{\partial t} = 0\right)\) and the Navier-Stokes equation

\[
\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{K}
\]

where \(\mathbf{u}\) is vector of flow velocity, \(\rho\) is fluid density, \(p\) is pressure, \(\nu\) is coefficient of kinematic viscosity, and \(\mathbf{K}\) is external force term.

Accurate interface tracking is vital in the MPS method to reproduce waveform changes of tsunami run-up in a non-uniform flow field and transport of drifting object. This study adopted the Improved MPS method, which can reduce pressure disturbances more greatly than the Standard MPS method can. Out of many approaches proposed by some domestic research groups (e.g., Khayyer et al., 2011) to improve the accuracy of pressure solution in the MPS method, the
CMPS (Corrected-MPS) scheme and the MPS-HL-HV scheme were employed in this study. The CMPS scheme replaces the pressure gradient term discretized by the MPS method in the Navier-Stokes equation below

$$\langle \nabla p \rangle_i = \frac{D_s}{n_0} \sum_{j \neq i} \frac{p_j - \bar{p}_i}{|\mathbf{r}_{ij}|^2} \mathbf{r}_{ij} w(|\mathbf{r}_{ij}|)$$  \hspace{1cm} (2)

with the following anti-symmetric expression

$$\langle \nabla p \rangle_i = \frac{D_s}{n_0} \sum_{j \neq i} \left( \frac{p_i + p_j}{2} - \frac{\bar{p}_i + \bar{p}_j}{2} \right) \frac{\mathbf{r}_{ij} w(|\mathbf{r}_{ij}|)}{|\mathbf{r}_{ij}|^2} \right)$$  \hspace{1cm} (3)

To obtain divergence of spatial gradient of physical quantity, the MPS-HL-HV scheme replaces the Laplacian model expressed by the following equation

$$\langle \nabla^2 \phi \rangle_i = \frac{2D_s}{n_0 \lambda} \sum_{j \neq i} \phi_{ij} w(|\mathbf{r}_{ij}|)$$  \hspace{1cm} (4)

with the expression below

$$\langle \nabla^2 \phi \rangle_i = \nabla \cdot \langle \nabla \phi \rangle_i = \frac{1}{n_0} \sum_{j \neq i} \frac{3\phi_{ij} r_e}{|\mathbf{r}_{ij}|^3}$$  \hspace{1cm} (5)

where $p_i$ is pressure of particle $i$, $n_0$ is standard particle number density, $\mathbf{r}_{ij}$ is relative position vector of particle $j$ toward particle $i$, $w(\mathbf{r}_{ij})$ is weight function, and parameter $\lambda$ ($\Sigma_{j \neq i} (w(|\mathbf{r}_{ij}|)|\mathbf{r}_{ij}|^2) / \Sigma_{j \neq i} |\mathbf{r}_{ij}|^2$) is model constant of the MPS method. Brackets $\langle \rangle_i$ indicate discretization by a particle interaction model of the original MPS method. The ice floe transport was reproduced using the Passively Moving Solid (PMS) model (Gotoh et al., 2006b), and the specific gravity of a rigid particle representing ice was set at 0.92 to adjust for the specific gravity of the polypropylene plates used in this experiment.

Due to the limitations of the computer, the diameter of the particles placed in the computational region was set at 1cm ($d_0$), which was the maximum value to resolve images of the thinnest ice floe ($h_i = 1$cm). The number of particles in the whole region for reproducing Figure 1 was approximately 70,000. As for the area of influence for calculating interaction, the effective radius $r_e$ was set at $d_0$ multiplied by 2.4, which is the value recommended by Khayyer et al (2011). Although wave generation boundary provided by coupling with the Boussinesq equation has been proposed by Gotoh et al. (2006a) for reduction of computation time in particle methods, it is difficult for this method to reproduce river flow and outflow at the downstream end. In line with the actual conditions of the flume, rigid body particles were set to provide forced displacement for simulating the wave paddle at the downstream end in this study, allowing waves to be generated by imparting the same displacement at the same time interval for the wave maker installed in the experimental flume. The computational region from the downstream end
to the point at $X=20.0$ was reproduced, and the computational particles were inpoured from the inflow boundary corresponding to the influx of the experiment in terms of the number of particles and the velocity.

Artificial turf was placed on the bed of the experimental flume. Anti-symmetric frictional drag force: $F_{ij}^D = -\gamma v(|r_{ij}|)(\mathbf{r}_{ij} \cdot \mathbf{u}_{ij})\mathbf{r}_{ij}$ is introduced in the MPS method by Khayyer et al. (2010) was applied for simulation of the bed. A relatively-large value ($\gamma = 1.5$) was used for the friction coefficient so that distribution of the longitudinal water level in the pre-wave-making condition could closely correspond to the actual distribution in the experiment. Using the above-described computational model, water levels during wave making were calculated at the installation points of the wave gauges.

Determination on water level and on water depth in a particle method such as the MPS has few practical examples due to the Lagrangian characteristics of the computational process. At present, provided by Gotoh et al. (2006a), very few examples that describe the process of water-level determination are available. To examine the process of bore formation in detail in this study, it was essential to accurately determine the water levels corresponding to the motions of computational particles as closely as possible. To achieve this, at the initial status ($t = 0$ s), the neighboring particle number (=12) of a single particle at the surface under the condition of $r_e = 2.4 \times d_0$ was set as a standard. Then, in the region around the setting point of the level gauge, the average on $y$ coordinates of the particles whose vicinity particle number was 12 was determined as the water level.

3. Results and Discussion

At first, it was investigated whether the presence of the drifting ice floe affected river tsunami's waveform. Figure 3 shows the results of measurement by the wave gauges at the two different setting points of ice floe at the lower reach ($d=5.0$) and at the upper reach ($d=15.0$) using the ice floe with the maximum dimensions of length ($L_i=12\text{cm}$) and thickness ($h_i=4\text{cm}$). In all the cases, the waveform changed in the same manner from the downstream end of Ch.1 toward Ch.5. Starting as a single-arch form, the river tsunami gradually tilted forward, and generation of a dispersive wave started at its front at Ch.2. Subsequently, dispersive waves formed at around Ch.3 and Ch.4. At Ch.5, the form of the first peak at the head collapsed when the wave broke, and the maximum water level was observed at the second peak.

When the discharge $Q$ is 0.0 L/s, the water-level fluctuation of the two setting points of the ice floe roughly coincide, and the run-up velocity is almost the same at both points. With the existence of downflow ($Q=2.5$ L/s or 5.0 L/s), although the run-up velocity for the ice floe at the lower reach is slightly lower than that for the ice floe at the upper reach, there is no significant disparity between the cases in terms of the waveform transformation process. Due to space limitations, this paper omits detailed descriptions of how the aforementioned trend was also observed in the cases of smaller $L_i$ and $h_i$.

The facts mentioned above imply that although there is a possibility to slightly decrease the propagation velocity, the existence of the ice floe in this experiment has little influence on the transformation process of the undular bore. In other words, in the transport of a single drifting ice
floe, generation of an undular bore has to be considered. Special mention is made that the length $L_f$ and the thickness $h_i$ of ice floe used in this study were set in reference to the actually measured values of ice floes left as traces of the tsunami in the event of the 2011 Off the Pacific Coast of Tohoku Earthquake.

An image analysis of single-form ice floes in our previous report (Abe et al., 2011) revealed that the analyzed waveform was similar in shape to the incident waveform. Examination was made at the same point in this paper. Figure 4 shows the results of comparison between the time series velocity of ice floe transport provided through the image analysis and the time series water level change $\eta$ measured at the setting points of the wave gauges.

Figure 4 (a) shows the ice floe transport process in still water without discharge. The single-arch form of the incident tsunami wave at Ch.1 corresponds to the form of the time series change of the transport velocity. Also, the slightly stepwise form at the wave head at Ch.2 corresponds to the form of the time series change of transport velocity.

At discharge $Q=0.0$ L/s, the development of the undular bore at Ch.3 can be clearly observed at least up to the third peak of the wave. In the case of $L_f=6$cm and $h_i=1$cm, the ice floe transport velocity reacted strongly to the incidence of the wave in the undular bore form. It is presumed that the ice floe was drastically accelerated by the action of the bore. Regarding the dimensions of the ice floe, in the case of the large dimensions ($L_f=12$cm, $h_i=4$cm), the change of ice floe transport velocity is observed in a single-arch form at Ch.1 ($d=5.0$) and at Ch.2 ($d=10.0$). At Ch.3, however, the response of the ice floe transport velocity to the incident waveform is slightly slower compared with the standard dimension ($L_f=6$cm, $h_i=1$cm) case.

As shown in Figure 4 (b), with the existence of discharge, the wave height observed at Ch.3 is about 1cm higher than in still water, due to the promotion of the bore. Corresponding to the phenomenon, the waveform of the transport velocity from the PTV analytical results shown in the three lower graphs strongly reacted. This implies that the promotion of undular-bore development by the action of river flow accelerates ice floe transport. The comparison between the lower two graphs shows that the maximum ice floe transport velocity and the amplitude of the bore waveform of the long ice floe ($L_f=12$cm, $h_i=1$cm) are greater than for the thick ice floe ($L_f=6$cm, $h_i=4$cm).

Then, to examine the reproducibility of tsunami run-up in a non-uniform flow field in a particle method, simulation was conducted on an experiment in which no ice floe was set in the flume. Figure 5 shows the transformation of the waveform at Ch.1, Ch.2, and Ch.3 resulting from the computational method applied in this study. In every different discharge case, the trend of the water-level change is well-reproduced at Ch.1 and Ch.2. The initial stage of dispersive wave formation is observed at the wave front at Ch.2, and the similar waveform can be observed to a small extent from the numerical result by the particle method. However, the waveforms after the peak water level at Ch.1 and at Ch.2 are overestimated.

At Ch.3 where the clear undular bore is formed, the calculated values show overestimation of the wave velocity in any discharge cases. In addition, regarding the formation of the dispersive wave train, three or four peaks are observed in the experiment whereas only two peaks can be observed in the simulation by the particle method. That is to say the simulation cannot provide accurate
reproduction of the undular bore observed in the experiment. One of the possible causes of the overestimation of the wave velocity is the numerical analysis in a two-dimensional vertical space with no consideration on the friction resistance of the acrylic sidewalls of the flume. In addition, out of the computational particles on the flume bed, only the re (effective radius) particles are intended for consideration of bed friction. This limitation may give the friction effect only to very thin layer. In other words, it is conceivable that the effect of the bed friction relatively decreases with increase of the wave height (increase of water depth) through the undular bore development caused by the run-up. Despite the fact that the tsunami run-up is generated after matching the distribution of the initial water level with the experimental values, the simulation cannot reproduce proper velocity distribution near the bed at the current situation. With this issue in mind, it is necessary to discuss the matters in the following way. Including examination on how to determine the parameters such as friction coefficient, more in-depth discussion should be carried out. There are few research cases that showed formation of undular bore using the MPS method. The numerical experiment in this study showed certain effectiveness of the method in analyzing the run-up of river tsunami in a non-uniform flow field.

Next, numerical simulation was carried out on the hydraulic experiment with the drifting ice floe set at \( d = 15.0 \). Snapshots of a movie taken from the side were compared with the positions of the ice floe reproduced using the water-surface form from the calculation results and rigid body models (Figure 6). For comparison, the simulation results under the same condition using the standard MPS method which does not employ higher order scheme were also shown. In Figure 6, (a) shows the time-series snapshots in the hydraulic experiment. Figure 6 (b) and (c) show the calculation results provided by the improved MPS method and by the standard MPS method respectively. As shown in the photos in (a), the channel flow comes from the right, and the tsunami run-up in the bore form from the left starts to transport the ice floe. The experimental snapshots are in time-series first from the left to the right in the upper layer and then from the left to the right in the lower layer. The position of the ice floe at different times is pointed with an arrow and by the text "Ice Floe", and the first and the second wave peaks of the undular bore are indicated by the text of (1st) and (2nd) respectively.

The experimental photos revealed that the ice floe is carried away by not only the first peak at the bore front. The ice floe slides on the water surface up to the first peak, falls in the wave trough between the first and the second peaks, and again run up to the second peak. The fact that the ice floe is transported by repeating such up-down motions was observed. This phenomenon was commonly observed in any forms of the ice floe used in this study.

The calculation results by the improved MPS method using the rigid body model in (b) shows clear reproduction of the motions of the ice floe on the whole. The ice floe runs up to the first peak, falls in the wave trough, and again runs up to the second peak. As pointed out previously, although the simulation could not provide a clear form of the third peak, good outcomes were obtained through the simulation for the first and the second peaks at around the front of the undular bore. Regarding the results obtained from the scheme of the standard MPS method (shown in (c)), wave breaking takes place before the wave reaches the point of \( X = 15.0 \) in the flume, and no formation of an undular bore is observed. Rapid run-up of the ice floe takes place with the motion of the breaking wave front. As in this figure, the potential to stably analyze intense interference between breaking wave and a floating object is one of the advantages of the
particle method. However, in view of both measuring results by the wave gauges and experiment snapshots, it can be said that the improved MPS method has remarkably improved reproducibility on transport of drifting objects. The improved MPS method has reduced pressure disturbance and improved reproducibility on internal distribution of vertical velocity in the head of tsunami. It is presumed that this feature realized the formation of the undular bore.

In the next step, the numerical analysis on the drifting motions of the ice floe was compared with the experiment results. As mentioned before, since the run-up velocity of the bore was overestimated using the calculation model, the time base for this comparison was the elapsed time from the start of the ice floe drift. Figure 7 shows the floating time of the ice floe, the ice floe transport velocity \(U_{ix}\) measured in the experiment, and the translational velocity vector \(T = (T_x, T_y)\) of the rigid body which is considered as a whole ice floe. The top, middle, and bottom graphs show the results of the different dimensional cases of \(L_i=6\, \text{cm and } h_i=1\, \text{cm}, \quad L_i=12\, \text{cm and } h_i=1\, \text{cm}, \quad \text{and } L_i=6\, \text{cm and } h_i=4\, \text{cm} \) respectively. With the incidence of the waveform in the case of discharge \(Q=5.0\, \text{L/s}\) shown in the bottom graph in Figure 5, \(T_x\) sharply rises at the beginning, and then while repeating decrease and increase, it gradually dwindles. Compared to the \(U_{ix}\) obtained from the experiment, the result for \(T_{ix}\) shows overestimation of the whole waveform including the peaks. As observed in Figure 3, it is conceivable that the overestimation of the waveform at Ch.3 caused this overestimating result. On the other hand, the time frame of the peaks and the troughs of the temporal waveform of the velocity were consistent between the experiment and the calculation on the whole.

Conclusively, despite using the rigid body models of simple one-way coupling, the particle method proved its potential for application in simulating transport of floating objects with violent interfacial change of this kind. More precise reproduction in a particle method requires not only study on higher accuracy of the calculation scheme and higher resolution in the computational domain but also modeling of the interaction force at the interface between ice and water.

4. Conclusion and Future Works

This paper presented a basic study focusing on the transport of a drifting ice floe in an undular bore that can be generated by run-up of a so-called L1 tsunami in a river. The obtained findings are as follows.

A hydraulic experiment was conducted using ice models whose specifications were set based on surveyed specifications of actual drifting ice floes in a river tsunami. The results suggest that the existence of ice floes that can appear in actual rivers gives little influence on the transformation of the river-tsunami’s waveform and that large, thin ice floes are more drastically accelerated from the greater influence of undular bore than are small thick ice floes.

Numerical simulation of the hydraulic experiment was conducted using a particle method. The simulation proved to have good reproducibility of the method of undular bore formation and on the transport process of a drifting ice floe carried by the undular bore on the whole.

Based on the analytical results obtained by the particle method, close examination was made on the internal structure of the undular bore that carries the drifting ice floes and on the transport
process of the ice floes. The examination provided some numerical basis for discussion on the transport mechanism of a drifting ice floe implied by the experiment.

The subject of this study was drifting ice floes, which form every year in rivers in cold regions through river freezing. Currently it is hard to take "soft" measures against issues of drifting ice floes. The insights obtained from this study are considered to be also applicable to other types of drifting objects possibly generated by river tsunamis, such as driftwood and ships. It has already been proved that a particle method can realize stable analysis even if a model of some structure, such as bridge, is laid in the computational region, because of the nature of the simulation scheme (Gotoh et al. 2006b). Hereafter, various analyses are intended to be carried out on river tsunamis and on related matters including the ice floe models provided in this study, in order to provide a practical numerical simulator for transport of objects drifting in river tsunamis through collection and summarization of analytical results.

Acknowledgements
The authors would like to express their gratitude to the Kushiro Development and Construction Department at the Hokkaido Regional Development Bureau for its support in providing video imagery data.

References
Photo 1 Tsunami in the Shinkushiro River (KP.5.2, near Kushiro-Shitsugen Bridge, at around 4:05 pm on March 11, 2011).

Figure 1 Dimensions of experimental equipment and positions of wave gauges (side view).

Figure 2 Picture and schematic diagram of ice floe model installation.
Figure 3 Measurement results using wave gauges with the maximum dimensions of ice floe.

Figure 4 Comparison between waveform observed at Ch.1, Ch.2, Ch.3 and time series change of ice floe transport velocity at setting points of ice floe at \(d = 5.0\) m, 10.0 m, and 15.0 m. Case of \(Q = 0.0\) L/s (left) and case of \(Q = 5.0\) L/s (right).
Figure 5 Comparison between measurement results (Exp.) using wave gauges in the hydraulic experiment and calculation results by high-precision particle method.

Figure 6 Qualitative comparisons between experiment and numerical analysis on ice floe transport process in tsunami run-up.
Figure 7 Comparison between experimental results and calculation results on time-series change of ice floe transport velocity (case of $Q = 5.0$ L/s).