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Comparative Testing of Border Ice Growth Prediction Methods

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This paper describes a simple border ice growth equation that was developed using all currently available data regarding border ice growth. The developed equation can reasonably predict border ice growth if model coefficients are calibrated to a particular reach and season. It is not possible to define a global set of coefficients that can be accurately applied to all rivers due to the large variation in flow and climate characteristics. Best results are obtained when both surface water velocity and the number of degree days of freezing are incorporated into the equation. However, the equation performs reasonably well using only the number of degree days of freezing, a quantity that is routinely measured. Generally, comparisons between the new border ice growth equations and more complicated existing models could not be made due to a shortage of data reported in the literature.

Keywords: border ice, river hydraulics, ice hydraulics, empirical modeling, field data

1. Introduction

Border ice growth is one of the least studied river ice formation phenomena. While many authors have speculated on border ice formation characteristics based on their qualitative observations, very little quantitative research has been completed. As a result, only a limited amount of border ice field data has been collected, leaving much uncertainty regarding its influence on the formation of ice covers and on frazil accumulation.

The studies that have been undertaken to date have helped define the prominent variables with respect to border ice growth. However, border ice growth relationships that resulted from these studies are largely empirical and calibrated to a particular river, making the transferability of each model between rivers questionable.

The purpose of this study is to combine all pre-existing knowledge of border ice growth on rivers in order to obtain a better understanding of the factors that influence this phenomenon and ultimately close the gap that exists between border ice growth models. Using all previously available field data, a new border ice growth equation will be developed that is more widely applicable for a variety of rivers with different flow characteristics and climatic regions while requiring only readily available data inputs.

2. Literature Review

As previously mentioned, several border ice growth studies have been undertaken, resulting in the development of three widely accepted border ice models.
The first comprehensive border ice growth study was undertaken by Newbury in 1968. Newbury conducted a field program on the Nelson River in northern Manitoba, and through this research noted that border ice growth was highly influenced by the presence of slush ice adjacent to the border ice cover. In addition, he observed that border ice growth was also influenced by channel geometry and flow conditions. From these observations Newbury proposed the following relationship for border ice growth in cold rivers:

$$W = (mn/2)\sum \Phi \qquad [1]$$

where W is border ice width, n is the number of boundaries from which border ice can grow, Φ is the net heat loss, and m is the adhesion parameter which represents the portion of slush ice that is retained and can be calculated as:

$$m = a/(AS)^b \qquad [2]$$

where A is the area of flow, S is water surface slope and a and b are empirical calibration coefficients which can be obtained through regression analysis.

Newbury's relationship was developed assuming that flow is uniform and the product of area and water surface slope remains relatively constant throughout the border ice growth period.

Michel et al. (1982) conducted a study on the Ste. Anne River in Quebec and agreed with Newbury that concentration and velocity of frazil slush ice were the main variables that influenced border ice growth. However, Michel did not believe that the parameters used in Newbury's model clearly defined the variables that influenced the growth of border ice. Thus, he developed a second relationship to predict incremental border ice growth in cold water:

$$\Delta W = R\Delta\Phi/(\rho L) \qquad [3]$$

where ΔW is the total width of border ice growth, $\Delta\Phi$ is the net heat loss from the water, ρ is the density of the water, L is the latent heat of fusion of ice, and R is a dimensionless growth factor relating growth rate to a given heat loss rate and is given by:

$$R = 14.1N^{1.08}/(V_S/V_C)^{0.93} \qquad [4]$$

where N is the frazil concentration, V_S is the surface velocity of frazil at the border ice edge, V_C is the critical velocity of adhesion (given as 1.2 m/s).

Michel acknowledged the difficulty of directly measuring heat loss from the water body and proposed it could be calculated as:

$$\Delta\Phi = kDD \qquad [5]$$

where k is a heat exchange coefficient and DD is the cumulative degree days of freezing.

Matousek (1984) built on Michel's work by carrying out his own field program on rivers and canals in Czechoslovakia. His research centred on the idea that border ice growth in warm water is a function of water temperature, heat loss rate, and surface velocity, as first noted by Michel (1971). However, Matousek built on Michel's work by also considering the surface effects caused by wind produced phenomena. From this work, Matousek developed a third relationship for border ice growth:

$$V_v = \mathrm{Q}/[1130(-1.1 - \mathrm{T_V})] - \mathrm{dV_W}/1130 \qquad [6]$$

where V_v is the average vertical velocity beyond which ice will not exist, Q is the heat loss rate, T_V is the average vertical water temperature at the ice edge, and V_W is the wind velocity at two meter elevation.

The term d is a factor of channel width and can be calculated as:

$$d = \begin{cases} 15 & ; B \leq 15 \\ -0.9 - 5.8\ln(B) & ; B > 15 \end{cases} \qquad [7]$$

where B is the open water width in the direction of the wind. Since this model was developed assuming thermal growth conditions, it is suitable only for use in warm regions or during the initial ice growth in cold regions.

Perhaps one of the simplest models was developed by Calkins and Gooch (1982) after conducting research on the Ottauquechee River. They concluded that there is a reasonable linear relationship between border ice growth and the number of degree days of freezing for steep shallow rivers. However, when Calkin and Gooch applied the same methodology to Newbury's data from the Nelson River, an empirical power relationship between flow velocity and border ice growth was shown.

Hirayama (1986) studied the formation of ice cover on small steep rivers. The procedure that was used included studying daily photographs to determine the rate of shore ice growth, which was ultimately determined to be a function of the daily air temperature and a condition for the production of frazil ice.

Miles (1993) studied the formation of ice cover on the Burntwood River in northern Manitoba, Canada. In addition to this, Miles was able to apply the Michel, Newbury and Matousek models for the Burntwood River. Miles also developed a further border ice growth equation from the data collected on the Burntwood River:

$$W = [a(1 - PI)^b / V^c] DD \qquad [8]$$

Where W is the growth of border ice, PI is the percentage of the water surface covered by ice, V represents the average water velocity, DD is the cumulative degree days of freezing, and a,b, and c are parameters determined through regression analysis.

Border ice growth data on the Rideau River near Ottawa, Ontario, Canada, has also been collected by Raymond Bourdages (Environment Canada). Although the data for the Rideau River is yet to be published, the data provided was useful for the purposes of this study. The border ice growth data for the Rideau River was obtained using a four-step camera calibration procedure. This calibration process consisted of matching surveyed points to the corresponding pixel location on photographs that were usually taken at an hourly time step. Using complex transformations to account for the distortion within the camera, accurate border ice growth information was obtained for the winter of 2008/2009.

3. Methodology

In order to develop a border ice growth equation that is easy to use and widely applicable to a variety of flow and climate characteristics, all available border ice growth data sets from previous studies were incorporated in the equation development process. These data sets, along with other available information are summarized in Table 1.

The original objective of this study was to apply the existing border ice growth models (Michel, Matousek, and Newbury) to the data sets collected on rivers other than those for which they were developed, with intentions of evaluating which model performed best. However, as shown in Table 1, several key pieces of information were not available from each data set, preventing the application of the different models. As a result, efforts were turned to providing a simpler model in which all data input would be readily available to the user.

Table 1. Available data on border ice growth.

Data Set	Michel	Hirayama	Calkins and Gooch	Miles	Newbury	Bourdages
Year	1981/1982	1985/1986	1980	1984/1985, 1985/1986, 1991/1992	1966/1967	2008/2009
River	Ste. Anne	Yubetsu, Japan	Ottauqueche	Burntwood	Nelson	Rideau
Number of Reaches	6	1	1	3	6	1
Velocity	Surface water velocity reported	Not reported	Not reported	Average velocity reported	1 seasonal average reported per reach	Not reported
Discharge	Not Reported	Reported	Reported	Reported	Not reported	Reported
DD	Reported	Reported	Reported	Not explicitly reported	Partial record	Not explicitly reported
Reach Cross Sections or A	Not Reported	Flow Area	Not reported for 1980	Reported	Not Reported	Reported
River Width	Range given for all reaches	Reported	Reported	Reported	Reported	Reported
N	Reported	Not reported	Not reported	Not reported	Not reported	Not reported
S	Not reported	Not reported	Not reported	Not reported	Reported	Not reported
Φ	Reported	Reported	Not reported	Modeled based on air temp.	Reported	Not reported
V_W	Not reported	Not Reported	Reported	Not reported	Not reported	Not reported

Intuitively, and as supported by previous research, one would expect border ice growth to be related to the number of degree days of freezing and/or water velocity at minimum. Previous research suggests that frazil slush concentration is also an important factor in predicting border ice growth. However, this variable is rarely measured and when it is, the accuracy is usually questionable. Thus, including the frazil slush concentration in a border ice growth equation would greatly diminish the equation's ability to be easily applied in a variety of applications. As such, four simple equations using only degree days of freezing and surface water velocity were selected to calculate the incremental border ice growth:

$$W = a \cdot DD \qquad [9]$$

$$W = a \cdot DD^b \qquad [10]$$

$$W = a \cdot DD^b \cdot V_s \qquad [11]$$

$$W = a \cdot DD^b \cdot {V_s}^c \qquad [12]$$

Where *W* is the border ice growth during a period, *DD* is the cumulative degree days of freezing during a period, V_s is the surface water velocity next to the border ice, and *a*, *b*, and *c* are empirical coefficients determined through optimization.

Where the surface water velocity was not available, a multiplying factor of 1.1 was applied to the average velocity (V_s = 1.1V_{avg}). If neither air temperature or degree days of freezing was available for a particular data set, the nearest Environment Canada meteorological gauge was used to estimate air temperature during the study period.

Each equation was applied to all of the data sets, excluding Calkins and Gooch for Eqn. 11 and Eqn. 12, as velocity data was not available. The coefficients *a, b,* and *c,* were determined by minimizing the root mean square error (RMSE) for each data set. These optimized coefficients for each equation were subsequently applied to their respective data sets to obtain modeled results.

4. Results

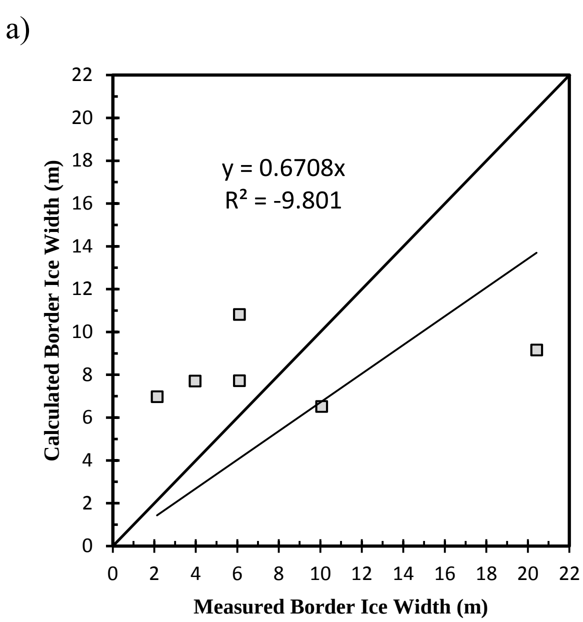
Model correlation was evaluated based on four parameters that were computed for each data set: The Nash-Sutcliffe coefficient, the RMSE, and the slope and R^2 values derived from plotting the measured and modeled data against each other (proportionality plot). These parameters are summarized in Table 2 for all equations and data sets.

The RMSE is an estimator of the difference between observed and modeled data. In this particular application, the RMSE was most useful for comparing different model results for a particular data set, rather than for a comparison between data sets as it is not dimensionless and thus, river size will influence the magnitude of this parameter.

The Nash-Sutcliffe coefficient assesses the predictive power of the model. It can range from $-\infty$ to 1 where a coefficient of 1 corresponds to a perfect match between modeled and observed data, a coefficient of 0 indicates that the model predictions are as accurate as the mean of the observed data, and a coefficient less than 0 occurs when the observed mean is a better predictor than the model.

Table 2. Correlation parameters for Eqn. 9 through Eqn. 12.

Data Set	Equation 9				Equation 10				Equation 11				Equation 12			
	RMSE	Nr	Slope	R^2	RMSE	Nr	Slope	R^2	RMSE	Nr	Slope	R^2	RMSE	Nr	Slope	R^2
Michel 1	0.280	0.105	0.856	-	0.280	0.106	0.856	-	0.280	0.105	0.856	-	0.278	0.113	0.857	-
Michel 2	1.042	0.000	0.971	-	1.042	0.000	0.971	-	1.137	-0.191	0.965	-	5.997	1.000	1.000	1.000
Michel 3	0.566	0.576	0.872	0.536	0.551	0.598	0.879	0.405	1.029	-0.401	0.578	-	0.295	0.885	0.966	0.883
Michel 4	0.099	0.994	0.997	0.994	0.066	0.997	0.999	0.997	0.073	0.997	0.998	0.997	0.035	0.999	1.000	0.999
Michel 5	0.959	0.571	0.793	-	0.447	0.907	0.953	0.901	0.574	0.846	0.926	0.858	0.447	0.907	0.955	0.901
Michel 6	1.336	0.261	0.621	-	1.335	0.262	0.621	-	1.630	-0.100	0.436	-	1.056	0.538	0.757	0.484
Hirayama	1.647	0.776	0.953	0.650	1.639	0.778	0.953	0.683	2.030	0.660	0.929	0.571	1.349	0.850	0.969	0.798
Calkins & Gooch	1.958	0.928	0.980	0.902	1.840	0.936	0.982	0.927	-	-	-	-	-	-	-	-
Miles 1	8.409	0.922	0.962	0.887	3.233	0.989	0.992	0.989	4.675	0.976	0.986	0.977	3.205	0.989	0.995	0.989
Miles 2	20.428	0.519	0.738	0.147	19.727	0.544	0.756	0.452	22.312	0.426	0.687	0.425	18.032	0.625	0.796	0.526
Miles 3	15.417	0.687	0.864	0.101	9.273	0.887	0.945	0.871	12.805	0.784	0.900	0.776	7.838	0.919	0.954	0.922
Newbury 1	52.532	-0.836	0.316	-	33.028	0.274	0.730	-	33.028	0.274	0.730	-	33.028	0.274	0.729	-
Newbury 2	21.614	-1.325	0.569	-	14.171	0.000	0.815	-	14.171	0.000	0.815	-	14.171	0.000	0.815	-
Newbury 3	6.120	0.890	0.890	0.464	5.284	0.918	0.918	0.108	5.284	0.918	0.918	0.108	5.284	0.918	0.918	0.108
Newbury 4	12.549	-0.482	0.450	-	10.305	0.001	0.629	-	10.305	0.001	0.629	-	10.305	0.001	0.629	-
Newbury 5	2.574	0.855	0.949	0.827	2.352	0.879	0.957	0.891	2.352	0.879	0.957	0.891	2.352	0.879	0.957	0.891
Newbury 6	8.809	0.240	0.240	-	5.799	0.671	0.671	-	5.799	0.671	0.671	-	5.799	0.671	0.671	-
Bourdages	1.513	0.978	0.985	0.976	0.887	0.992	0.994	0.993	1.727	0.971	0.976	0.971	0.779	0.994	0.995	0.994



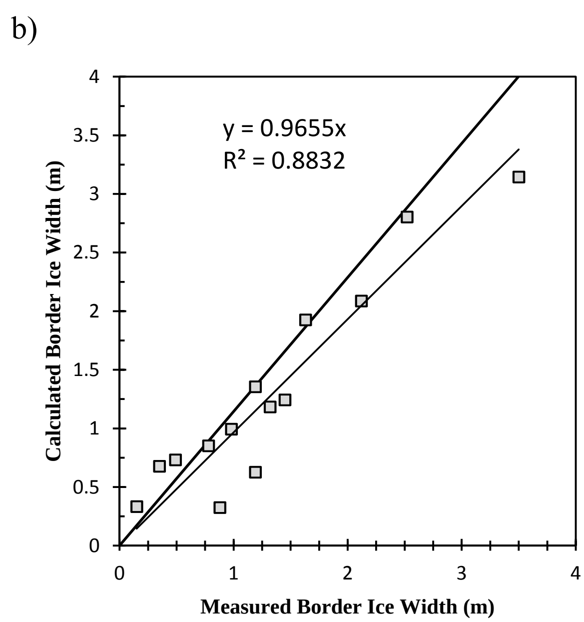
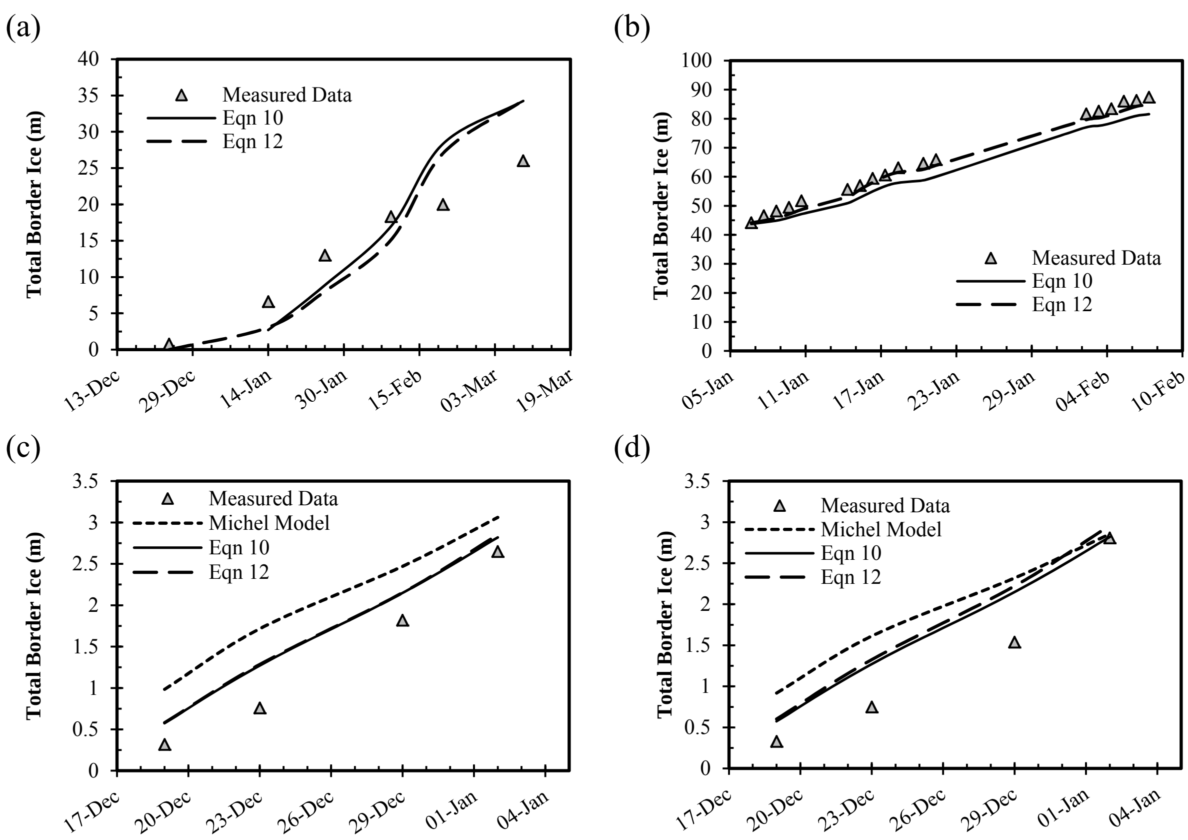


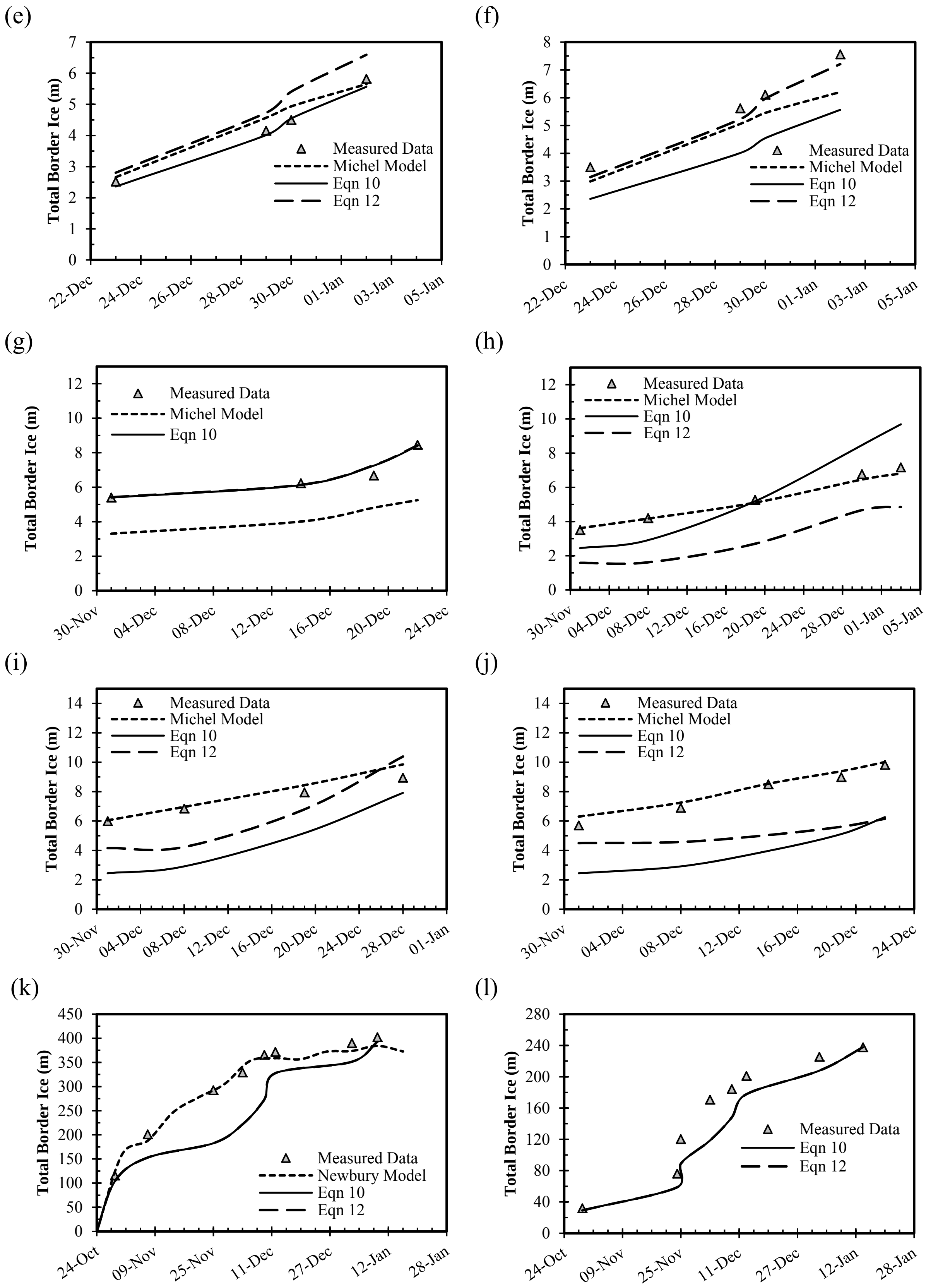
Figure 1. Proportionality plots for a) Newbury 6 data set b) Michel 3 data set.

Proportionality plots were also used to determine how well modelled border ice widths correlated to the observed border ice widths and to determine any model bias or trends in model error. The R^2 value of the data was determined via a linear trend line that was forced to pass through the origin. Where R^2 values are not reported in Table 1 they had negative values, indicating that a horizontal line is a better fit to the data than a trend line forced through the origin. Further, the slope of the trend line was measured for means of comparison with the 1:1 proportionality line. Two selected proportionality plots for Eqn. 12 are shown in Figure 1.

As depicted by Figure 1, there was a large range in model performance. Further, it was found the apparent correlation in a proportionality plot did not necessarily dictate an equation's ability to predict border ice growth trends over the season. Thus, due to the previously discussed shortcomings of using the RMSE and proportionality plots to assess model error, the Nash-Sutcliffe coefficient was judged the most reliable and objective means of quantifying model performance.

As a result, Eqn. 12 was deemed to perform the best over most data sets, as determined by the above-mentioned correlation parameters. As shown by Figure 2, nearly all data sets can be reasonably reproduced using optimized coefficients *a*, *b*, and *c* in Eqn. 12. However, the implication of using Eqn. 12 to predict border ice growth is that it requires one to have knowledge of the degree days of freezing and the surface water velocity. However, the latter is not often measured and would require extra resources to do so. Thus, one might choose to use Eqn. 10, as it requires only the number of degree days of freezing and can usually still provide a reasonable estimate of border ice growth (shown in Figure 2).




(e)
Total Border Ice (m)
Measured Data
Michel Model
Eqn 10
Eqn 12
22-Dec
24-Dec
26-Dec
28-Dec
30-Dec
01-Jan
03-Jan
05-Jan
(f)
Total Border Ice (m)
Measured Data
Michel Model
Eqn 10
Eqn 12
22-Dec
24-Dec
26-Dec
28-Dec
30-Dec
01-Jan
03-Jan
05-Jan
(g)
Total Border Ice (m)
Measured Data
Michel Model
Eqn 10
30-Nov
04-Dec
08-Dec
12-Dec
16-Dec
20-Dec
24-Dec
(h)
Total Border Ice (m)
Measured Data
Michel Model
Eqn 10
Eqn 12
30-Nov
04-Dec
08-Dec
12-Dec
16-Dec
20-Dec
24-Dec
28-Dec
01-Jan
05-Jan
(i)
Total Border Ice (m)
Measured Data
Michel Model
Eqn 10
Eqn 12
30-Nov
04-Dec
08-Dec
12-Dec
16-Dec
20-Dec
24-Dec
28-Dec
01-Jan
(j)
Total Border Ice (m)
Measured Data
Michel Model
Eqn 10
Eqn 12
30-Nov
04-Dec
08-Dec
12-Dec
16-Dec
20-Dec
24-Dec
(k)
Total Border Ice (m)
Measured Data
Newbury Model
Eqn 10
Eqn 12
24-Oct
09-Nov
25-Nov
11-Dec
27-Dec
12-Jan
28-Jan
(l)
Total Border Ice (m)
Measured Data
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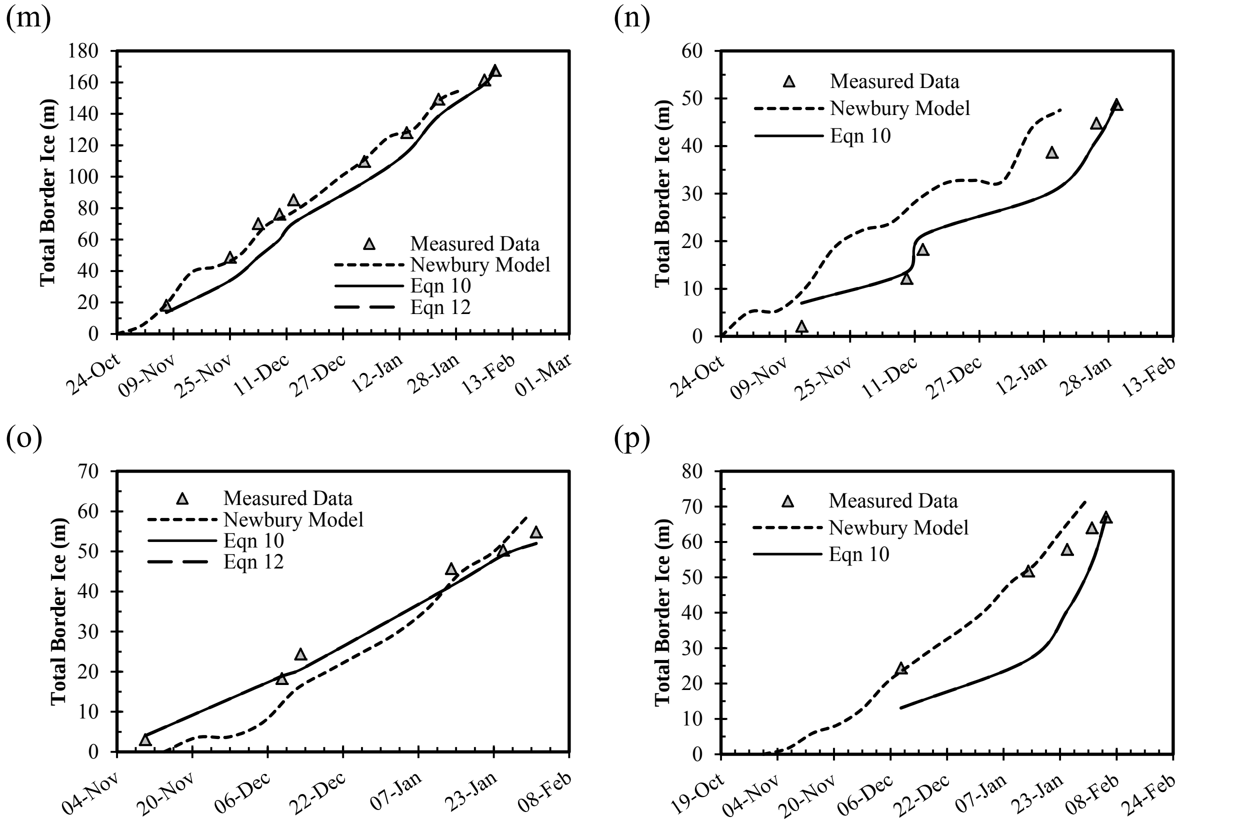

Figure 2: Model results using Eqn. 10 and Eqn. 12 for a) Hirayama data set, b) Bourdages data set, c) Michel 1, sub-reach 4 data set, d) Michel 1, sub-reach 5 data set, e) Michel 3, sub-reach 3 data set, f) Michel 3, sub-reach 4 data, g) Michel 5, sub-reach 3 data set, h) Michel 6, sub-reach 1 data set, i) Michel 6, sub-reach 2 data set, j) Michel 6, sub-reach 3 data set, k) Newbury 1 data set, l) Newbury 2 data set, m) Newbury 3 data set, n) Newbury 4 data set, o) Newbury 5 data set, p) Newbury 6 data set.

For those data sets in which the researcher has developed his own model to fit the data (Michel and Newbury), the previously developed model has also been plotted for comparison. It was not possible to apply these models to data sets besides the one for which they were derived due to a shortage of data reported in the literature.

Figure 2 also reinforces the conjecture that the proportionality plots were not the best indicator of model performance. Although the Newbury 6 data set seems to have a weak correlation between observed and modeled data (shown in Figure 1), when plotted as a time series, it is clear that both Eqn. 10 and Eqn. 12 can reasonably predict border ice growth over the season. This is consistent for most other data sets that had low (or negative) R^2 values.

While there is generally good agreement between observed and modeled data, the practicality of equation 12 is severely inhibited by the need to calibrate the coefficients *a*, *b*, and *c* on a reach by reach basis. Thus, efforts to develop a variation of Eqn. 12 with a global coefficient set were undertaken. However, the range of *a* values obtained in this study was on the order of 10^7

making it very difficult to recommend a global set of coefficients that would be suitable for all rivers.

Another factor that largely governs the practicality of using Eqn. 10 or Eqn. 12 to predict border ice growth is the ability to use the same calibrated coefficients for a particular reach from season to season. Within the available data, Miles was the only researcher who has collected data over more than one year at the same location.

Miles has recorded border ice growth in three reaches over three years: 1984/85, 1985/86, and 1991/92. In order to assess whether Eqn. 10 or Eqn. 12 can be used in a predictive manner, coefficients *a*, *b*, and *c* were optimized for each reach using only the 1991/92 data, since this was the season that contained the most observed data points. Once these coefficients were established, Eqn. 10 and Eqn. 12 were applied to the data from the remaining years without further optimization of the coefficients. The modeled border ice growth for select seasons and reaches is shown in Figure 3 and statistical parameters are reported in Table 3.

Table 3. Correlation parameters for calibration and validation of Eqn. 12 on the Burntwood River.

Data Set	1991/1992							1984/1985		1985/1986	
	Equation 10			Equation 12				Eqn 10	Eqn 12	Eqn 10	Eqn 12
	a	b	Nr	a	b	c	Nr	Nr	Nr	Nr	Nr
Miles 1	3.778E-06	2.419	0.912	2.491E-04	1.891	0.908	0.991	-	-	0.757	0.938
Miles 2	2.261E-01	0.765	0.279	3.540E-01	0.607	-1.319	0.604	0.274	0.568	0.669	0.750
Miles 3	2.686E-06	2.471	0.989	2.265E-05	2.083	-2.685	0.997	0.178	-3.351	0.817	0.622

Figure 3 indicates that attempting to validate reach and seasonally calibrated coefficients with a second data set in the same reach resulted in significant model error. Often the first measurement of the season is severely over or underestimated, causing a large offset in the border ice growth trend. It is possible that this offset could be accounted for within the model, but without further validation data this is only speculative. Thus, it is evident that while Eqn. 10 and Eqn. 12 have been proven to reasonably model border ice growth in a time and reach specific application, it is unlikely that either model can be used in a predictive mode.

Miles (1993) was able to apply the Michel, Matousek, and Newbury models to the data collected on the Burntwood River. All three models showed good agreement with the measured data over all three seasons. Since the Michel, Matousek, and Newbury models were able to predict border ice growth well without yearly calibration, Miles' results suggest that accurately predicting border ice growth requires the consideration of additional variables other than only degree days of freezing and surface water velocity.

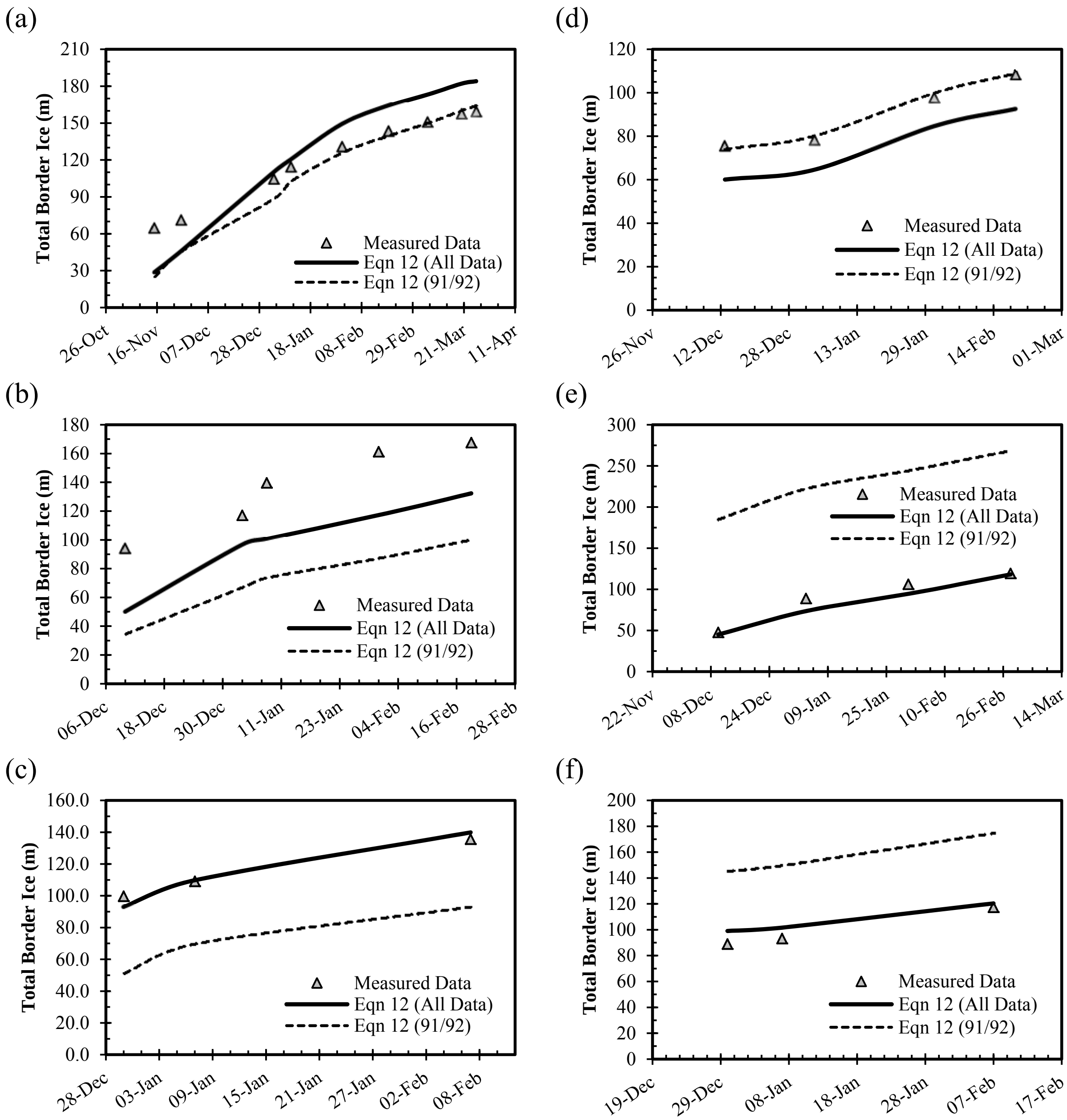

Figure 3: Calibration and validation results using Eqn. 12 for a) Miles 2 (91/92) data set. b) Miles 2 (84/85) data set. c) Miles 2 (85/86) data set. d) Miles 3 (91/92) data set. e) Miles 3 (84/85) data set. f) Miles 3 (85/86) data set.

5. Conclusion

Border ice growth is still a phenomenon that is not well understood. Relationships have been developed to predict border ice growth, but they are too complicated and require too many obscure variables to be deemed useful. It is important that a border ice growth model be simple enough to use without undertaking a vigorous field program to collect data. Installing instrumentation to measure complex variables is costly, time consuming and impractical, especially in remote locations. Thus, the need for a border ice growth model that requires only readily available data is evident.

In this study, simple relationships were developed to predict border ice growth. It was found that a simple equation requiring only the number of degree days of freezing could reasonably model border ice growth if model coefficients were calibrated on a reach and season specific basis. In addition, if surface water velocity is known, the model can be revised to incorporate this parameter and considerably improve model performance.

However, it is not possible to derive a set of equation coefficients that can be used to reasonably model border ice growth in all rivers or even all reaches in a single river due to differences in flow and climate characteristics.

In addition, equation coefficients that have been optimized for a particular reach were unable to be validated in the same reach during an alternate time period without significant model error. This indicates that a border ice growth model requires additional parameters if it is to be applied in a predictive manner. This conjecture is supported by the literature, as most of the existing border ice growth models (Michel, Newbury, Matousek) require further model parameters including frazil slush concentration, water surface slope, heat loss, wind velocity, and water temperature. Currently, these parameters are not regularly measured which is a major drawback of the presently available border ice growth models. Initiating field programs to start measuring these variables would be a good first step to improve the understanding of border ice growth.

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List of Symbols

A	Area of flow
a, b, c	Empirical calibration coefficients
B	Open water width
d	Factor of channel width
DD	Cumulative degree days of freezing
k	Heat exchange coefficient
L	Latent heat of fusion of ice
m	Adhesion parameter
n	Number of boundaries from which border ice can grow
N	Frazil concentration
PI	Percentage of water surface covered by ice
Q	Heat loss rate
R	Dimensionless growth factor relating growth rate of ice to a given heat loss rate
S	Water surface slope
T_V	Average vertical water temperature at the ice edge
V	Average water velocity
V_C	Critical velocity of ice adhesion
V_S	Surface Velocity
V_V	Average vertical velocity beyond which ice will not exists
V_W	Wind velocity two meters above the water surface
W	Border ice width
ρ	Density of water
Φ	Net heat loss