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Alternative Method for Synthetic Frequency Analysis of Breakup-Jam Floods

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River ice jams can cause extreme flood events with major socio-economic and ecological impacts. A major practical question is how to quantify and assess ice-jam flood risk. Ideally, this question is answered via historical ice-influenced water level peaks. More commonly, however, the available historical information is scarce and determination of ice-jam flood frequencies or probabilities must rely on a synthetic method. After noting that empirical evidence does not support the assumption of discrete stage outcomes, which is central to the existing methodology, an alternative synthetic method is developed for breakup ice jams. It recognizes that the peak breakup stage can take on any value between discharge-dependent envelopes, and is therefore termed the distributed-function method or DFM for short. A key element of the DFM is the conditional probability of stage, given a value of the breakup discharge. This probability is first expressed in terms of a dimensionless parameter that involves not only stage, but also maximum and minimum values that apply to the particular discharge being considered. It is then quantified empirically, assuming a similarity property as a working hypothesis, which is eventually substantiated by successful application to three case studies. The existing, discrete-function, approach is also applied to enable a comparison of the results of the two methods. Practical limitations are outlined, and future research avenues identified.

1. Introduction

Ice jam floods can be devastating in terms of social disruption (e.g. mass evacuation, loss of human life) and damage to property and infrastructure (e.g. homes, buildings, ships, bridges, roads, railway lines). They also have major ecological impacts, which can be both beneficial (e.g. replenishment of floodplain ecosystems with river water and sediment) and detrimental (e.g. fish mortality, loss of spawning grounds). Evaluating the risk of ice-jam flooding is an essential step in regulating floodplain development, identifying effective ice-jam mitigation measures, and assessing the economic and ecological impacts of regulation or de-regulation, and of climate change.

As with open-water flooding, the risk at a particular location is quantified by developing a stage-frequency (or stage-probability) relationship. However, ice-influenced flood events are not readily amenable to traditional stage-frequency analyses. The complex hydro- meteorological and structural processes that lead to ice jam formation, progression, and release are highly site-specific. Therefore, parametric regional equations such as those developed for open-water floods do not apply. Moreover, gaps in historical records are much more pronounced for ice-related events because hydrometric gauges are often damaged by ice, usually when an ice jam forms nearby. Not only does this shorten the stage record, but the missing data include those associated with extreme events. Often, flood risk assessment is required at sites where there are few or no historical data on past ice-influenced peaks. In such instances, the only alternative is to develop synthetic flood-frequency relationships (Gerard, 1989; FEMA, 1995; USACE, 2002; White and Beltaos, 2008).

The main objective of this paper is to present a new method for generating synthetic relationships, based on the empirical observation that any stage between flow-dependent lower and upper bounds is possible. The current approach for synthetic frequency analysis, herein called the discrete function approach, is outlined in the next section, where associated assumptions are evaluated in the context of available data and understanding of ice jam processes. A distributed function method (DFM) is subsequently introduced and mathematical expressions are developed to calculate ice jam flood stage probability. Case studies illustrating the efficacy of the DFM are presented, followed by a comparative application of the discrete-function approach. This is followed by a discussion of the physical significance of DFM parameters, as well as of limitations and future research avenues.

2. Discrete-function approach

Where the available historical stage data are sparse, it may be possible to generate the stage-frequency relationship indirectly (Gerard, 1989). Historical flow data are far more readily available than peak ice-influenced stages because the spatial variability of flow is much smaller than that of ice-jam water levels. As a result, the flow at an ungauged site may often be deduced from records of upstream and downstream gauges, or even from regional estimates. By contrast, ice-jam stage data cannot be meaningfully transposed from other sites. Once the frequency of peak ice-influenced flows is established, the frequency of corresponding stages can be determined by introducing the empirically assessed local probability of jam occurrence in any one year, P(J).

According to Gerard and Calkins (1984), if a jam does form, the peak breakup stage H_m can be assumed to be equal to the stage caused by an equilibrium jam that fully affects the site of interest (Fig. 1) at the prevailing discharge, Q_m. If a jam does not form, H_m is assumed to be equal to the stage associated with flow under a sheet-ice cover. The probability of H_m is then calculated using Fig. 1 in conjunction with the separately-established probability of the discharge, and the probabilities of jam occurrence, P(J), and non-occurrence, P(NJ) = 1-P(J). This approach is herein termed the “discrete-function approach” because at any given flow, H_m can only take one of two possible values (Fig. 1).

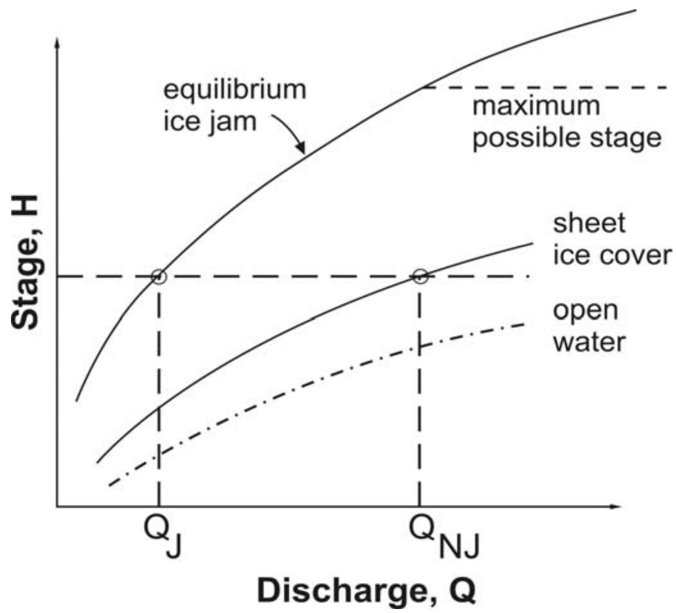

Fig. 1. Schematic illustration of discharge-dependent upper and lower envelopes of peak breakup stages.

The “rating curves” of Fig. 1 can be improved by consideration of additional physical constraints (Gerard and Calkins, 1984; Tuthill et al., 1996). For instance, careful inspection of the site may reveal extensive floodplains where ice and water can escape, thus limiting H_m to a constant, rather than flow-dependent, value (dashed line segment on right upper corner of graph in Fig. 1). The same applies where an ice-clearing discharge can be determined, such that no jam can remain in place beyond this value. Tuthill et al (1996) also considered the minimum flow that can actually mobilize and break up a sheet ice cover [P(J) = 0 below this threshold] and the maximum flow above which a sheet-ice cover cannot remain in place (H_m reverts to the open-water stage). The possibility of such refinements should be kept in mind when conducting in situ reconnaissance and resident interviews.

3. Distributed function concept

Contrary to the discrete-function concept, plots of observed peak breakup stage versus prevailing discharge do not exhibit clustering of data points about upper and lower envelopes. Rather, the data points fill the area enclosed by these curves, as illustrated in Fig. 2. The upper and lower envelopes in that figure were determined by equilibrium-jam and sheet-ice cover calculations

(Beltaos et al., 2006), and slightly adjusted to ensure that they adequately envelop the data points. Bathymetric data and detailed site inspection indicated that the maximum possible stage (MPS) at this site is about 222 m (amsl).

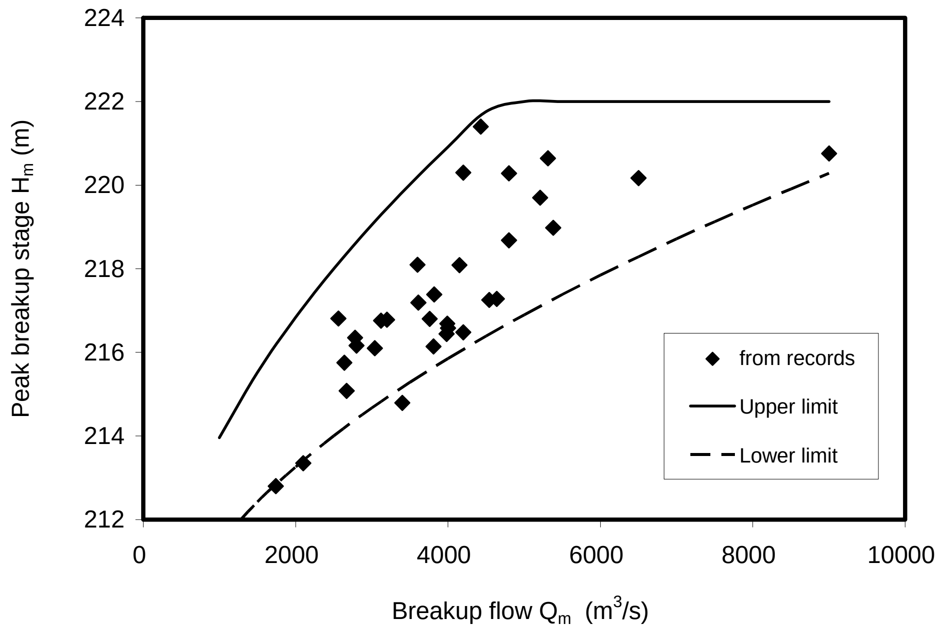

Fig. 2. Plot of H_m verus Q_m, along with upper and lower envelopes, Peace River at Peace Point. Partially based on Water Survey of Canada archived data; http://www.wsc.ec.gc.ca/applications/H2O/index-eng.cfm; accessed Jan. 17, 2011.

The type of plot illustrated in Fig. 2 is very common, while the writer has never encountered a discrete-type graph. Experience, therefore, suggests that H_m is a distributed function that can take on any value between upper and lower envelopes, respectively denoted by H_{max} and H_{min}. Current understanding of breakup and jamming processes fully explains this finding, as is illustrated in Fig. 3. It is well known that ice jams do not form at exactly the same location each year. Consequently, their backwater effect on the water level at the site of interest (points 1-4 in Fig. 3) will vary from nil to full (i.e. H_m will vary from H_{min} to H_{max}). At point 4, the effect will be manifested in the form of the wave that follows jam release.

The "distributed-function method" (DFM) begins by recognizing that all values between H_{min} and H_{max} are possible. For any given discharge (Q_m), these values will then conform to a continuous cumulative probability distribution, such that $P(H_m < H$ for the given $Q_m)$ varies between 0 and 1 as H varies from H_{min} to H_{max}. For $H < H_{min}$, $P(H_m < H) = 0$ and for $H > H_{max}$, $P(H_m < H) = 1$.

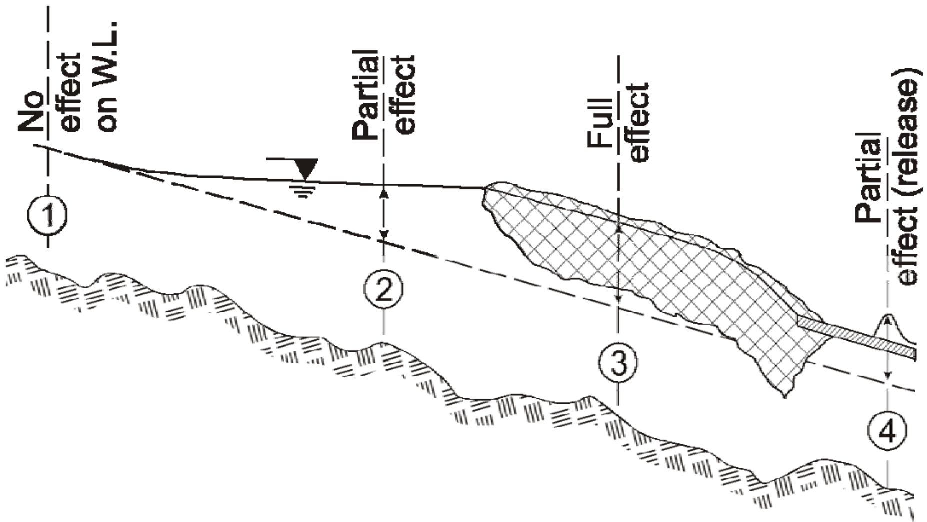

Fig. 3. Upstream and downstream effects of an equilibrium ice jam on peak breakup stage.

With this background, the DFM can be formulated as follows:

$$P\{(H_m<H)/(Q<Q_m<Q+dQ)\} = \frac{P\{(H_m<H)\cap(Q<Q_m<Q+dQ)\}}{P(Q<Q_m<Q+dQ)} \quad [1]$$

The denominator of the right-hand-side (RHS) of Eq. 1 is equal to the probability density function of the breakup flow, [p(Q)], times the interval dQ:

$$P(Q<Q_m<Q+dQ) = p(Q)dQ=\frac{dP_Q}{dQ}dQ \quad [2]$$

where $P_Q = P(Q_m < Q)$. Introducing the dimensionless variable $\eta = (H-H_{min})/(H_{max}-H_{min})$, one can express the conditional probability on the LHS of Eq. 1 as:

$$P\{(H_m< H)/(Q <Q_m< Q+dQ)\} = P(\eta_m< \eta) \equiv \phi(\eta) \quad [3]$$

$P(\eta_m < \eta)$, or $\phi(\eta)$ for short, varies from 0 to 1 as η varies from 0 to 1; moreover, $\phi = 0$ for negative values of η while $\phi = 1$ for $\eta > 1$.

With these results, P_i [defined as $P(H_m < H)$] can be obtained as:

$$P_i \equiv P(H_m< H)=\int_0^1\phi(\eta)dP_Q \quad [4]$$

This relationship shows that P_i is governed by two probability distributions: (a) the probability distribution of the flow Q_m; and (b) the probability distribution of the dimensionless stage variable, η.

In general, the function ϕ could change with changing discharge. However, the fact that $\phi(0) = 0$ and $\phi(1) = 1$ regardless of discharge, suggests the possibility of similarity, i.e. that $\phi(\eta)$ is independent of Q_m. Herein, similarity or approximate similarity is assumed as a working hypothesis; it cannot be tested directly, because there are too few data points within any small interval of Q_m (Fig. 2) to permit calculation of respective probabilities. However, if similarity is assumed to apply, ϕ can be evaluated using all data points. If the resulting P_i distribution (Eq. 4) agrees with that of the historical peaks, then the similarity hypothesis can be considered sound.

With reference to Fig. 2, any plotted pair (Q_m, H_m) implies a value of η_m [use the corresponding H_{max} and H_{min} to calculate η_m as $(H_m-H_{min})/(H_{max}-H_{min})$]. This operation will generate as many η-values as there are years of record. These can be ranked and respective probabilities assigned via a plotting-position formula [rank/(number of years + 1) is used herein].

4. Data sources

The DFM has been tested in three case studies, involving the Restigouche River in E. Canada and the Peace River in N.W. Canada. Historical data derive from Water Survey of Canada (WSC) records of the following hydrometric gauges: Restigouche River above Rafting Ground Brook (1968-2008); Peace River at Peace River (1972-2009); and Peace River at Peace Point (1972-2009). Basin area and channel width range from 7,740 km^2 and 160 m (Restigouche) to 293,000 km^2 and 650 m (Peace). The flow regimes of the Peace River sites are affected by regulation, resulting from the construction of the Bennett Dam in British Columbia.

Flow data (daily mean values) are available online. On occasion, daily mean water levels may also be found online, but instantaneous stage peaks cannot be reliably obtained from daily values. However, WSC archives contain much more detailed information (e.g. recorder charts for older data; 15-minute digital records after ~ 1995), which is made available on request.

5. Results of the distributed function method

A key element of the DFM is the function $\phi(\eta)$ and its assumed non-dependence on discharge (similarity property). For Peace River at Peace Point (Fig. 2), this function is illustrated in Fig. 4. The solid line represents a second degree polynomial of the form:

$$\phi(\eta) = (k+1)\eta - k\eta^2 \qquad [5]$$

in which the coefficient k = 0.70. The form of Eq. 5 satisfies the end conditions $\phi(0) = 0$ and $\phi(1) = 1$. Similar results were obtained for the other two sites, but the values of k were different. As will be shown later, the difference is linked to how prone a site is to jamming. Once the function ϕ is at hand, the probability P_i can be determined by means of Eq. 4, which has been conveniently programmed in an Excel spreadsheet, as illustrated in Table 1.

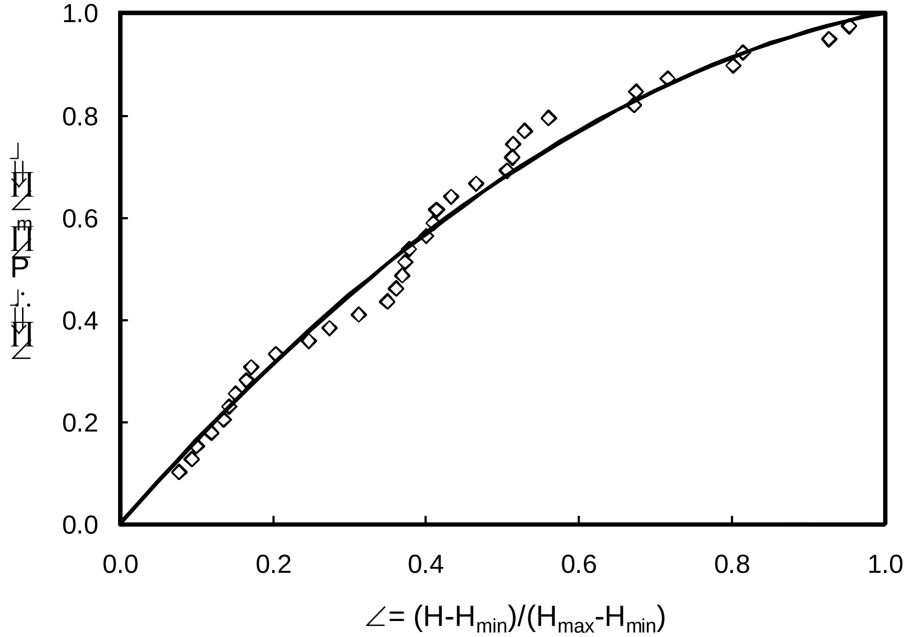

Fig. 4. Dimensionless plot of distributed function, Peace River at Peace Point.
The solid line is based on Eq. 5 with k = 0.70.

Table 1. Calculation of $P_i = P(H_m < H)$ for any given value of H, using Eq. 4.

Q	P_Q	dP_Q	H_{min}	H_{max}	η	$\phi(\eta)$; from Eq. 13	ϕdP_Q
Q_1 $=Q_{Start}$ [1]	P_{Q1} (= 0)	NA	…	…	Varies from 0 to 1	ϕ_1	NA
$Q_2 =$ $Q_1 + dQ$	P_{Q2}	$dP_{Q2} = P_{Q2} - P_{Q1}$		…	…	ϕ_2	$\frac{\phi_1 + \phi_2}{2} dP_{Q2}$
…	…	…	…	…	…	…	…
Q_{End} [2]	1	…	…	…	…	…	…
$\int_0^1 \phi(\eta) dP_Q \approx$ sum of values in the cells of the last column							$P_i = \Sigma\phi dP_Q$

(1) small value, certain to be exceeded at breakup; could be zero
(2) large value, certain to not be equaled or exceeded at breakup

Any one value of H_m is treated as a constant when calculating the integral (Eq. 4) over the entire ranges of Q_m and P_Q. Values of P_Q are obtained graphically using the available historical discharge data (e.g. Fig. 5). Upon changing the value of H_m, a new P_i is instantly computed by Excel. A sufficient number of pairs (H_m, P_i) are then generated to define the probability distribution of H_m.

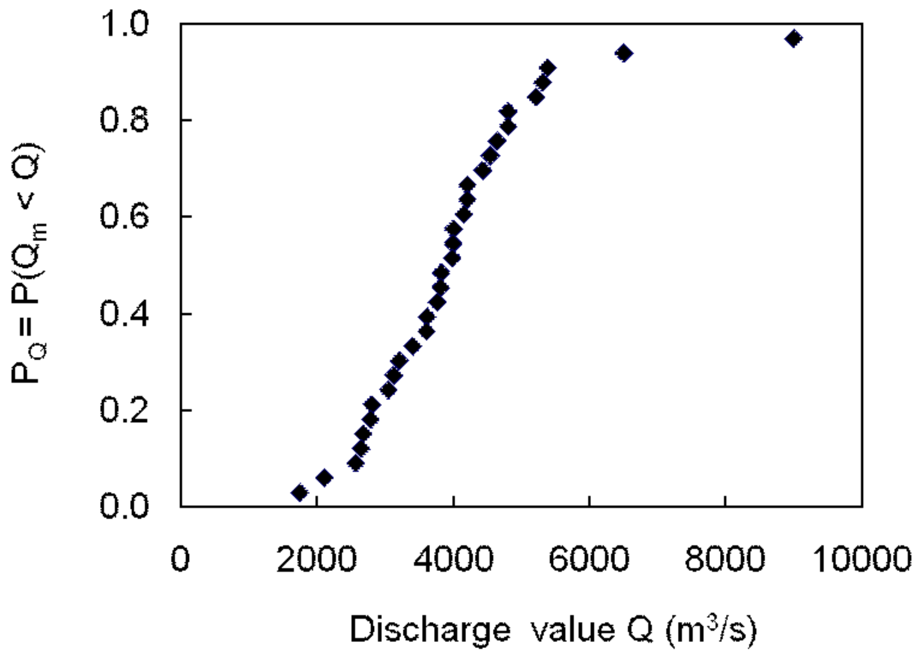

Fig 5. Cumulative probability distribution of breakup discharge, Peace River at Peace Point. Based on Water Survey of Canada archived data.

For Peace River at Peace Point, the synthetic probability distribution of H_m is compared in Fig. 6 to the distribution that derives directly from the historical record. It is seen that the DFM produces a reasonable description of the data points. Similar results were obtained for the other two sites. The effectiveness of the DFM in generating synthetic probabilities supports the working hypothesis of similarity with respect to the function $\phi(\eta)$. Had this hypothesis been off the mark, the synthetic predictions would not have agreed with the historically-derived probabilities.

The maximum possible stage (MPS) was estimated on the basis of local morphology and historical data. As a check, calculations were also carried out assuming that there is no MPS. This revealed significant differences near the upper end of the probability curve in two of the three cases (Peace River sites). The error was on the conservative side (as expected), but could result in costly decisions via assigning finite probabilities of exceedance (e.g. 1 or 2 percent) to near-impossible events. This finding underscores the importance of careful field inspections and well-designed data collection programs.

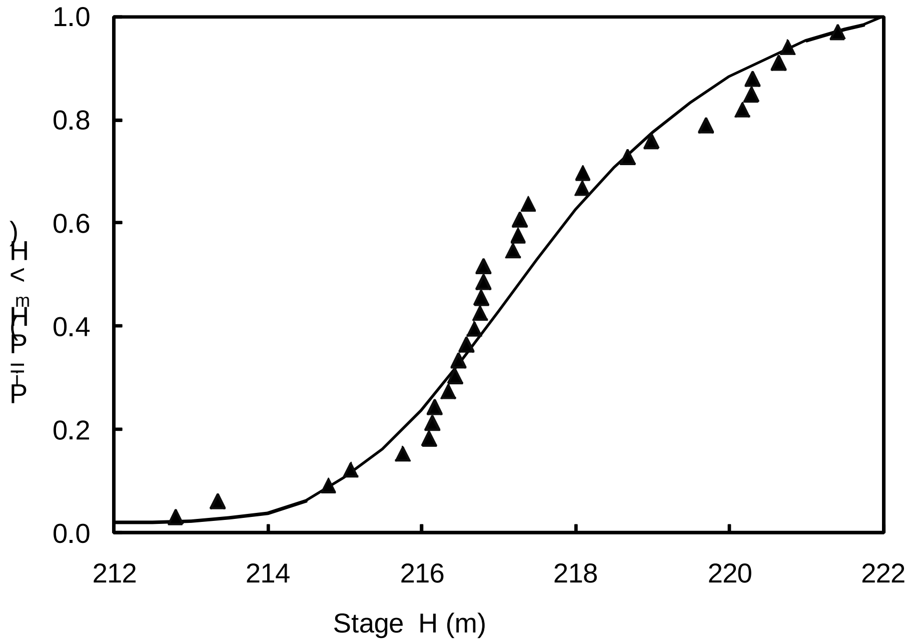

Fig. 6. Probability distribution of maximum breakup stage, Peace River at Peace Point. Data points: from historical station records; solid line: from DFM.

6. Discrete-function results

Normally, the synthetic approach is applied where there are no historical stage data that could be used to generate the stage-probability relationship directly. Consequently, there are very few instances in which a synthetic relationship has been actually compared with existing historical information (e.g. Gerard and Calkins, 1984; Gerard, 1989; Tuthill et al., 1996). As all of the requisite data are available in the case studies discussed here, it is of interest to apply the discrete-function approach. The results of such calculations are illustrated in Fig. 7 for different assumed values of the probability of jamming, P(J). It appears that no synthetic line, regardless of P(J), can provide a satisfactory description of the data points; similar results were obtained for the other two case studies. Interestingly, the probabilities of the higher stages are better matched with low values of P(J) and vice versa. This is in accord with the concept of ice clearing discharge and with a related reduction in the probability of jamming with increasing breakup flow (Grover et al, 1999; Beltaos, 2010).

7. Discussion

The preceding results lend considerable support to the DFM and the assumption of similarity regarding the function $\phi(\eta)$, but the physical significance of the coefficient k has not yet been demonstrated. For two sites of different susceptibility to jamming, the higher exceedance probabilities would apply to the site that is more ice-jam prone. Since the probability of exceedance is equal to $1-\phi(\eta)$, it can be expected that ϕ should lie closer to the η-axis at sites that

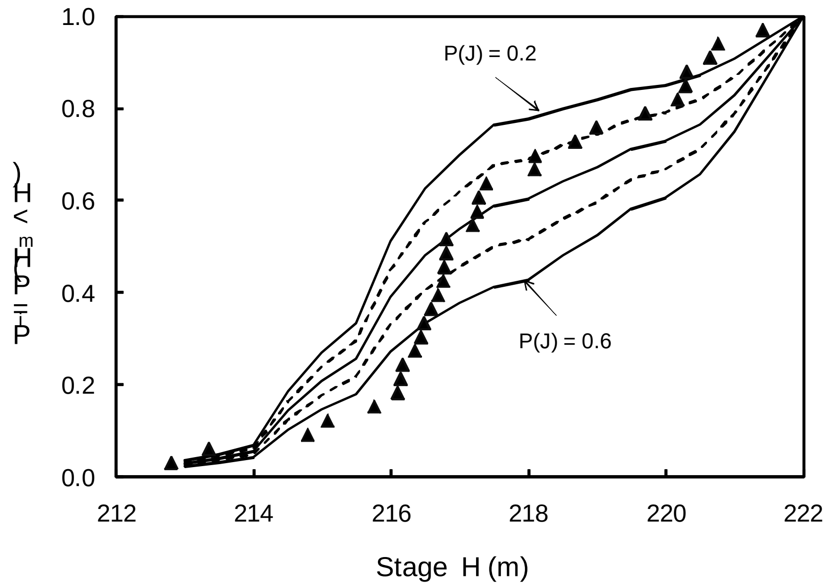

Fig. 7. Comparing discrete-function probability distributions with historical data. Peace River at Peace Point. Data points: from records; continuous lines: calculated for P(J) = 0.2 to 0.6.

are more prone to jamming. With reference to Eq. 5, it can be shown that this trend is represented by a decreasing value of k. Consequently, k may be viewed as an inverse index of susceptibility to jamming. This is supported by the data of Table 2 where it can be seen that the more susceptible sites are associated with the lower values of k.

Table 2. Optimal value of the coefficient k and susceptibility to jamming.

River and location	Ranking of susceptibility to jamming, based on historical observations	Value of k	Comments
Peace R. at Peace Point	2	0.7	Major dam upstream, significant regulation effects on flow regime
Peace R. at Peace River	3	0.9	Major dam upstream, large regulation effects on flow regime
Restigouche R. above Rafting Ground Brook	1	0.6	Natural flow regime; breakup often triggered by rainfall

The primary limitation of the DFM (and of any synthetic analysis method) lies in the selection of the breakup discharge, Q_m, which has been defined as the flow value at the time of the peak breakup stage, H_m. In practical applications, daily flow data will be available, but few, if any, stage data. Consequently, it will be difficult to pinpoint the time of H_m in any one year of the record and thence select the correct value of Q_m. At best, one could identify a time period during which the breakup event would have taken place and select the highest flow for this period, Q_{max}, which may or may not be equal to Q_m. The effect on the synthetic probability distribution would then depend on the relative magnitudes of Q_m and Q_{max}. Example calculations suggest that the error in estimated probabilities is always on the conservative side and diminishes as the range of rare events (exceedance probability = a few %) is approached.

A secondary source of error is the lack of a unique sheet-ice cover rating curve, owing to interannual variations in ice cover thickness and roughness. This is sometimes evinced by data points that plot beneath the computed sheet-ice curve in such graphs as Fig. 2. A check on the Restigouche River data set revealed that associated errors in calculated probabilities are small and conservative, while being negligible in the range of rare events. Similar considerations apply to the ice-jam rating curve which is based on equilibrium-jam calculations. In the writer's experience, such curves are rarely exceeded by historical data points. However, exceedance would result in non-conservative errors; this should be taken into account when calculating equilibrium-jam stages.

Where the maximum ice-influenced stages occur during freezeup, the DFM could also be applied, provided upper and lower stage-discharge relationships can be defined. This would certainly be possible where freezeup jamming is dominated by the equilibrium-type of jam that forms by collapse and shoving of an ice-floe accumulation. On the other hand, the formation of a hanging dam is governed by the transport of frazil slush under a sheet-ice cover and there is no general analytical expression to relate stage to flow, as in the case of the equilibrium jam. Peak water levels caused by hanging dams may also be controlled by discharge, but derivation of a rating curve would require site-specific numerical modelling applications (Beltaos et al., 2007; KGS Group, 2007).

The present results indicate that the DFM is a promising approach, but more case studies are needed to develop a more complete understanding of the behaviour of $\phi(\eta)$ and the coefficient k. This would not only enhance confidence in the use of the DFM, but also lead to refinements concerning how ϕ and k may change among different river sites. As already intimated, such changes reflect the susceptibility to jamming, which is a site-specific property that is governed by local morphology and climate.

9. Summary and conclusions

A new method (DFM) for generating synthetic probability distributions of ice jam flood levels has been developed after noting that empirical evidence does not support the assumption of discrete stage outcomes. Consistent with this evidence, it was recognized that the peak breakup stage can take on any value between discharge-dependent upper and lower envelopes. This concept led to a simple formulation of the probability of any given stage, which depends on that of the breakup discharge, as well as on a conditional probability function, ϕ. The normalized version of the latter is an important feature of the DFM, and evidence from three case studies points to a similarity property. Consequently, it was shown that the DFM gives good results in both case studies considered herein, while application of the discrete-function method gave less satisfactory results. Practical limitations arise primarily from the need to use the maximum breakup discharge in place of the unknown-in-practice value that prevails at the time of the peak stage. Resulting errors in probability estimates are conservative and diminish in the range of rare events. Analysis of additional case studies is recommended as a means of linking the structure of the function ϕ to the site-specific jamming susceptibility.

10. Acknowledgments

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