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Developing winter flow rating relationships using slope-area hydraulics

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Despite recent advances in instrumentation and modelling methods, accurate determination of river discharge under an ice cover still requires direct measurement. Published flows at hydrometric gauging stations are based on interpolation between a few measurements that are carried out during each winter. The feasibility of using slope-area hydraulics to develop discharge-stage rating relationships is explored at two stations, Peace River at Peace Point and Mackenzie River at Arctic Red River. Records at both gauges contain key information for understanding local ice jamming processes, which are known to control the long-term maintenance of the aquatic ecosystems in the respective deltas. For each site, the variations of reach-average hydraulic parameters with stage are first determined from several nearby cross-sections. This information is then used to calculate hydraulic resistance characteristics during the ice season based on archived discharge measurement data, which also include ice cover thickness. The Peace River flow measurements indicate a well-defined seasonal variation in hydraulic resistance, excepting years with large slush deposits under the solid-ice sheet. Slush effects are negligible at the Mackenzie River gauge site, but the stage-flow relationship is complicated by a variable water surface slope, owing to downstream control by the Beaufort Sea. This feature is most pronounced during the pre-breakup period when flows are rising sharply, and renders flow estimation uncertain. A nearby water-level gauge would help quantify the slope and increase confidence in spring flow estimates.

1. Introduction

Flow discharge is a key hydrologic parameter that largely controls river regime, hydraulic extremes such as flooding or low flows, and various hydro-ecological processes. The need for accurate determination of river flow is underscored by the issue of climate change, given the close links between climate and hydrology. Increasing resource exploration and development in Canada's northern and arctic regions has resulted in a growing need for reliable flow data. Under open-water conditions, flow determination at various hydrometric stations is based on empirically developed rating tables, which express discharge as a function of stage or "gauge height".

Because an ice cover modifies channel conveyance, it is not generally possible to uniquely relate flow to stage. Consequently, winter flow at any one gauge site is inferred from a few measurements that are performed during each winter period, using one of several - often subjective - interpolation methods (Rosenberg and Pentland, 1983; Melcher and Walker, 1992; Shiklomanov et al., 2006). Interpolation errors often exceed 25% and, on occasion, 50%. None of the commonly used methods invoke basic hydraulic principles of flow under an ice cover. Hicks and Healy (2003) explored the potential of using numerical hydraulic modelling for winter discharge and concluded that reliable applications would require stage data at a minimum of two closely-spaced gauges, thus enabling calculation of the water surface slope. .

Herein, the efficacy of the simple slope-area method is examined with reference to routinely obtained hydrometric gauge data. The motive for this work originates in studies of ice breakup processes in the Peace-Athabasca and Mackenzie River Deltas (Beltaos, 2009), and their controlling influence on peak spring water levels and maintenance of local ecosystems. River flow is a major driving factor in ice breakup and jamming, but concomitant flow data are uncertain. Following background material on the slope-area method and its application to natural streams, data sources are described and results of the analysis presented. The seasonal variation of hydraulic resistance is explicitly established in the case of one station but obscured by a concurrent change in the water surface slope at the other station. Implications of the present findings to winter flow estimation and measurement are discussed.

2. Quasi-Uniform Flow in Natural Streams

Steady flow in a prismatic channel of constant bed slope, such as a laboratory flume, will assume a uniform condition if the channel is long enough to contain a reach where the effects of upstream and downstream controls are no longer felt. This condition is characterized by constant flow area, width, and depth, and by a constant water surface slope that is equal to the bed slope of the channel. Uniform flow cannot occur in a natural stream because cross-sectional shape and area change from transect to transect even where the water surface slope, S_W, remains constant along the stream. Nevertheless, hydraulicians recognize conditions of "quasi-uniformity", characterized by a lack of trend in hydraulic parameters over a sufficiently long channel reach, despite local fluctuations. Many gauge sites do meet these conditions, and uniform-flow equations can then be applied to reach-averaged quantities (e.g. Kellerhals et al., 1972). Where bathymetric data are available for a representative set of river cross sections, and the water surface profile is known, reach-average width (W), area (A), mean flow depth and slope (S_W) can be calculated. For known flow (Q), one can further determine reach-average values of mean flow velocity (V) and Manning resistance coefficient (n), or Darcy-Weisbach friction factor (f). As the flow changes, the water surface moves up or down, bur S_w remains the same, unless there are hydraulic controls nearby. Highly dynamic conditions would also modify S_w, but are unlikely to occur during the winter period.

3. Data Analysis and Results

Two Water Survey of Canada (WSC) gauge sites were selected for this study (Fig. 1): Peace River at Peace Point (herein abbreviated as PAPP) and Mackenzie River at Arctic Red River (abbreviated as MARR). Table 1 summarizes salient features of the selected gauges and data sets. River bathymetry was documented by surveying several cross-sections within reaches centred at the respective gauge sites (Figs. 2a and 2b). Water surface elevations obtained during these surveys enabled determination of the water surface slope in each reach.

3.1. Peace River at Peace Point (PAPP)

The effect of the ice cover on stage is illustrated in Fig. 3, where the only definable stage-flow relationship is that of the open-water condition. Using the measured slope of 0.064 m/km along with reach-averaged channel bathymetry (surveyed in 1999), the composite-flow Manning coefficient, n_c, was calculated for each flow measurement. To determine net flow area, the average ice-cover thickness, as measured at the flow metering site, was applied to the entire gauge reach. Presence of significant slush deposits often resulted in highly implausible values of n_c, owing to the irregular slush underside and its variability in space and time. As Figs. 4a and 4b suggest, it seems reasonable to extrapolate the measured solid-ice thickness to the entire reach, but uncertain for the slush.

Restricting attention to cases where h_s (laterally-averaged slush thickness) < 0.3 m, the calculated value of n_c varies during the ice season in a very plausible manner (Fig. 5a). The same applies to n_i (Manning coefficient of the ice underside), as shown in Fig. 5b. The gradual decrease in ice roughness during the winter and the pre-breakup increase in the spring are in accord with earlier findings [Nezhikhovskiy (1964) and Carey (1966), respectively]. As a check on the present findings, the 3^{rd}-degree polynomial function shown in Fig. 5a was used to calculate discharge on the dates of available flow measurements using the applicable water levels. Ice thickness was estimated from the graph of Fig. 6, where thickness is plotted against Julian Day. Values of n_c and thickness can also be conveniently obtained from Table 2. The effect of slush on net flow area and width was simply ignored, as there is no way of knowing its thickness without in situ measurements. Even with this drastic assumption, discharge prediction error was under 23% nine times out of ten, and had a maximum of 30%.

3.2. Mackenzie River at Arctic Red River (MARR)

This hydrometric station is located ~ 25 river km above the entrance to the Mackenzie Delta. Local river bathymetry was surveyed in the summer of 2005. At that time, the water surface slope (S_W) was 0.029 m/km and the corresponding bed Manning coefficient (n_b) was calculated as 0.025. However, when the survey data were applied to other open-water stages and flows, it became apparent that S_w would have to increase with discharge (Fig. 7). Otherwise, one would have to accept a highly improbable four-fold decrease in n_b over the range of flows covered by WSC's rating table ($Q = 5900$ to 48000 m^3/s). The slope variation with discharge is easily explained by Gradually-Varied-Flow hydraulics (Henderson, 1966): it is a property of the "M1" water surface profile, which asymptotically joins a reservoir level (in this case the Beaufort Sea) to the uniform-flow level far upstream of the reservoir (Fig. 8).

With this background, the effect of ice on stage is explored in Fig. 9. While the data points exhibit the expected scatter, the pattern of the departure from the open-water curve is unusual. Normally, this departure (commonly referred to as backwater) should increase with the flow. However, the backwater indicated in Fig. 9 only increases up to a flow of 5000 to 6000 m^3/s and decreases steadily as the flow increases further. This pattern can be explained by an increase in the water surface slope relative to that of the open-water condition. Thus, the slope appears to depend not only on flow but also on whether there is an ice cover or not. The thickness and roughness of the cover could also be relevant in this regard.

Since it is not possible to determine resistance and slope characteristics, data analysis is limited to a combined parameter ($Q/AR^{2/3}$), which is equivalent to $S_W^{1/2}/n_c$ for ice-covered flow. This parameter can be readily calculated from the available data, and its evolution during the ice season is illustrated in Fig. 10. There seems to be no change, or at most a very slight increase, during the period November to April (Julian Day = -60 to 120), followed by a sharp rise in May (Julian Day =121 to 151). Slush was frequently present, with an average thickness of up to 0.7 m. However, the slush did not appear to significantly influence the results, likely owing to the large depth of the Mackenzie River (typical under-ice flow depth ~ 11 m; compare with ~4 m for PAPP).

For the pre-breakup period, which occurs in May, it is not advisable to base flow estimates on the trend shown in Fig. 9 because the plotted values depend on discharge in an unknown manner. Figure 11 shows published daily May flows, plotted versus gauge height during the years for which such data are available online at the time of writing (WSC archives; 2002-2007; web link: http://www.wsc.ec.gc.ca/hydat/H2O/index_e.cfm?cname=graph.cfm&RequestTimeout=300 as of April 2009). Days when stage is influenced by ice jams (such as in 2005 and 2006) are excluded. Measured May flows are also shown for comparison. The published values are in general agreement with measured ones, but scatter increases considerably as the flow rises above ~ 10,000 m^3/s. To develop reliable melt-driven spring breakup hydrographs at the entrance to the Mackenzie Delta, one would need the water surface slope in addition to the stage at the MARR gauge. This could be achieved with installation of an auxiliary gauge, suitably located to ensure that a representative value of the slope is obtained between it and the existing gauge.

4. Discussion

For the Peace River gauge site, which is free of nearby stage controls, the slope-area method yields plausible results that can be readily implemented in practical applications [see Beltaos (2009) for details]. The number of Canadian gauge sites where similar conditions apply is not known but experience suggests that they should represent a significant fraction of the total. Savings could be realized by developing local winter rating tables and reducing the number of measurements per season. This would require an initial investment of resources for: (a) surveying reach bathymetry and slope; and (b) analyzing past data to develop seasonal variations of ice thickness and resistance characteristics.

A key element of the slope-area method is the use of reach-averaged bathymetry, as opposed to relying on single-station data. This point is illustrated in Fig. 12, where the composite Manning coefficient n_c has been calculated from measured area and width at the flow-metering site. The resulting scatter is much larger than that of Fig. 5a, which is based on reach-average hydraulic parameters.

Though many hydrometric stations are located in reaches that meet the constant-slope requirement, there are some that do not. The MARR gauge is a good example of such a station. Here, slope increases with flow. It does so predictably under open-water conditions but in unknown fashion when an ice cover is present. Considering the importance of this particular station to many hydro-ecological issues and studies pertaining to Canada's Arctic regions, it would be highly desirable to install an auxiliary gauge. Though year-round operation would be ideal, knowledge of the slope is only critical to estimating discharge during the months of May and June.

The presence of sizeable slush deposits hinders the utility of slope-area hydraulic analysis. This stems from the large spatial and temporal variability of the thickness of the deposit and could also interfere with "index velocity" methods. In such methods, mean flow velocity is linked via calibration to measured point or vertically-integrated velocity at a fixed location across the channel. The mean velocity is then multiplied with flow area, which is determined from known cross-sectional bathymetry. Under an ice cover, flow area is deduced from net stage, which can be determined with upward-looking acoustic devices such as described by Morse et al. (2005). With a single device, the average elevation of the bottom of the ice cover across the channel would be assumed to be equal to that of the ice-water interface at the point of measurement. Figures 3a and 3b indicate that this assumption could lead to serious errors in the presence of slush deposits.

5. Summary and Conclusions

The efficacy of the slope-area method has been explored for winter conditions at two gauge sites, both situated near the entrances to major freshwater deltas of northern Canada. In both cases, only measured flows have been used, starting in the early 1990s and ending in 2008. Good results were obtained for the first site, Peace River at Peace Point, which is free of downstream controls on the water level. The composite-flow Manning coefficient, n_c, as deduced from numerous winter flow measurements, exhibits a stable and highly plausible seasonal variation. If the ice cover thickness is known or can be estimated from past measurements, the discharge can be determined for a measured water level via reach-average bathymetric data and the applicable value of n_c. Severe slush conditions reduce the accuracy of the slope-area method. Where the quasi-uniform flow concept applies and slush is not severe, operational costs could be moderated by reducing the number of winter measurements per season and rely more on slope-area interpolation, coupled with suitable interpolation of the ice thickness. Analysis of open-water data at the second site, Mackenzie River at Arctic Red River, indicated that the water surface slope increases with increasing stage, owing to the control exercised by the Beaufort Sea. The same applies to ice-covered flows but now the slope also depends on the thickness and type of the ice cover. Therefore, knowledge of stage alone is not sufficient for determining the discharge. This deficiency is particularly pronounced during the spring melt, when the flow is rising sharply, and eventually triggers the breakup of the ice cover. Improved flow estimates require an additional gauge, which would record the stage concurrently with the existing one during, at least, the May-June period. Considering the importance of spring flows to the maintenance of the Mackenzie Delta ecosystem, installation of a secondary (water level only) gauge near the end of the Mackenzie River would be beneficial to several ongoing scientific studies. Apart from hindering slope-area calculations, slush could also generate large errors in flow areas obtained from upward-looking instrumentation that may be deployed to determine discharge using the index velocity approach.

Acknowledgments

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Table 1. Gauge details and data sets used in this study

WSC GAUGE	Peace River at Peace Point (PAPP)	Mackenzie River at Arctic Red River (MARR)
Gauge number	07KC001	10LC014
Latitude (N)	59° 07' 02"	112° 26' 13"
Longitude (W)	67° 27' 22"	133° 44' 13"
Drainage area (km^2)	293000	910600
Period covered by data	Dec. 1993 to Apr. 2007	Feb. 1996 to Apr. 2008
Number of measurements	1 to 5 per season	1 to 9 per season
Average number of measurements	2.9 per season	4.1 per season

Table 2. Composite-flow Manning coefficient and ice thickness; PAPP reach

Date	Value of n_c ($m^{-1/3}s$)	Ice thickness (m)
December 1	0.035	0.30
January 1	0.031	0.54
February 1	0.026	0.77
March 1	0.023	0.92
April 1	0.022	0.98
May 1	0.024	0.89

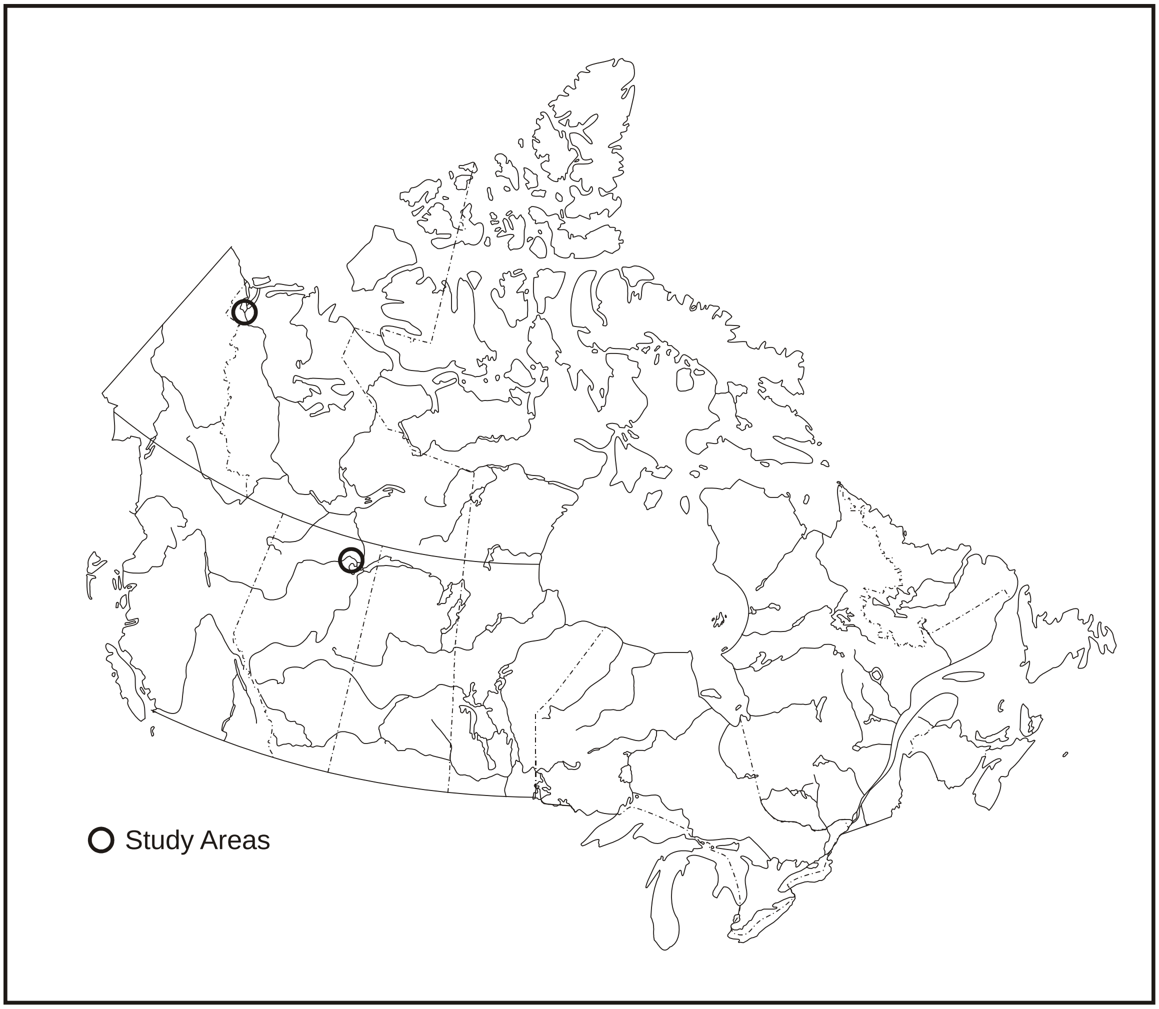

Figure 1. Locations of the two study areas in Canada.

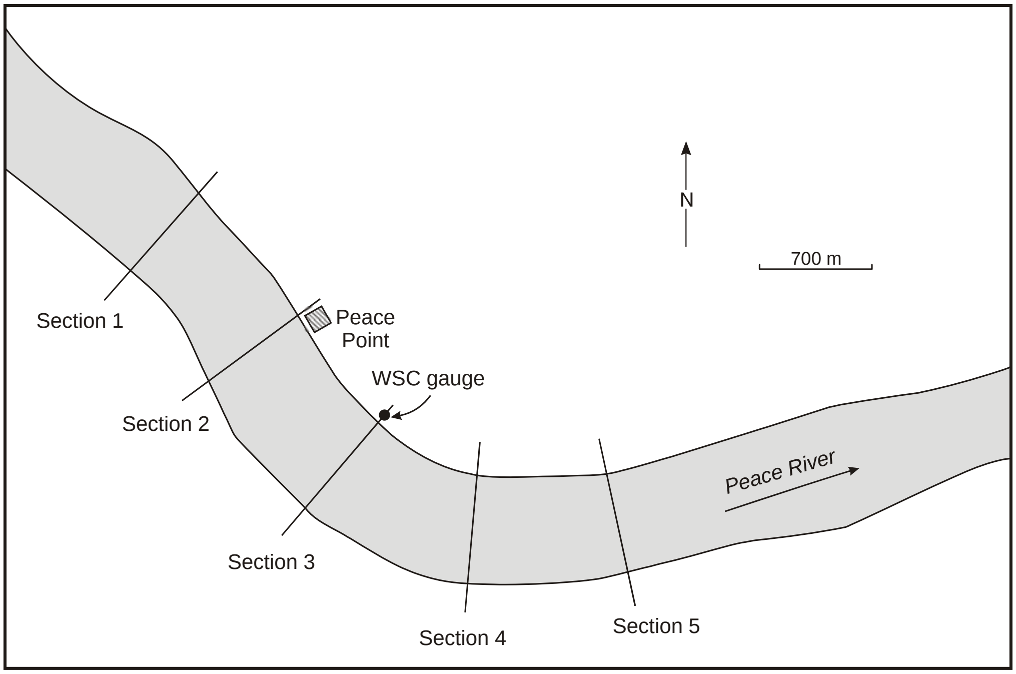

Figure 2a. Study reach, Peace River at Peace Point. Cross sections were surveyed in the summer of 1999.

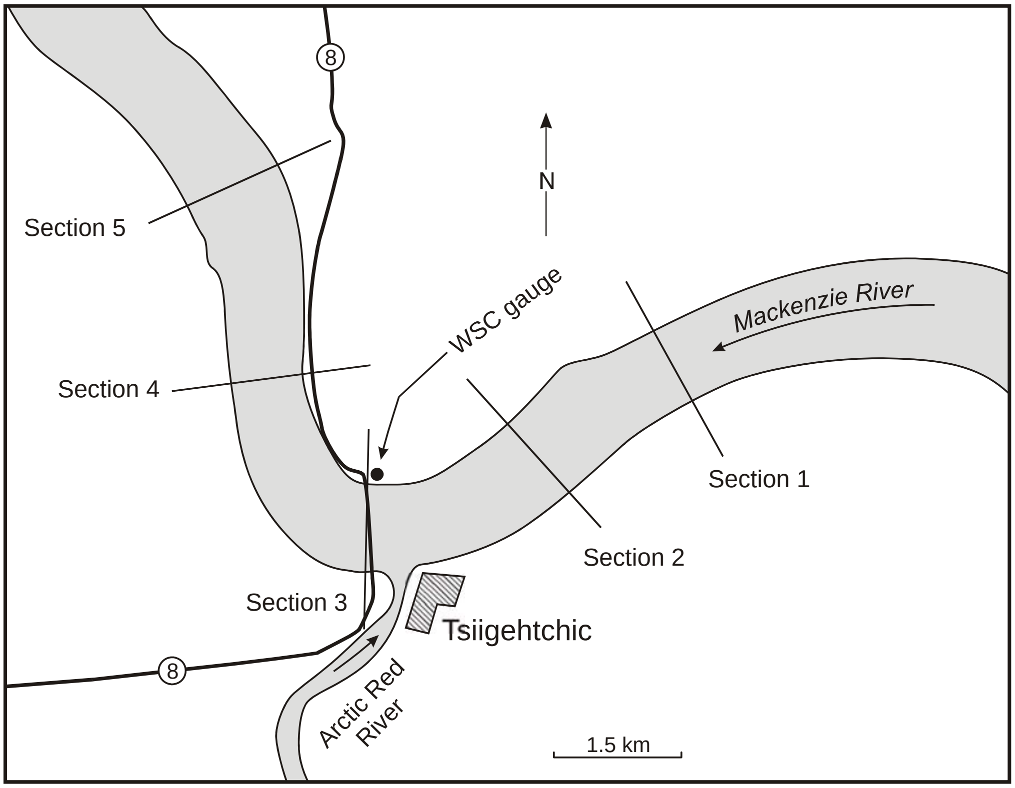

Figure 2b. Study reach, Mackenzie River at Arctic Red River. Cross sections were surveyed in the summer of 2005.

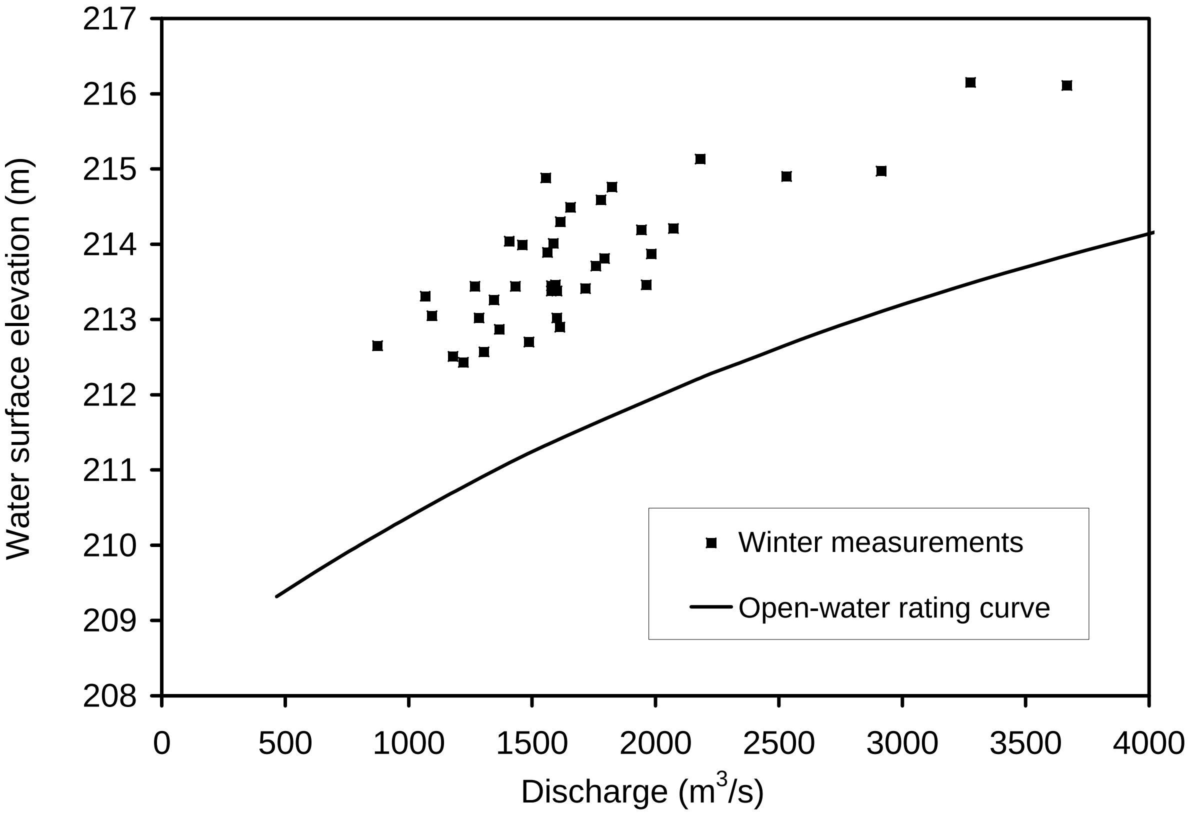

Figure 3. Effect of winter ice cover on stage; PAPP gauge.

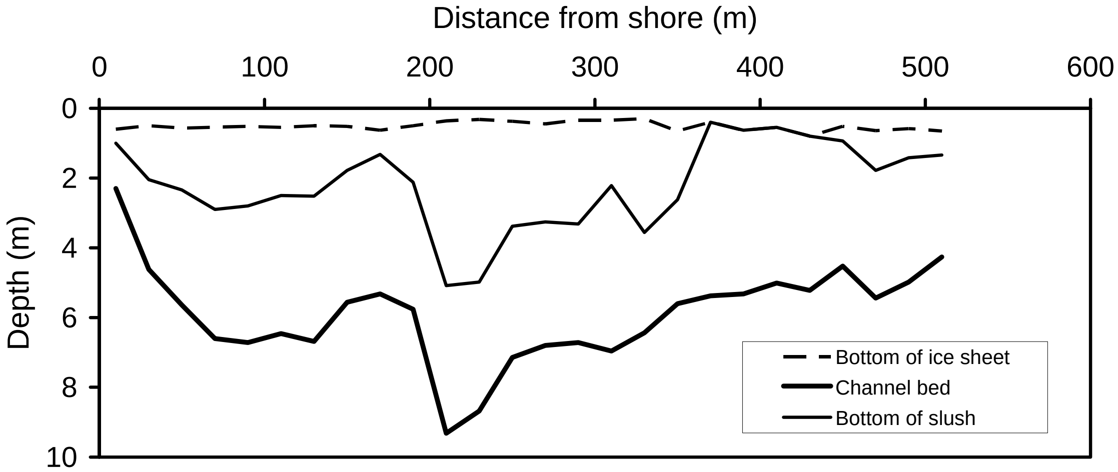

Figure 4a. Transverse variations of ice thickness and slush deposit; PAPP gauge; Jan. 30, 2002.

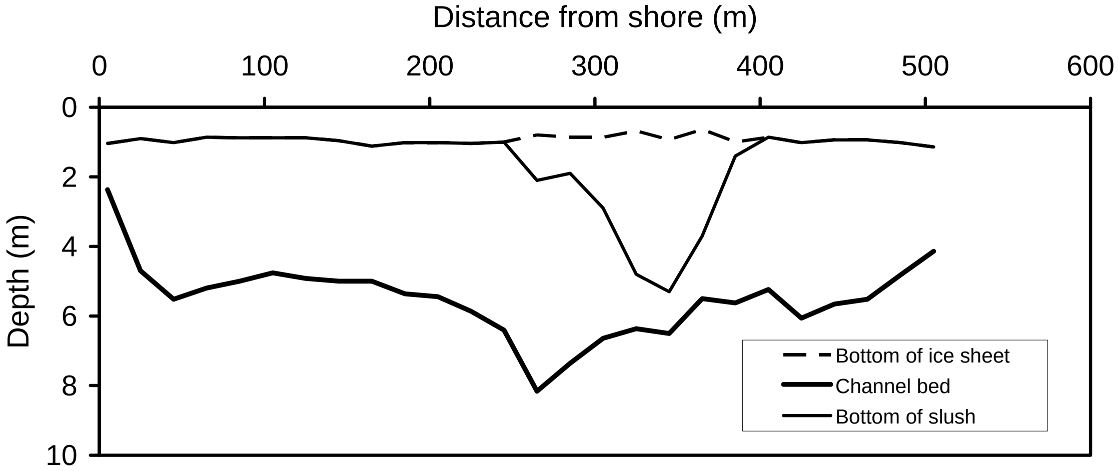

Figure 4b. Transverse variations of ice thickness and slush deposit; PAPP gauge; Apr. 25, 2002.

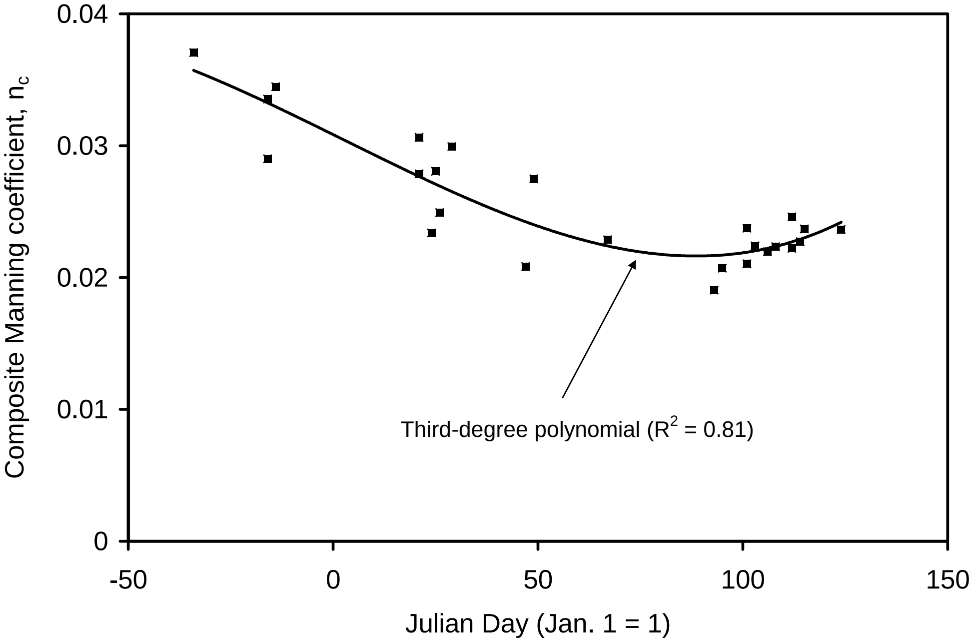

Figure 5a. Variation of composite-flow Manning coefficient during the ice season ($h_s < 0.3$ m); PAPP gauge. The value of n_c is expressed in metric units ($m^{-1/3}s$).

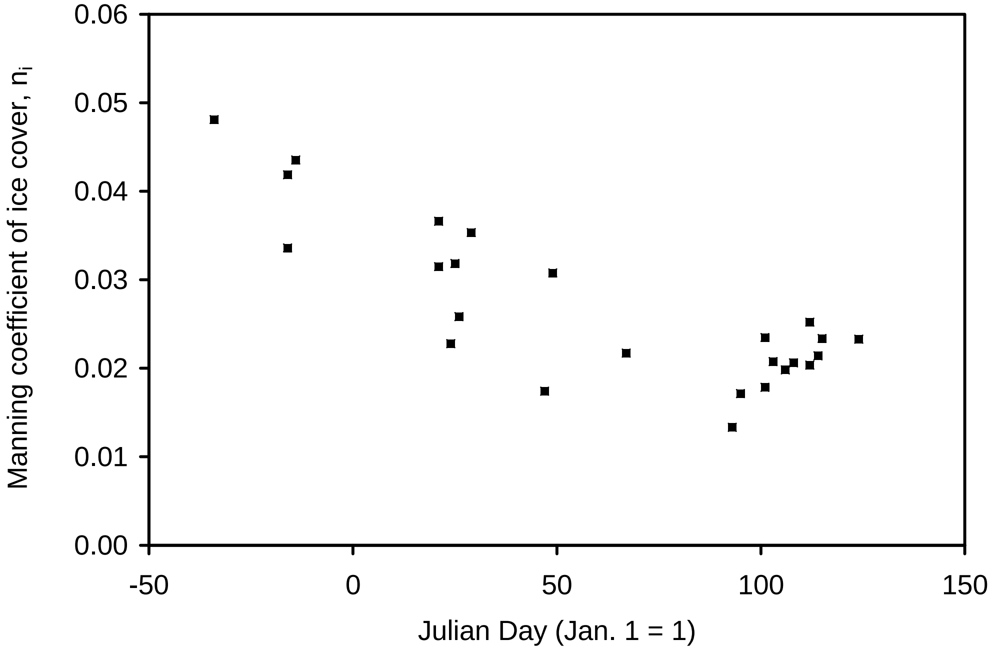

Figure 5b. Variation of ice cover Manning coefficient during the ice season ($h_s < 0.3$ m); PAPP gauge. The value of n_i is expressed in metric units ($m^{-1/3}s$) and was determined from the Sabaneev equation, using a bed coefficient of 0.024 (Beltaos, 2009).

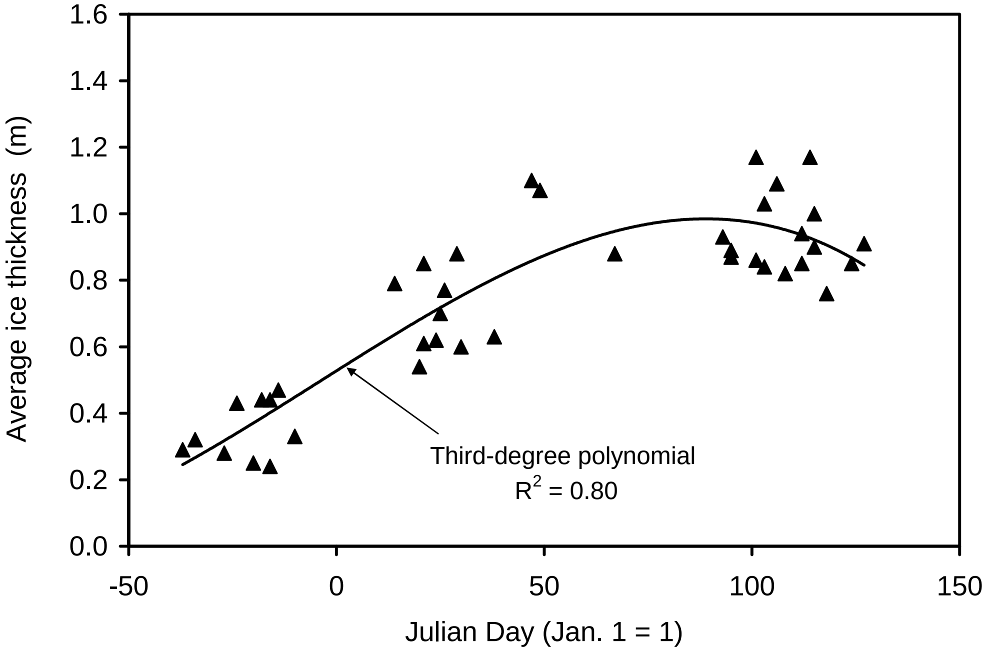

Figure 6. Seasonal thickness variation of the solid-ice sheet; PAPP gauge.

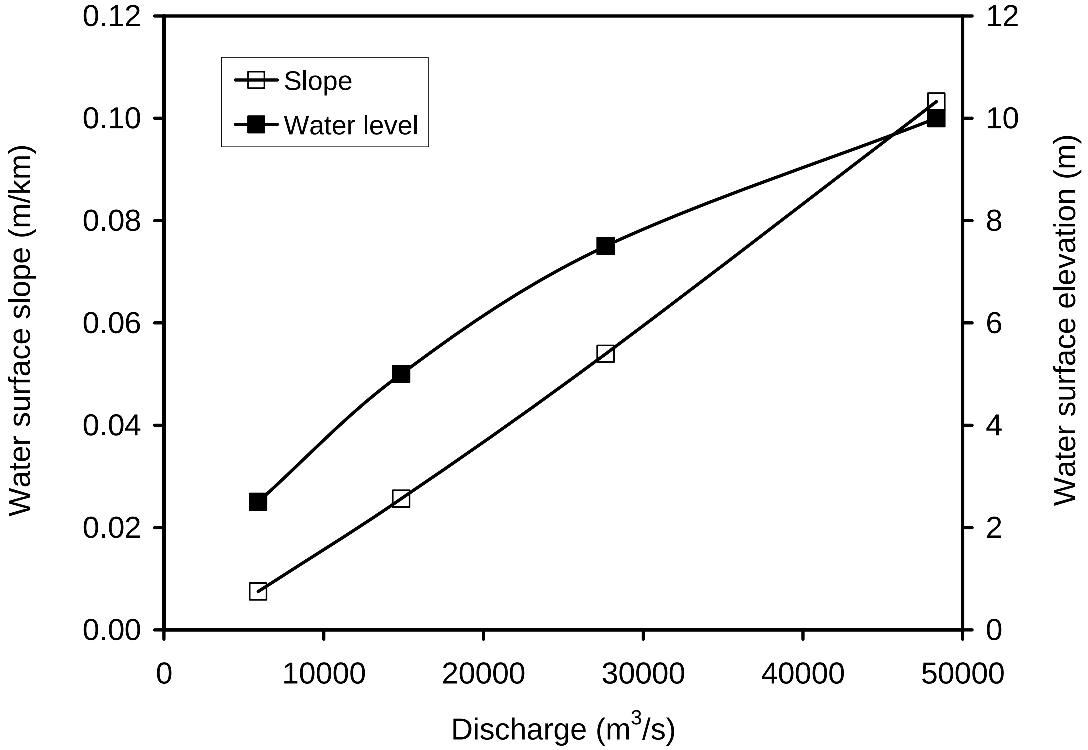

Figure 7. Variation of water surface slope and elevation with discharge under opne-water conditions; Mackenzie River at Arctic Red River. The slope is a calculated value, assuming $n_b = 0.025$.

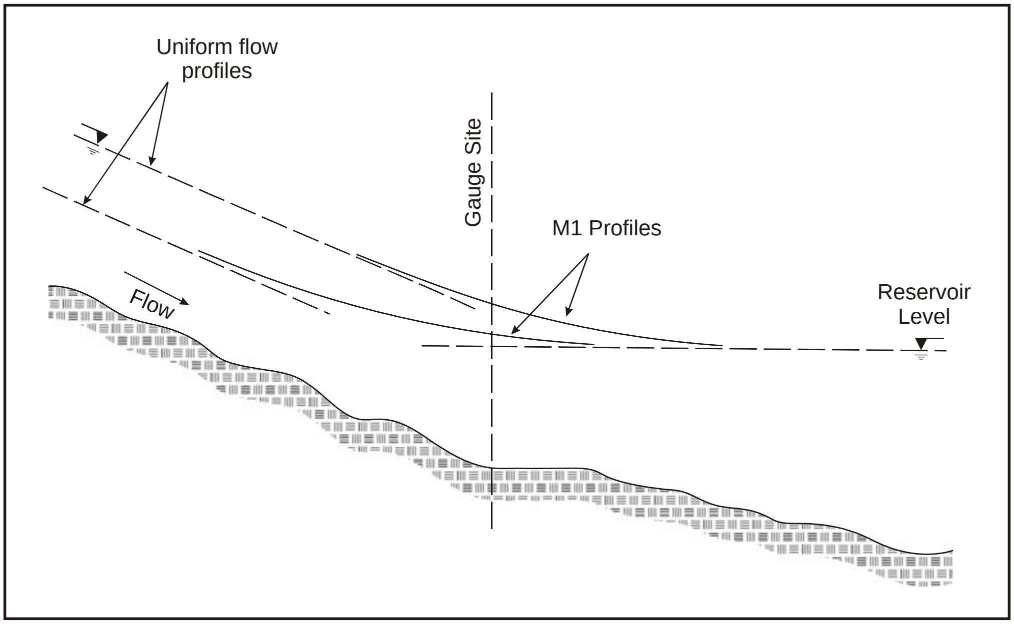

Figure 8. Illustration of M1 water surface profiles and increase in slope with discharge at sites where the water level is influenced by a downstream reservoir or sea level. An open-water condition is assumed. A similar configurations also applies in winter, provided the ice cover thickness is approximately uniform along the river.

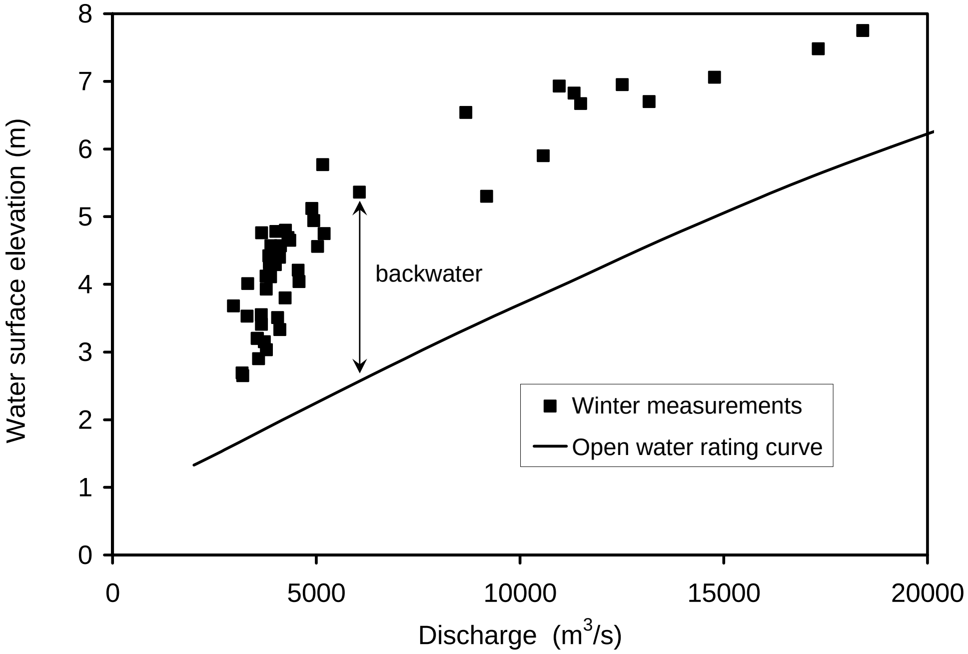

Figure 9. Effect of ice cover on stage; MARR gauge.

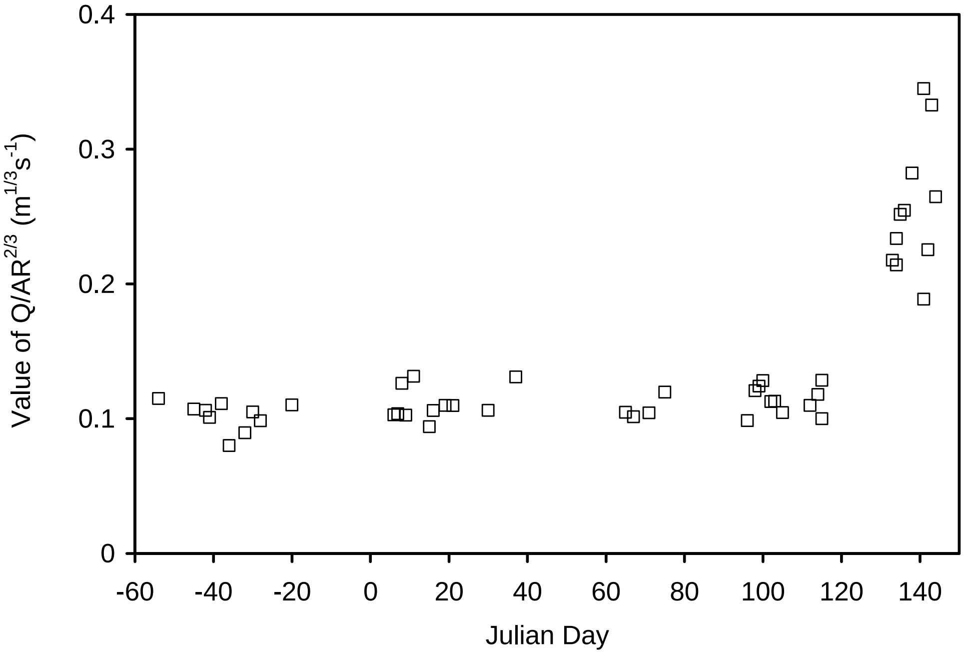

Figure 10. Variation of $Q/AR^{2/3}$ (or $S_W{}^{1/2}/n_c$) during the ice season; Mackenzie River at Arctic Red River.

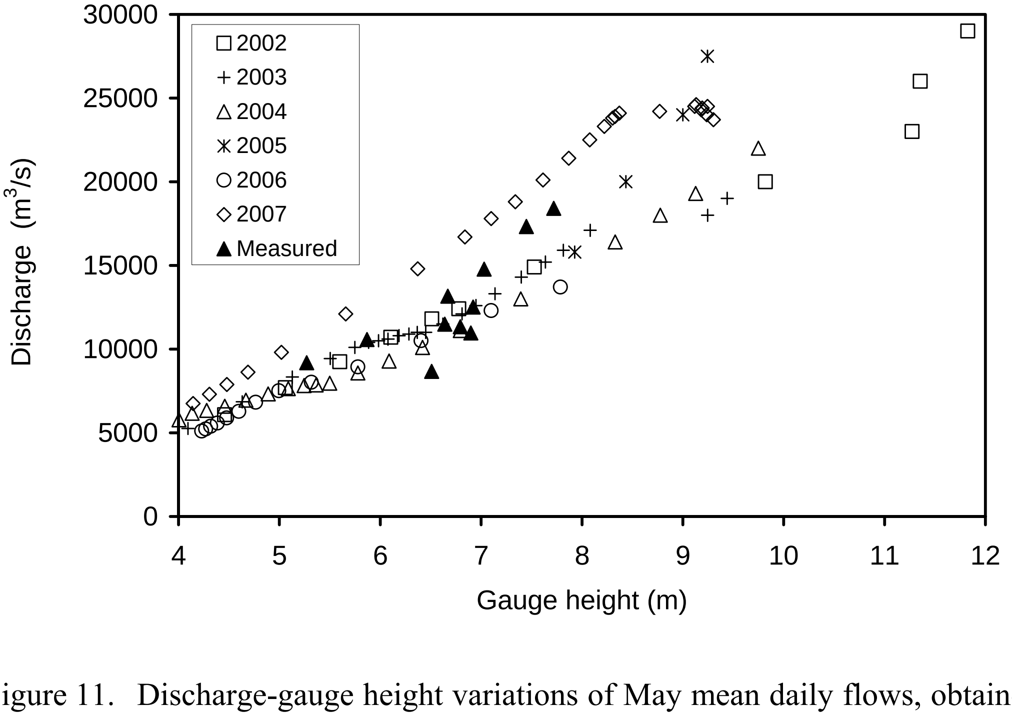

Figure 11. Discharge-gauge height variations of May mean daily flows, obtained from archived WSC data; MARR. Gauge height = Water surface elevation – 0.03 m.

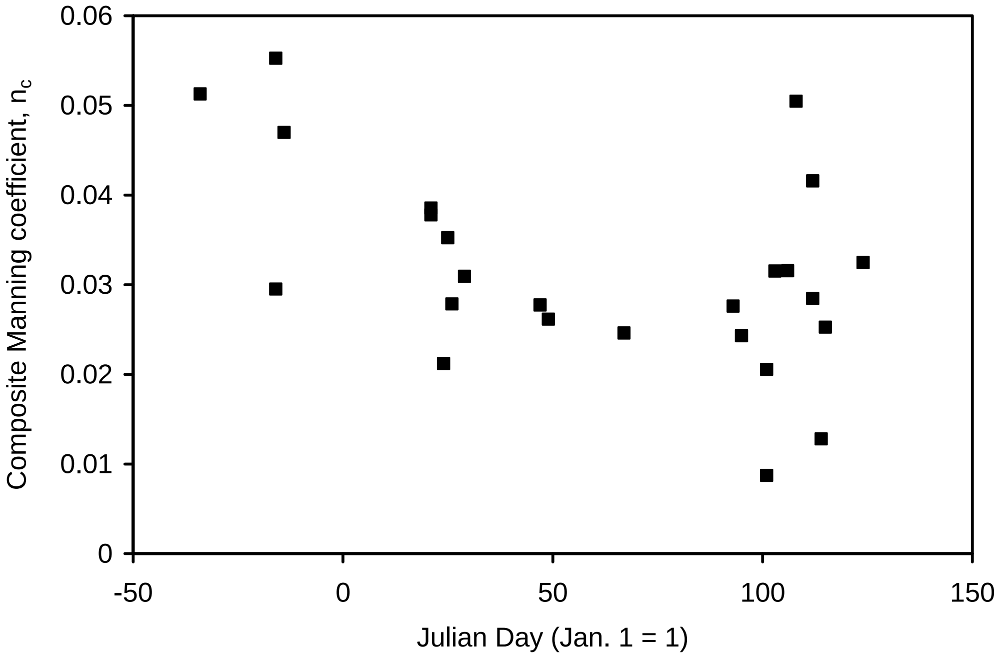

Figure 12. Composite Manning coefficient, as calculated from single-transect area and width ($h_s < 0.3$ m); PAPP gauge.