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Development of Site Specific Ice Growth Models for Hydrometric Purposes

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The Water Survey of Canada (WSC) has put together a database containing hydrometric measurements from various sources including data collected from routine field operations in both open water and under ice, and FUI (Flow Under Ice), a joint project between WSC and USGS containing winter hydrometric measurements. Field records of water surface to bottom of ice measurements, under ice discharge, meteorological records, and other pertinent information about river ice contained in this database were examined. Analysis of the data revealed information about the effect of climate variables such as snow cover, snow density, cumulative freezing degree-days, and solar radiation on river ice growth. The influence of the weight of a snow cover on top ice growth was also examined. Sites that were investigated include sites in Alberta, Ontario, and the Northwest Territories. A site specific statistical ice growth model was developed for each of 11 hydrometric stations based on water surface to bottom of ice records found in the database, current ice growth theory, and climate records. These ice growth models were evaluated using statistical analyses and they were also validated against measured data collected during the winters of 2003-2004 and 2004-2005. Six of the models developed were found to adequately predict river ice growth. Five of the models did not predict river ice growth adequately. The inadequacy of some of the models is primarily due to the fact that insufficient data were available for these locations, there was uncertainty in the accuracy of the climate data, and there were inconsistent measurement locations. In order to develop more accurate models, it is recommended that measurement locations be recorded, that climate sensors be installed at hydrometric stations located far from climate stations, and that the effect of velocity and other hydraulic parameters on ice thickness be examined.

1. Introduction

The Water Survey of Canada (WSC) currently uses seven methods for streamflow computation under winter ice conditions to obtain published data. The methods are: the Backwater Method, the Adjusted-Discharge Method, the Interpolated-Discharge Method, the Effective Gauge-Height Method, the Recession Curve Method, the k-Factor Method, and the Winter Rating Curve Method. These methods are described by Hicks (2002). Presently there is a need to develop more effective methods of estimating river discharge under ice, such as using hydraulic models or other models that are more accurate than the traditional approaches mentioned above. In order to estimate flow under ice using hydraulic models, it is necessary to have a method for the estimation of river ice thicknesses. Currently, the WSC does not use a method for the estimation of river ice thickness. Upon researching literature sources, the only method found that is widely used to determine ice growth is the Stephen equation, which relies only on temperature values (accumulated sum of freezing degree-days) and correction factors to account for snow accumulation and wind exposure.

$$h_i = \alpha (D_f)^{\frac{1}{2}} \qquad [1]$$

Where D_f is the accumulated sum of freezing degree-days (AFDD) and α is a coefficient varied to account for conditions of exposure and surface insulation.

A hydrometric database, called Measurement Database, has been put together by L. Liu at the Water Survey of Canada containing hydrometric measurements (including winter measurements) from various sources including routine field operations and FUI (Flow Under Ice). FUI contains winter hydrometric measurement data collected under a project conducted by USGS and WSC in the early 1990's containing both summary and panel information (Walker, 1997). HFC is the output from the hydrometric field computer containing both summary information and detailed panel information. Field records of water surface to bottom of ice measurements from 1985-2003, meteorological records, and other variables such as average velocity, average width, and average depth were examined. Variables that were thought to influence ice thickness were examined including snow cover, snow density, solar radiation, as well as cumulative freezing degree days. Most of the sites examined in this study were sites located in Alberta, because this data was readily available in the database. There was one site located in Ontario and one site in the Northwest Territories that were also examined closely. The objective of this study was to develop site specific ice growth models based on records found in the database and climate data, using current ice growth theory.

2. Theories on Ice Growth

Several different theories on river ice growth and river ice properties have been researched and examined in this study. Data from the ice measurement database as well as data collected from an Acoustic Doppler Velocity Meter (ADVM) installation have been examined closely. Cross sectional profiles of the river bed and ice cover were examined for several different sites and years to provide insight into ice cover growth and characteristics.

When a hole is drilled in the ice cover, it is common to see the water level rise up in the hole to a level equivalent to 8% of the thickness below the top of the ice. This is known as the phreatic surface, which is located at 92% of the ice thickness above the bottom of the ice. (Hicks, 2002) When this occurs, the ice cover is floating. The Water Survey of Canada does not currently collect measurements of the distance from the water surface to the top of the ice; therefore it is not clear whether there was a completely floating ice cover when all of the ice thickness measurements were done. Hydrometric technicians have observed that it is quite common for the water level to rise to the phreatic surface when a hole is drilled into the ice. But they have also observed situations where the water spills out of the hole onto the ice surface. This could indicate pressure flow or it may also be caused by the additional weight of the technicians, snow, and equipment on the ice cover. It has also been observed in some situations (for example on the Châteauguay River in Québec) that the water rises in the hole to a level below the phreatic surface. This could indicate a situation where the ice cover is partially floating and partially supporting itself. The following is a summary of three different theories on river ice growth:

a) One theory is that ice covers float with 92 % of their thickness submerged. In this situation, the ice cover is attached to the banks, but it is flexible and is able to shift upwards and downwards with moderate changes in water levels and discharges. Therefore it is always supported by the water and does not support its own weight. In this case the ice cover can be viewed as a "plastic" material, able to withstand a certain amount of tension and compression caused by the fluctuating water levels and being attached to the banks. If the change in water level is very large (as in the case of mechanical break-ups), the ice cover will not be able to withstand the forces of tension or compression and will therefore form cracks and eventually dislodge itself from the river banks.

b) Hydrometric technicians have also observed that in very narrow rivers and streams, the ice cover is rigid and becomes strong enough to support itself completely. In this situation, the ice cover does not move with the changing water levels and discharges except during break-up when the discharge rises to its highest levels, and the ice cover begins to float again. In order for the ice cover to support itself completely, the ice must be very thick, and the water level must recede such that it is no longer supporting or partially supporting the ice cover. In this situation, ice growth would stop because there is no longer contact between the water and the bottom of the ice. It is unlikely that a situation with a rigid ice cover was encountered with any of the measurements used in this study because only measurements taken during the ice growth period in this study were considered. According to the observations from hydrometric technicians, this situation is very rare and only occurs in narrow rivers and with very cold temperatures.

c) Also it is possible that an ice cover is partially rigid and partially floating; in this case the measurement of water surface to the bottom of the ice is below the phreatic surface and above the bottom of the ice. Since measurements of the distance from the water surface to the top of the ice were not recorded, it is not known whether or not this situation may have occurred with any of the ice measurements. In a situation with a partially rigid and partially floating ice cover, the phreatic surface would not be very close to the top of the ice; this would introduce a significant amount of error to the results of the models. Since no substantial evidence of a partially rigid and partially floating ice cover has been documented, it will be assumed that a floating ice cover was present during all of the ice measurements.

At hydrometric station 02EB013 (East River near Huntsville), data collected from an Acoustic Doppler Velocity Meter (ADVM) installation during the winter of 2004-2005 provided some insight into the nature of the ice cover (i.e. whether it was floating or rigid). The ADVM data was particularly useful because it provided velocity, bottom of the ice depth, and temperature data at frequent time intervals (every 15 minutes). The location of the bottom of the ice was compared with water level data from the gauge; this is shown in Figure 2.1. The ice thickness shown in Figure 2.1 was estimated by subtracting the bottom of the ice recorded by the ADVM from the water level recorded by the gauge. Vertical shifts in the entire ice cover are observed, with shifts in the bottom of the ice and water level occurring simultaneously. This indicates that a floating ice cover is present at this site, since a rigid or a partially rigid ice cover that supports its own weight would not exhibit vertical movement with changes in the water level. Furthermore, the water level measured by the gauge is a function of pressure head and therefore responds to changes in discharge or flow area. This indicates that changes in discharge resulted in vertical shifts in the floating ice cover at this site.

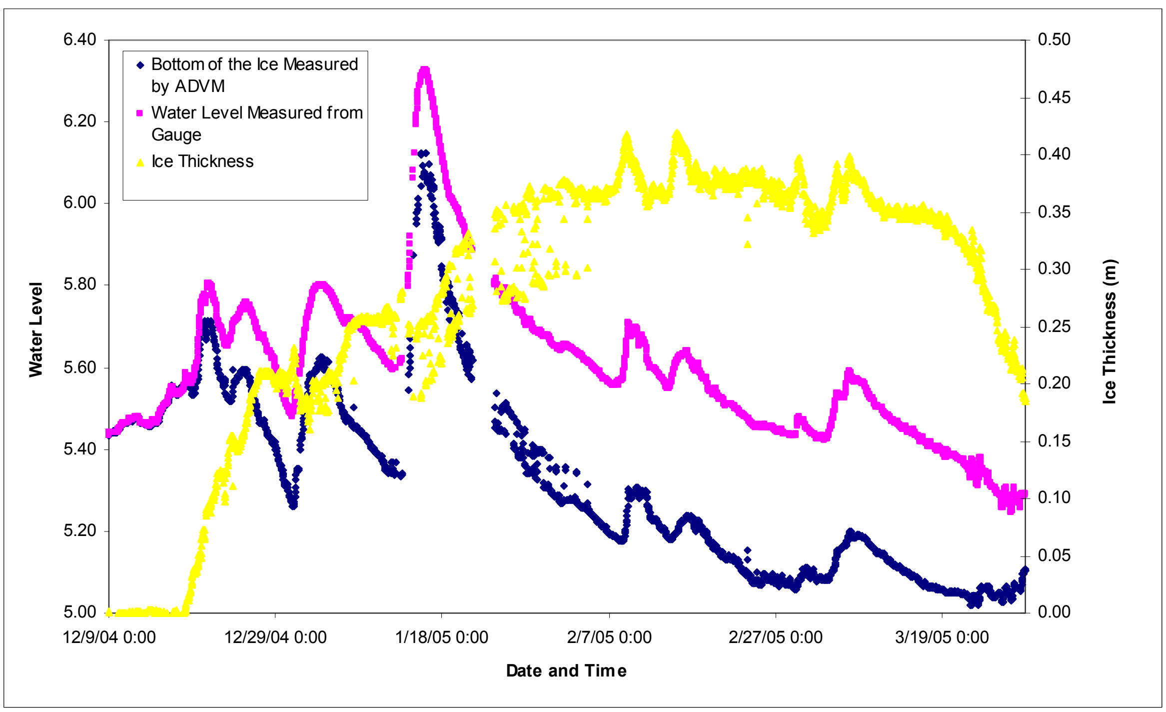

Figure 2.1 – Water Level, Bottom of Ice, and Ice Thickness Measurements for the 02EB013 ADVM Installation During the Winter of 2004-2005

Changes in velocity were also correlated with changes in pressure head and vertical shifts in the ice cover, this is shown in Figure 2.2.

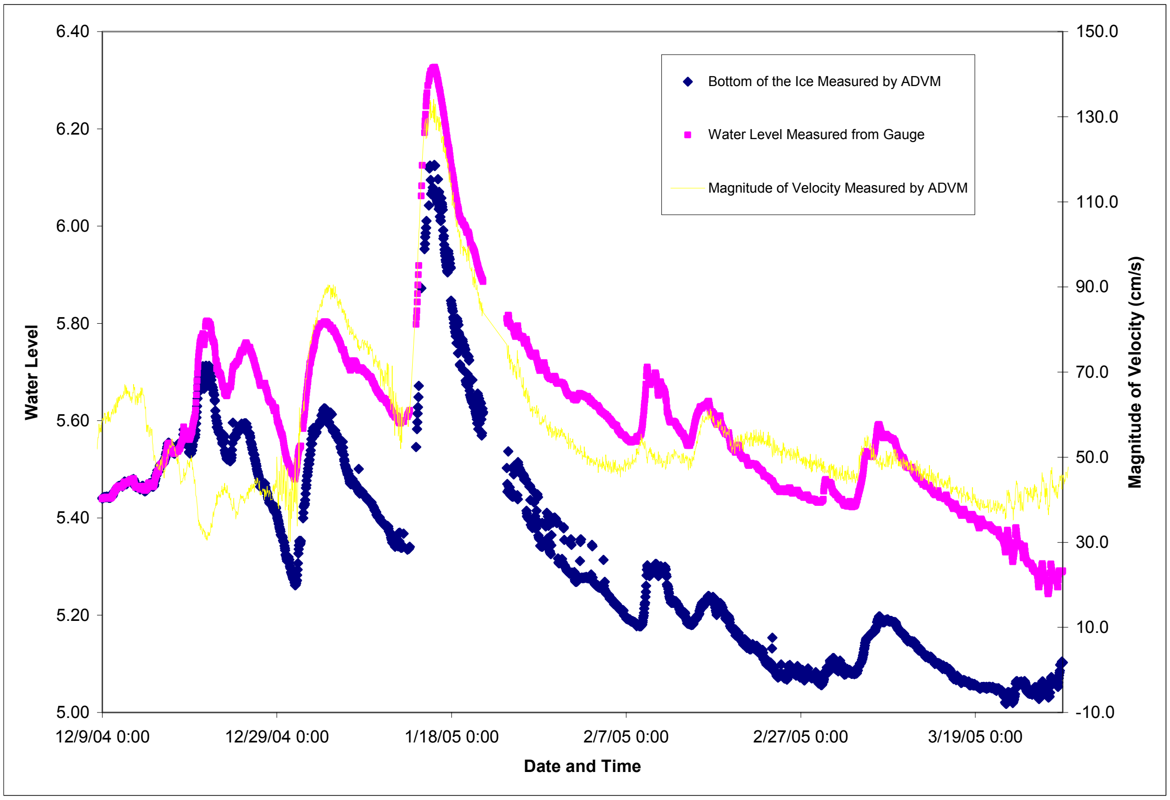

Figure 2.2 – Water Level, Bottom of Ice, and Magnitude of Velocity Measurements for the 02EB013 ADVM Installation During the Winter of 2004-2005

Figure 2.2 shows that magnitude of velocity is proportional to fluctuations in water level and bottom of ice, which also indicates a floating ice cover. We observe that as discharge increased, magnitude of velocity increased and there was an upward vertical shift in the position of the ice cover (except during freeze-up and break-up). The large spikes in water level and magnitude of velocity occurred during rain events

3. Assumptions

a) The ice thickness measurements recorded in the Water Survey of Canada's ice measurement database were actually measurements of the distance from the water surface to the bottom of the ice (WSBI). There are no records containing measurements of water surface to the top of the ice, therefore the actual ice thickness of these measurements is not known. Theoretically ice floats with 92% of its thickness submerged, therefore theoretically the measurement of the WSBI represents 92 % of the actual thickness of the ice (Hicks, 2002). In this study it was assumed that WSBI represents 92% of the ice thickness. But as discussed above it can be possible for the water level to be found below the phreatic surface if the ice cover is partially rigid or above the phreatic surface in situations of pressure flow. If either of these scenarios occurred at the hydrometric sites involved in the study, this could be a possible source of error in the accuracy of the model results. From observations made by hydrometric technicians, WSBI representing 92 % of ice thickness is an accurate assumption in most situations. Huntington (2003) also found in a recent study of the Piscataquis River near Dover-Foxcroft in central Maine that WSBI and ice thickness were strongly correlated. A valid regression model (with an R^2 of 0.91) was developed to describe the relationship between ice thickness and WSBI on the Piscataquis River. When this regression relationship is used to describe the ratio between WSBI and ice thickness, WSBI ranges from 84% to 97% of the ice thickness.

b) In this study it was assumed that the average WSBI across the reach is an accurate reflection of the WSBI at the maximum discharge. This assumption was verified at hydrometric station 06AD006 (Beaver River at Cold Lake, Alberta). For each ice measurement from 1997 to 2003, the average of the 3 panel WSBI at the maximum discharge were compared to the average WSBI for all of the panels. It was found that there was a significant difference (up to 35 cm) during the late winter break-up period. During early winter (right after ice formation) the differences ranged from approximately 2 to 8 cm. During the mid-winter period, differences ranged from 5 mm to 6 cm. It was concluded that the average WSBI across the reach is approximately equal to the WSBI at the maximum discharge during early and mid winter, but this is not true for late winter. All of the data used in this study was collected during the ice growth period in early and mid-winter.

c) It was assumed that the climate data obtained from the climate station located closest to the hydrometric station was an accurate representation of the climate conditions at the hydrometric station. This may not be an accurate assumption depending on the distance between the hydrometric station and the climate station.

4. Description of the Statistical Model

The statistical models were developed using StatGraphics Plus 5 statistical software. A Multiple Linear Regression Analysis was used along with a subset model approach called "Stepwise Regression" (Draper, 1981) to determine the best model for river ice growth. Both a Forward Stepwise Regression and a Backwards Stepwise Regression were performed and compared to assess the overall stability of the models. The procedure for the stepwise regression involved the insertion of variables into the regression equation one by one until the regression equation was satisfactory, or the best possible regression equation was obtained with the available variables. The order of insertion was determined by using the partial correlation coefficient as a measure of the importance of variables not yet in the equation. A significance level of 0.05 was used for all of the multiple regression analyses. After each variable was inserted into the equation, the overall regression was verified for significance, the improvement in the R^2 value was noted, and the partial F-values for all variables in the equation were examined. The partial F criterion for each variable in the regression at any stage of calculation was evaluated and compared with 5% of the F-distribution. For a more rigorous discussion of the stepwise regression method, refer to Draper (1981) or any Applied Regression Analysis textbook.

The models that were developed were evaluated using statistical methods. The models were examined for collinearity, which occurs when the individual regressors are related to one another (linearly dependent). Collinearity can lead to an unstable model since the model becomes overspecified. Indications of collinearity between regressors occurred when the overall model was significant according to the F-test, but some or all of the individual t-statistics were not significant. The correlation matrix for the estimated coefficients was then examined to see if there was a strong relationship between any of the regressors (Vining, 1998). Residual plots were also examined in order to get an idea of how well the model explained the data used in the model's estimation. Plots of residuals against predicted values, residuals against regressors, and residuals against row number were examined. These plots allowed for the testing of systematic model misspecification and the verification of the assumptions that are always made in multiple regression analysis (constant variance assumption and independence assumption) (Vining, 1998). In addition, the R^2 adjusted values (the amount of variability in the data explained by the model relative to the amount of variation left unexplained) were examined. The R^2 adjusted value is more appropriate than the R^2 value for a multiple linear regression model, because it adjusts the R^2 for the degrees of freedom used in the model, since the R^2 cannot decrease as more regressors are added to the model (Vining, 1998).

In addition to the statistical methods used to evaluate the stability and accuracy of the ice growth models, the ice growth models were also validated against measured data. WSBI values predicted by the models were compared with measured data collected in the field from 2003-2005 (data that was not used to develop the models).

5. Climate Data

The models were developed using readily available climate data from Environment Canada's National Climate Data and Information Archive which contains official climate and weather data (http://climate.weatheroffice.ec.gc.ca/climateData/canada_e.html). The climate data used was retrieved from the closest climate station that contained complete climate data for the specified period of time. In some cases a combination of data from more than one climate station was used depending on the availability of data. In the models that were developed, the distance between the hydrometric station and the climate station varied from 4 km to 100 km. The larger the distance between the hydrometric station and the climate station, the greater the uncertainty associated with the climate data and the model developed using the climate data. The following climate data was used to develop the ice growth models: mean daily temperature, snow cover, daily snowfall, daily precipitation, global solar radiation, and hours of daily sunshine. In some cases, there was missing snow cover data and hours of daily sunshine data. These values were estimated based on data collected from nearby stations or using daily snowfall, maximum daily temperature, recorded hourly weather conditions, and sunrise and sunset times.

The following variables were examined as regressors in the multiple linear regression analysis: Accumulated Freezing Degree Days, Σ (Snow Depth x Snow Density), Cumulative Solar Radiation, and Σ (Sunshine Hours x Solar Altitude). Ice growth (or snow ice growth) on the top of the ice cover was also examined and included in the ice growth model.

Cumulative values of the climate parameters were used instead of instantaneous values due to the fact that the ice thickness is a function of changes in the climate parameters over time.

Cumulative parameters were a way of representing the changes in the climate data over time. Since WSBI measurements were only done approximately monthly during the winter months, it was necessary to utilize cumulative climate parameters that would incorporate in their values the changes that had occurred in the climate data over the course of the winter period.

a) Accumulated Freezing Degree Days

To calculate Accumulated Freezing Degree days (AFDD), the daily mean temperature data for each day during the winter season must be known. AFDD represents the sum of the mean daily temperatures below zero throughout the winter; if a daily mean temperature above freezing is recorded, this is subtracted from the AFDD. AFDD do not begin accumulating until the first sustained period of cold temperatures.

$$AFDD = \sum FDD \qquad [2]$$

Where FDD is the daily mean temperature in degrees Celsius, a negative freezing degree day value represents a temperature warmer than freezing, while a positive freezing degree represents a temperature below freezing (White, 2004).

b) Σ (Snow Depth x Snow Density)

Σ (Snow Depth x Snow Density) or Cum Snow, represents the cumulative daily sum (beginning once ice cover formation has occurred) of the product of the depth of the snow cover on the ice and the average snow density. This parameter was used to account for the insulating effects of the snow cover on the ice. Surface snow cover, especially a snow cover consisting of newly-fallen light-density snow, can significantly retard the growth of static ice by slowing the heat loss to the atmosphere (Davar, 1996).

$$CumSnow = \sum \rho_s \mathrm{H} \qquad [3]$$

Where ρ_s represents the average density of the snow cover, and H represents the depth of the snow cover on the ice.

i. Snow Density

The average density of the snow cover was estimated from the height of the snow cover on the ice and the cumulative precipitation which has occurred since the formation of the ice cover.

$$\rho_s = \frac{\sum P}{H} \qquad [4]$$

Where $\sum P$ represents the cumulative precipitation that has occurred since the formation of the ice cover. The density of snow is the fraction of snow volume occupied by its water equivalent. Snow density is computed as the depth of the measured water equivalent of the snow cover divided by the depth of the snow cover (Singh, 1992).

ii. Snow Cover on the ice

Snow depth on the ground is available from the Environment Canada's climate database. The snow cover on the ice was set to zero on the first day of ice formation; subsequent changes to the snow cover on the ground were added to the snow cover on the ice.

c) Cumulative Solar Radiation

Complete daily global solar radiation data was only available for one hydrometric station that was examined (10LC014 – Mackenzie River at Arctic Red River). Solar radiation is the measurement of radiant energy from the sun, on a horizontal surface. The standard metric unit of radiation measurement is the Mega Joule per square metre (MJ/m^2). Global solar radiation is a measure of the total incoming direct and diffuse short-wave solar radiation received from the whole dome of the sky on a horizontal surface (from National Climate Archives Glossary at:
http://www.climate.weatheroffice.ec.gc.ca/prods_servs/glossary_e.html#s).
A cumulative sum of the daily global solar radiation was used in the regression model for Mackenzie River at Arctic Red River.

d) Σ (Sunshine Hours x Solar Altitude)

Due to the lack of global solar radiation data available from Environment Canada's weather office, another parameter related to radiation, namely Σ (Sunshine Hours x Solar Altitude), was developed and included in the multiple linear regression analysis for 3 hydrometric stations (06AD006, 05CK004, and 05AJ001).

i. Daily Hours of Bright Sunshine

Daily Sunshine hours data were available for only a few climate stations. Bright sunshine observations are made using the Campbell-Stokes sunshine recorder. The recorder measures only "bright" sunshine which is less than visible sunshine. For example sunshine immediately after sunrise, just before sunset, or the presence of a cloud cover would not be bright enough to register.

ii. Solar Altitude

To calculate the average solar altitude, the sun's altitude at solar noon (maximum altitude) was divided by 2. The solar altitude at solar noon was calculated using the following formula:

$$\theta = 90^\circ - |\alpha - \beta| \qquad [5]$$

Where θ is the sun's altitude above the horizon at solar noon, α is the latitude of the site, and β is the sun's declination (Nelson, 2000). β is given by the following equation from Pillay:

$$\beta = 23.45 \sin\left[\frac{360(n-80)}{365.25}\right] \quad [6]$$

Where n is the day of the year with Jan 1 being n=1.

e) Top Ice Growth due to Snow Weight

An ice cover will tend to be weighed down by the presence of a large amount of snow. If there is a large amount of snow present on the cover, this may cause the ice to be submerged below the phreatic surface. In this case, it is likely that water will seep up through cracks in the ice cover, saturate the lower portion of the snow layer, and cause the formation of snow ice (Hicks, 2002). Calculations were done to estimate the extent of this snow ice growth using the depth of the snow cover on the ice, the estimated density of the snow cover, and the estimated WSBI of the ice.

First of all, the height of the ice equivalent of the snow cover (i.e. depth of the snow cover if it was converted into ice) was calculated by dividing the cumulative precipitation by the theoretical density of ice (0.92 g/cm^3).

$$H_s = \frac{\sum P}{\rho_i} \quad \textbf{[7]}$$

Where H_s is the height of the ice equivalent of the snow cover, $\sum P$ represents the cumulative precipitation since the formation of the ice cover, and ρ_i represents the density of ice.

Secondly, H_s is added to the model's estimate of WSBI from the previous day (t_{n-1}) and multiplied by 8%, the theoretical percentage of the ice cover found above the water's surface. This represents t_a, the theoretical thickness of the ice cover above the water surface when the weight of the snow cover is converted into an ice thickness.

$$t_a = 0.08(t_{n-1} + H_s) \quad [8]$$

To determine the height of the ice surface above the water surface (H_i), the ice equivalent of the snow cover (H_s) is subtracted from t_a.

$$H_i = t_a - H_s \quad [9]$$

If the value for H_i is negative, this amount represents the distance below the water surface that the ice cover would be submerged. It is assumed that if the ice cover is submerged below the water surface, water will seep through cracks, saturate the bottom snow layer, and freeze on the surface of the ice forming ice or snow ice. Therefore if the value of H_i is negative, this value represents the amount of snow ice growth that would occur on the ice surface. A positive value for H_i indicates no snow ice growth on the ice surface.

It must be stressed that the calculated values of top ice growth are **estimates** of the amount of snow ice growth on the ice surface. These values are based on the assumption that the ice cover is floating with 92% of its thickness below the water surface. These values are also based on estimates of ice thickness from the previous day, therefore depending on the accuracy of the model there could be some error associated with these values. In most cases the contribution of the top ice growth to the overall thickness of the ice was estimated to be minimal (< 8 cm in most cases).

f) Ice Cover Formation Dates

Estimates of the ice cover formation dates for all of the hydrometric stations were obtained from the HYDAT database published by the Water Survey of Canada and available at the following internet address:
http://www.wsc.ec.gc.ca/products/main_e.cfm?cname=products_e.cfm .
These estimates of ice cover formation dates were used along with WSBI data to develop the ice growth models. It was assumed that the ice cover formation dates represent an ice thickness of 0. In reality the ice cover may have an initial thickness on the date that it is formed. There may also be some error associated with the ice cover formation dates, as they are estimated (based on temperature data as well as changes in stage) and not physically observed. Following an analysis of the ice growth model at 06AD006 (Beaver River at Cold Lake, Alberta) it was determined that small variations in the ice formation dates (i.e. +/- 4 days) did not significantly affect the model. The multiple linear regression model obtained using the existing ice formation dates was compared with a multiple linear regression model using ice formation dates 4 days earlier and 4 days later than the ice formation date recorded in the HYDAT database. The results indicated that the differences in the models were insignificant (differences in the coefficients on the order of 1-3%, and differences in the constants of less than 5 mm).

6. Model Results

Statistical multiple linear regression models were developed for 11 sites out of the 30 sites that were examined in this study. Ten of these sites are located in Alberta and one site is located in the Northwest Territories. A wider spatial variation in sites was not chosen due to the lack of available data in provinces other than Alberta. The sites that were chosen in Alberta were chosen to reflect a wide variation in geographic location, river width, river depth, and river velocity. The sites in which statistical models were developed are summarized in Table 6.1 below.

Table 6.1 Hydrometric Stations Chosen for Development of Statistical Ice Growth Models

Hydrometric Station	Station Name	Latitude of Station	Longitude of Station	Location	Average Width of River (m)	Average Velocity (m/s)	Average Depth below ice (m)	Distance to Closest Climate Station (km)
05CK004	Red Deer River near Bindloss, Alberta	50.9	-110.3	South-East AB	69	0.37	0.64	20.9
06AD006	Beaver River at Cold Lake, Alberta	54.4	-110.2	North-East AB	16	0.30	0.40	8.4
10LC014	Mackenzie River at Arctic Red River, NWT	67.5	-133.7	North NWT	915	0.60	10.90	94.7
07GD004	Red Willow River near Rio Grande, Alberta	55.1	-119.7	North-West, AB	8	0.16	0.29	23.4
05DA007	Saskatchewan Crossing, Alberta	51.9	-116.7	South-West AB	12	0.20	0.30	54.3
05CB004	Raven River near Raven, Alberta	52.1	-114.5	South-Central AB	11	0.40	0.36	24.0
05AA023	Old Man River near Waldron's Corner, Alberta	49.8	-114.2	South-East AB	32	0.26	0.25	21.7
05AJ001	South Saskatchewan River at Medicine Hat, Alberta	50.0	-110.7	South-East AB	148	0.45	1.21	4.0
07AF002	McLeod River above Embarras River, Alberta	53.5	-116.6	Central-West AB	40	0.39	0.43	16.2
07BC002	Pembina River at Jarvie, Alberta	54.5	-114.0	North-East AB	56	0.37	0.43	20.4
05DC006	Ram River near the Mouth, Alberta	52.4	-115.4	South-West AB	26	0.40	0.30	30.6

Only WSBI measurements not containing slush or frazil ice were used in the development of the statistical ice growth models. This was due to the fact that the presence of slush and frazil ice at the bottom of the ice will confound the estimation of ice thickness (Huntington, 2003).

Estimated top ice growth was subtracted from the average WSBI measurements, before the multiple linear regression analysis was performed. This was done in order to separate the effects of the top ice growth due to the weight of the snow cover and the insulation effects of the snow cover (represented by Cum Snow). The multiple linear regression analysis was then performed on AFDD, Cum Snow, and in some cases Cumulative Solar Radiation (Cum Rad) or Σ (Sunshine Hours x Solar Altitude). The calculated value for top ice growth was added back into the model after the regression analysis was performed.

It was found in all of the cases that ice growth was a quadratic function of AFDD, therefore both the parameters AFDD and $AFDD^2$ were present in all of the models. The $AFDD^2$ parameter is likely due to the insulation effects of the ice cover in slowing down its own growth as the AFDD (and therefore ice thickness) increases. Cum Snow was found to be significant in some but not all of the cases. Cum Snow was found to have a much lower influence on ice thickness compared to AFDD. The parameters Cum Rad and Σ (Sunshine Hours x Solar Altitude) were tested at 4 different sites where radiation or daily sunshine hours data was available. Cum Rad and Σ (Sunshine Hours x Solar Altitude) were not found to be significant parameters in the models. It was found that both Cum Rad and Σ (Sunshine

Hours x Solar Altitude) were highly negatively correlated with AFDD. This caused the models to become overspecified and therefore inaccurate when both parameters were included. AFDD was the more significant parameter, therefore the stepwise regression analysis discarded Cum Rad and Σ (Sunshine Hours x Solar Altitude) from the models.

Table 6.2 shows a summary of the statistical models for all of the sites examined. These models are valid only for early and mid-winter periods when the ice growth is increasing, the models will not accurately predict the degradation of the ice cover. The adjusted R^2 values are included in the table to give an indication of how well the models explain the variability in the data. An analysis of residual plots indicated a valid regression model in all of the cases. Plots of residuals versus row order did not reveal any significant pattern; therefore there was no evidence of serial correlation. Based on the statistical analyses, the models developed for hydrometric stations 07GD004 and 05AJ001 are not considered to be accurate enough to be used to predict ice thickness at these sites since the adjusted R^2 values are quite low. The inaccuracy of the model at station 05AJ001 is likely due to insufficient data. The inaccuracy of the model at station 07GD004 is likely due to inconsistent measurement locations.

Table 6.2: Statistical Ice Growth Models for 11 Hydrometric Stations

Hydrometric Station	Constant Term	AFDD Coefficient	$AFDD^2$ Coefficient	Σ (Snow Cover x Snow Density) Coefficent	Adjusted R^2	Significant parameters in Model
05CK004	0.0579	0.00092	-2.73E-07	-0.00101	86.1	AFDD, $AFDD^2$, Cum Snow
06AD006	0.0287	0.00083	-1.41E-07	-0.00058	90.2	AFDD, $AFDD^2$, Cum Snow
10LC014	-0.0594	0.00063	-7.66E-08	Not significant	97.5	AFDD, $AFDD^2$
07GD004	0.0509	0.00069	-3.10E-07	Not significant	67.5	AFDD, $AFDD^2$
05DA007	0.0340	0.00169	-8.53E-07	-0.00174	91.2	AFDD, $AFDD^2$, Cum Snow
05CB004	0.0895	0.00155	-7.71E-07	Not significant	85.0	AFDD, $AFDD^2$
05AA023	0.0794	0.00172	-1.38E-06	Not significant	77.6	AFDD, $AFDD^2$
05AJ001	-0.0021	0.00141	-8.11E-07	Not significant	67.2	AFDD, $AFDD^2$
07AF002	0.0245	0.00103	-2.36E-07	-0.00035	87.6	AFDD, $AFDD^2$, Cum Snow
07BC002	0.0350	0.00090	3.04E-07	-0.00038	85.8	AFDD, $AFDD^2$, Cum Snow
05DC006	0.0386	0.00165	-1.03E-06	Not significant	78.6	AFDD, $AFDD^2$

The models and validations are summarized in the following sections. The comparisons between the WSBI values predicted from the model and the WSBI measurements are shown in the accompanying figures. Within the model equations, t_{avg} represents the average WSBI predicted at the site. All models have been validated against data from winters 2003-2004 and 2004-2005, with the exception of hydrometric station 10LC014 which was validated with data from winters 1998-1999, 2001-2002, 2002-2003, and 2003-2004.

a) 06AD006 Ice Growth Model

Based on 33 average WSBI data records and climate data from the winters of 1989-1990, 1997-1998, 1998-1999, 1999-2000, 2000-2001, and 2002-2003, the following ice thickness model was developed for hydrometric station 06AD006 (Beaver River at Cold Lake, Alberta).

$$t_{avg} = 0.0287 + 0.00083AFDD - 0.000000141AFDD^2 - 0.00058CumSnow \quad [9]$$

Figures 6.1 and 6.2 demonstrate that the model predicts WSBI within +/- 10 cm in all of the cases except for the measurement taken on February 4, 2004. This measurement appears to be a possible outlier as it does not follow the expected trend that the ice thickness continues to increase during extended cold periods. The measured WSBI on February 4, 2004 is actually lower than the WSBI on January 4, 2004 which is unexpected due to the extremely cold temperatures recorded during this time period. The change in AFDD between January 6, 2004 (previous measurement) and February 4, 2004 is 534.7.

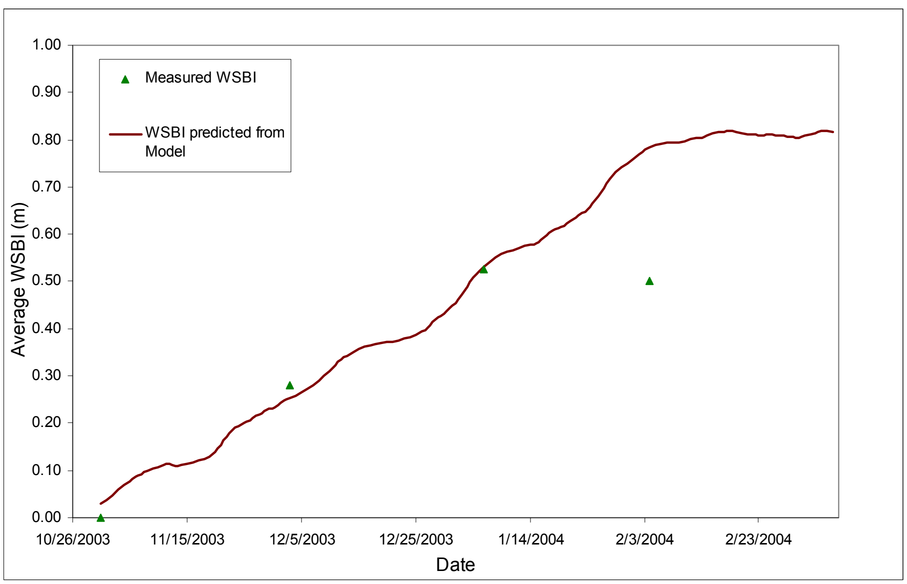

Figure 6.1 – Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 06AD006 During the Winter of 2003-2004

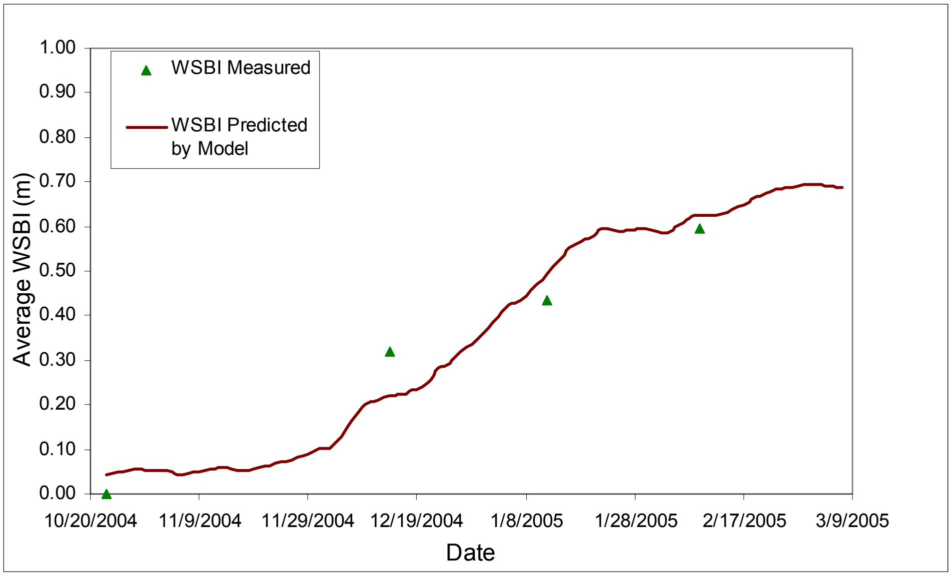

Figure 6.2 – Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 06AD006 During the Winter of 2004-2005

b) 05CK004 Ice Growth Model

Based on 32 average WSBI data records and climate data from the winters of 1996-1997, 1998-1999, 1999-2000, 2000-2001, and 2002-2003, the following ice thickness model was developed for hydrometric station 05CK004 (Red Deer River near Bindloss, Alberta).

$$t_{avg} = 0.0579 + 0.00092AFDD - 0.000000273AFDD^2 - 0.00101CumSnow \quad [10]$$

Figures 6.3 and 6.4 demonstrate that the model predicts WSBI within +/- 6 cm in all of the cases.

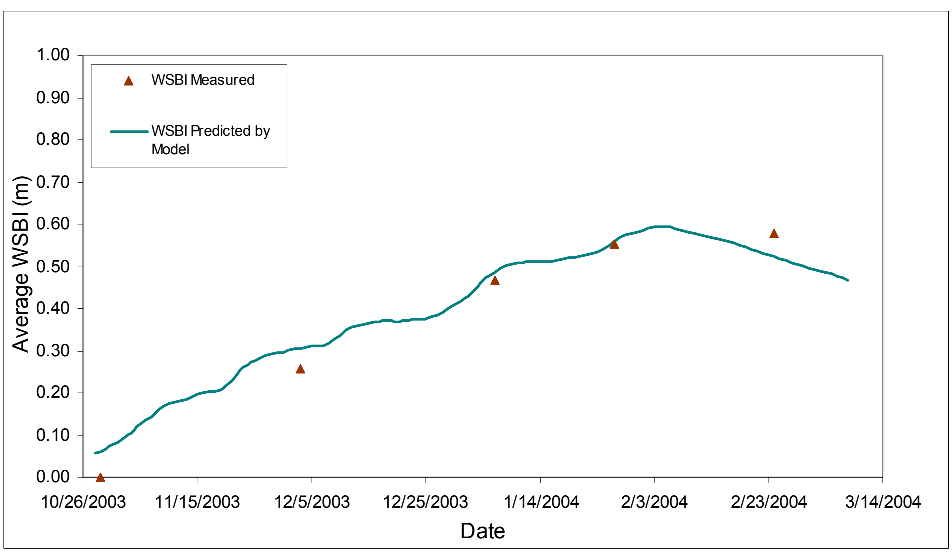

Figure 6.3 - Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 05CK004 During the Winter of 2003-2004

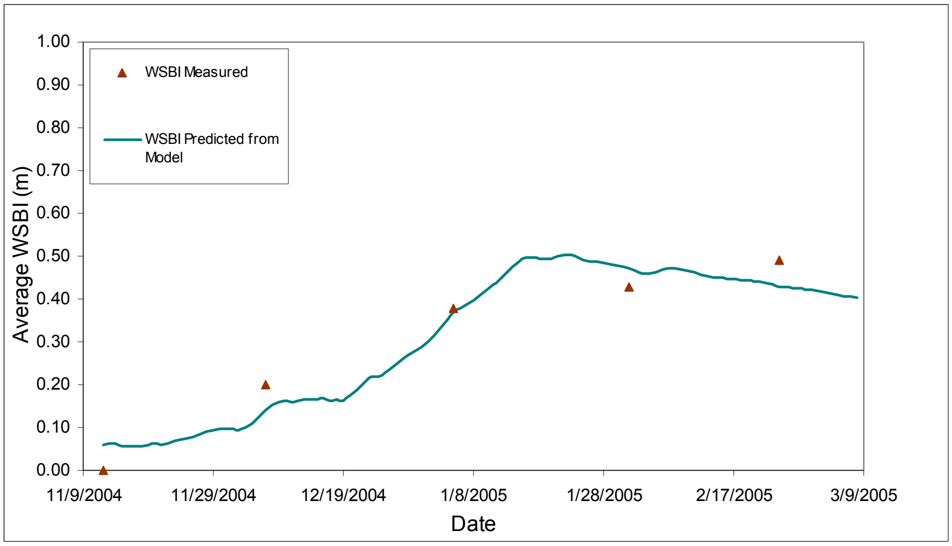

Figure 6.4 – Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 05CK004 During the Winter of 2004-2005

c) 05DC006 Ice Growth Model

Based on 29 average WSBI data records and climate data from the winters of 1997-1998, 1998-1999, 1999-2000, 2000-2001, and 2002-2003, the following ice thickness model was developed for hydrometric station 05DC006 (Ram River near the Mouth).

$$t_{avg} = 0.0386 + 0.00165 AFDD - 0.00000103 AFDD^2 \qquad [11]$$

Figures 6.5 and 6.6 demonstrate that the model predicts WSBI within less than +/- 14 cm in all of the cases. The accuracy of the model could likely be improved if more data were included in the model or if more accurate climate data were available.

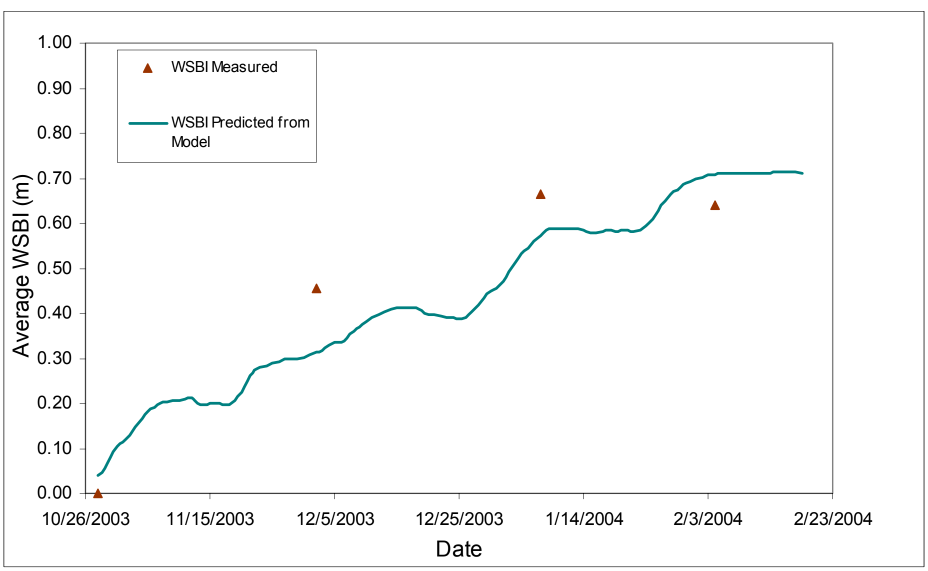

Figure 6.5 - Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 05DC006 During the Winter of 2003-2004

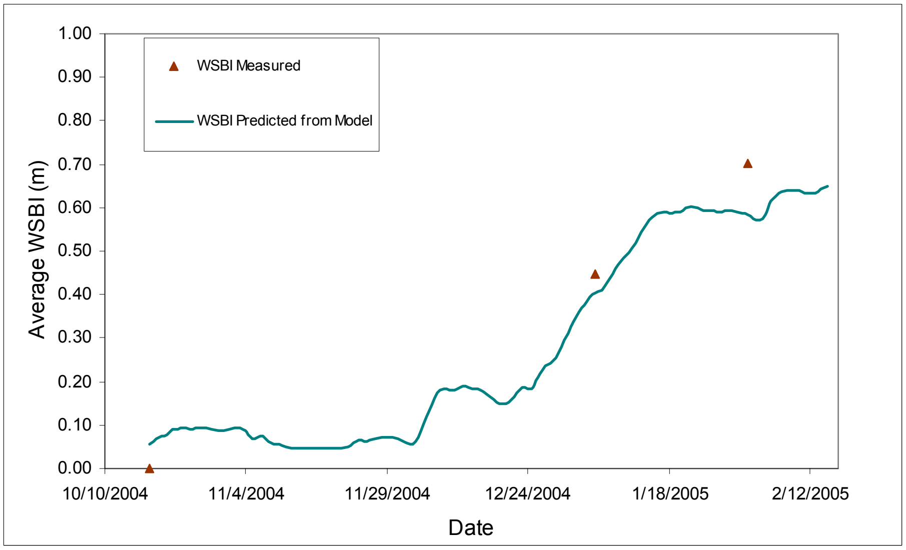

Figure 6.6 - Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 05DC006 During the Winter of 2004-2005

d) 05CB004 Ice Growth Model

Based on 31 average WSBI data records and climate data from the winters of 1997-1998, 1998-1999, 1999-2000, 2000-2001, and 2002-2003, the following ice thickness model was developed for hydrometric station 05CB004 (Raven River near Raven).

$$t_{avg} = 0.0895 + 0.00155AFDD - 0.000000771AFDD^2 \quad [12]$$

Figures 6.7 and 6.8 demonstrate that the model does not predict WSBI very well, particularly for the winter of 2004-2005. This model could likely be improved if more accurate climate data were available.

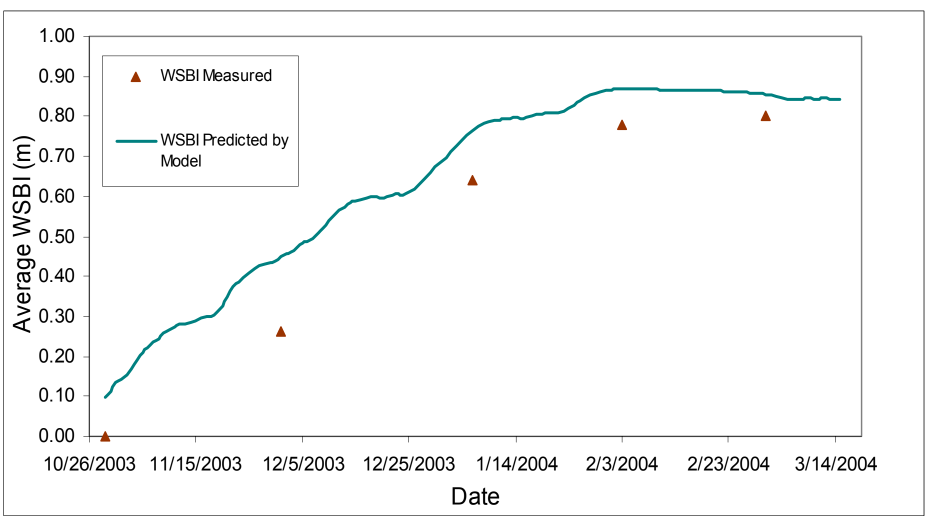

Figure 6.7 - Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 05CB004 During the Winter of 2003-2004

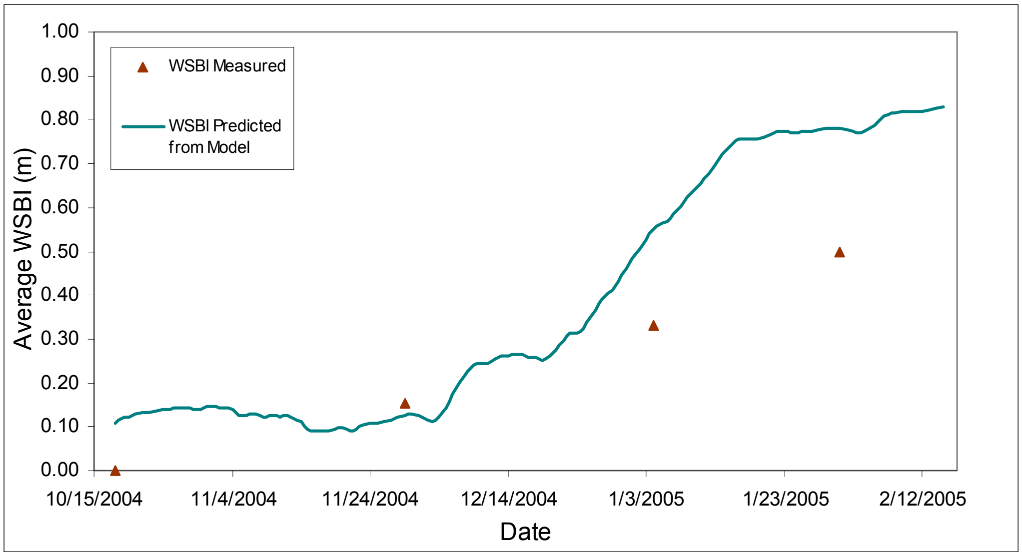

Figure 6.8 - Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 05CB004 During the Winter of 2004-2005

e) 10LC014 Ice Growth Model

Based on 23 average WSBI data records and climate data from the winters of 1984-1985, 1985-1986, 1995-1996, and 1996-1997, the following ice thickness model was developed for hydrometric station 10LC014 (Mackenzie River at Arctic Red River).

$$t_{avg} = -0.0594486 + 0.00063 AFDD - 0.0000000766 AFDD^2 \quad [13]$$

Figure 6.10 demonstrates that the model predicts the WSBI within +/- 10 cm for 2001-2002. Figure 6.11 demonstrates that the model predicts the WSBI within +/- 13 cm for 2002-2003. Figures 6.9 and Figures 6.12 show that the model is an accurate predictor of growth in the early stages of winter, but a poor predictor of growth in the later stages of winter. One possible reason for some of the inaccuracies in this model is the fact that the closest climate station is 95 km away from the hydrometric station. The installation of a temperature and precipitation sensor at the hydrometric station would improve the accuracy of the climate data and therefore the precision of the model.

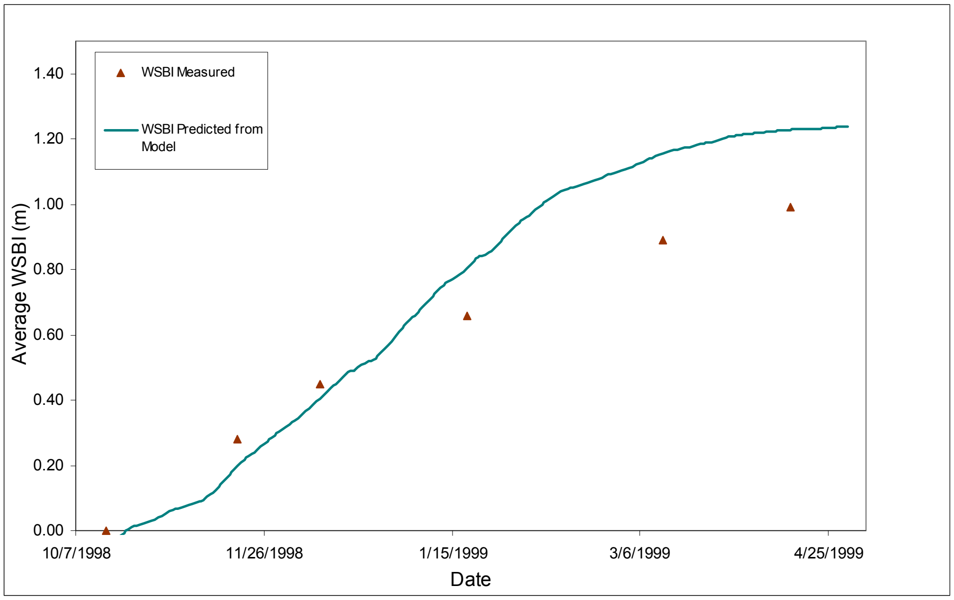

Figure 6.9 - Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 10LC014 During the Winter of 1998-1999

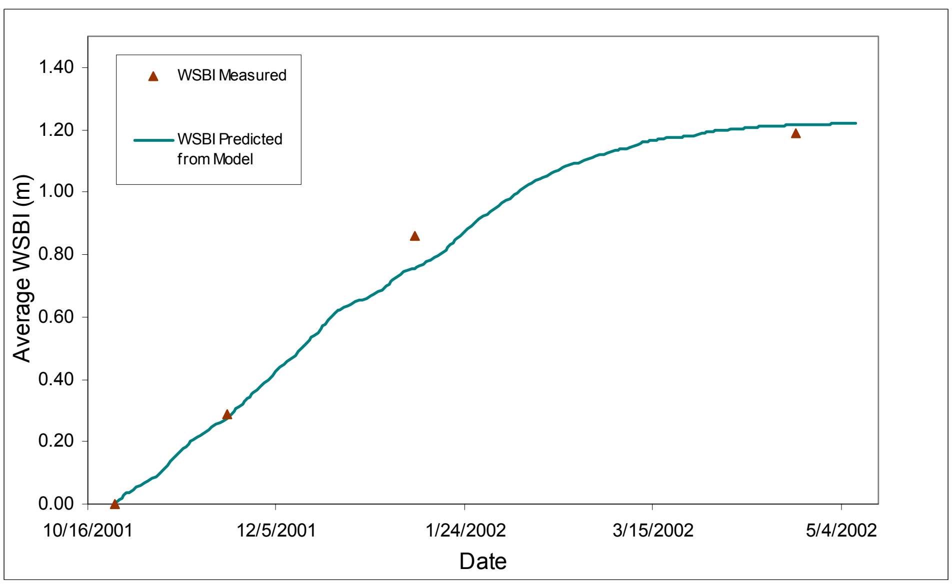

Figure 6.10 - Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 10LC014 During the Winter of 2001-2002

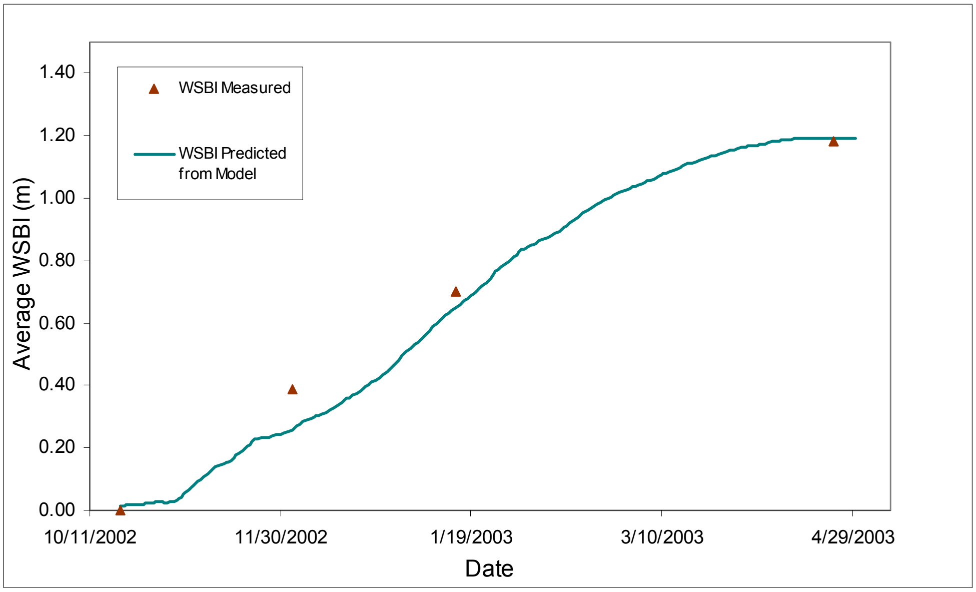

Figure 6.11 – Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 10LC014 During the Winter of 2002-2003

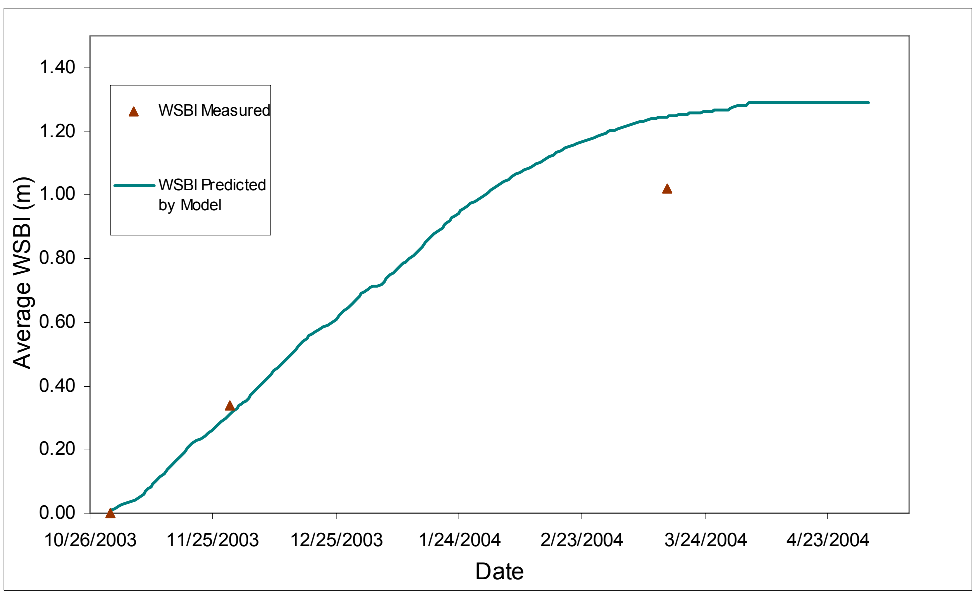

Figure 6.12 – Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 10LC014 During the Winter of 2003-2004

f) 07GD004 Ice Growth Model

Based on 22 average WSBI data records and climate data from the winters of 1998-1999, 2000-2001, 2001-2002, and 2002-2003, the following ice thickness model was developed for hydrometric station 07GD004 (Red Willow River near Rio Grande).

$$t_{avg} = 0.0509 + 0.00069AFDD - 0.00000031AFDD^2 \quad [14]$$

The measurements collected during the winter of 1999-2000 were not included in the model because upon examination of the data, it was concluded that these measurements were likely done at a different cross section. The river width data collected in 1999-2000 indicated an average river width of 1.5 m, whereas the data collected during the other years shows an average river width of 7.65 m. The velocity recorded during 1999-2000 was much larger than the velocity recorded in other years. This indicates that the WSBI data collected during the years 1999-2000 was likely measured at a different, narrower reach with larger velocities compared to the data collected during the other years. The WSBI measurements recorded during 1999-2000 were much lower in magnitude than the WSBI measurements recorded in other years at similar times during the winter. This also indicates that velocity could be a significant factor affecting ice thickness and contributing to spatial variations in ice thickness at this site.

Figures 6.13 and 6.14 demonstrate that the model is inadequate in estimating the WSBI at hydrometric station 07GD004. Upon further examination of the recorded WSBI values, it was observed that there were large variations in the average width measurements and the average velocity measurements of the WSBI measurements used to validate the model.

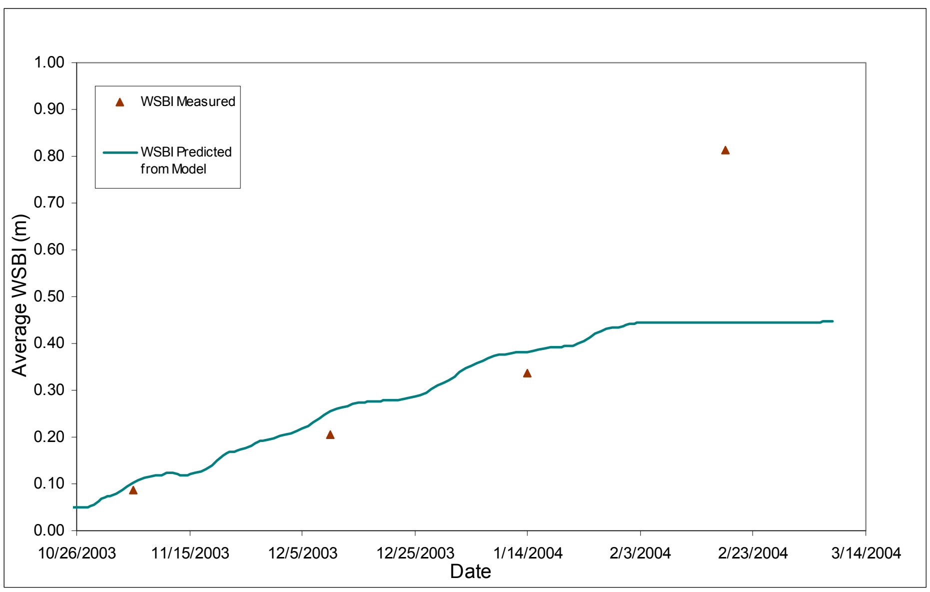

Figure 6.13 – Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 07GD004 During the Winter of 2003-2004

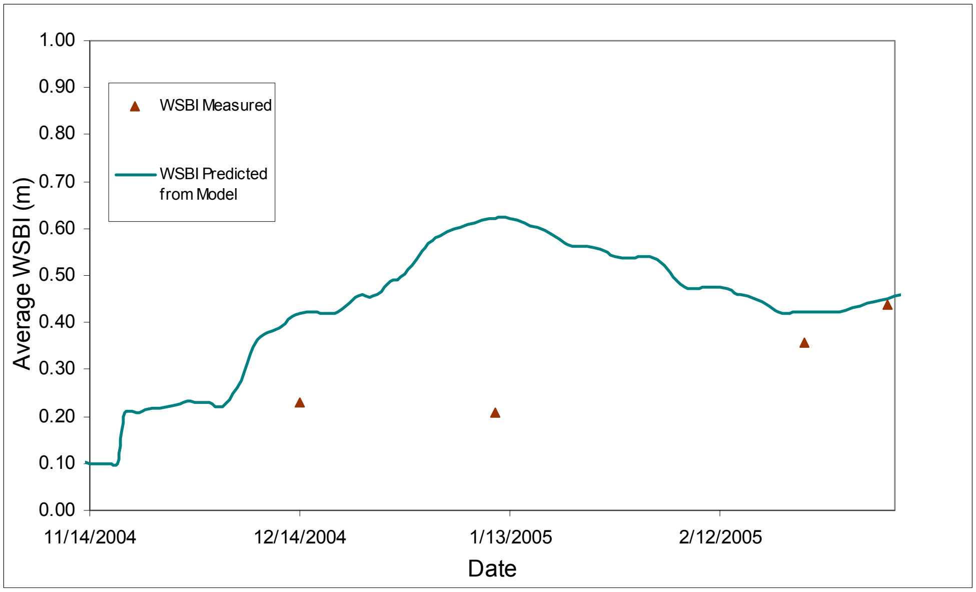

Figure 6.14 – Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 07GD004 During the Winter of 2004-2005

g) 05AA023 Ice Growth Model

Based on 33 average WSBI data records and climate data from the winters of 1989-1990, 1997-1998, 1998-1999, 1999-2000, 2000-2001, 2001-2002, and 2002-2003, the following ice thickness model was developed for hydrometric station 05AA023 (Oldman River near Waldron's Corner).

$$t_{avg} = 0.0794 + 0.00172 AFDD - 0.00000138 AFDD^2 \quad [15]$$

Figure 6.15 demonstrates that the model predicts WSBI within +/- 5 cm for 2003-2004. In Figure 6.16 it appears that there was a mid-winter ice break-up in early February (due to above zero mean daily temperatures in the last two weeks of January. A mid-winter break-up in February is extremely rare for this site (this type of event has not been recorded before in February at this site since records have been kept). The ice growth model is only valid during the ice growth period and is not robust enough to predict mid-winter ice break-up events. But Figures 6.15 and 6.16 indicate that other than the mid-winter ice break-up, the ice growth model was able to predict ice thickness to within +/- 5 cm.

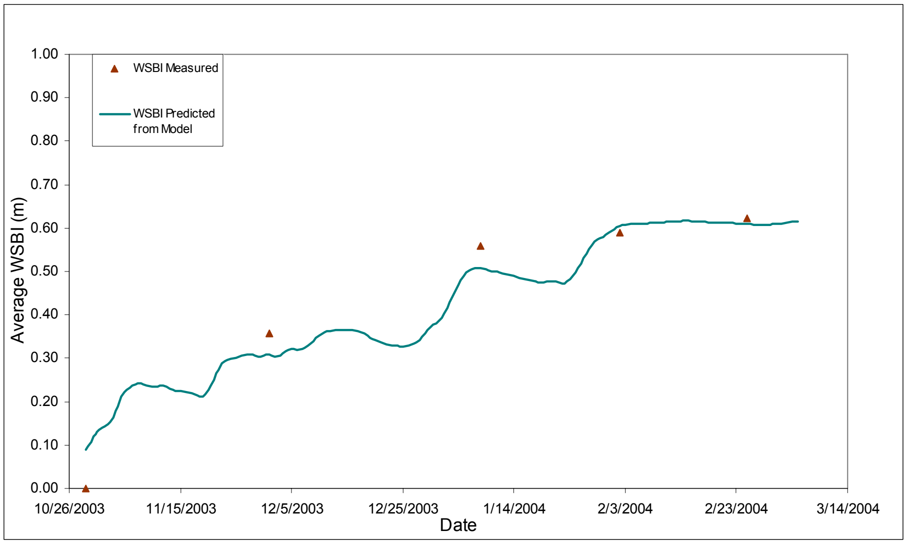

Figure 6.15 - Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 05AA023 During the Winter of 2003-2004

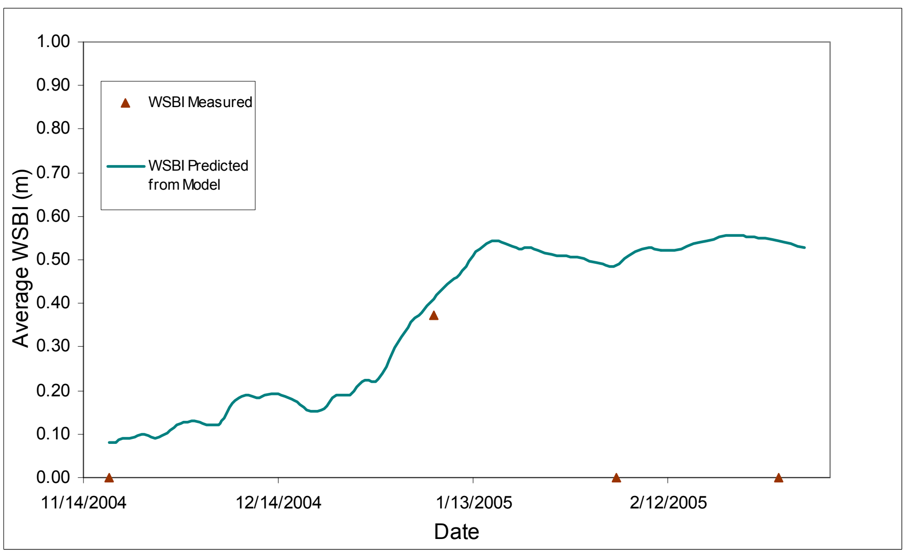

Figure 6.16 - Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 05AA023 During the Winter of 2004-2005

h) 07BC002 Ice Growth Model

Based on 31 average WSBI data records and climate data from the winters of 1989-1990, 1998-1999, 1999-2000, 2000-2001, 2001-2002, and 2002-2003, the following ice thickness model was developed for hydrometric station 07BC002 (Pembina River at Jarvie).

$$t_{avg} = 0.0350 + 0.00090\,AFDD - 0.000000304\,AFDD^2 - 0.00038\,CumSnow \quad [16]$$

Figures 6.17 and 6.18 show that the model predicts WSBI very poorly for both winters. The model overestimates WSBI by up to 37 cm for the winter of 2003-2004 and up to 22 cm for the winter of 2004-2005. The model predictions of WSBI tend to be more accurate in early stages of the winter, and as the winter progresses the predictions become less accurate.

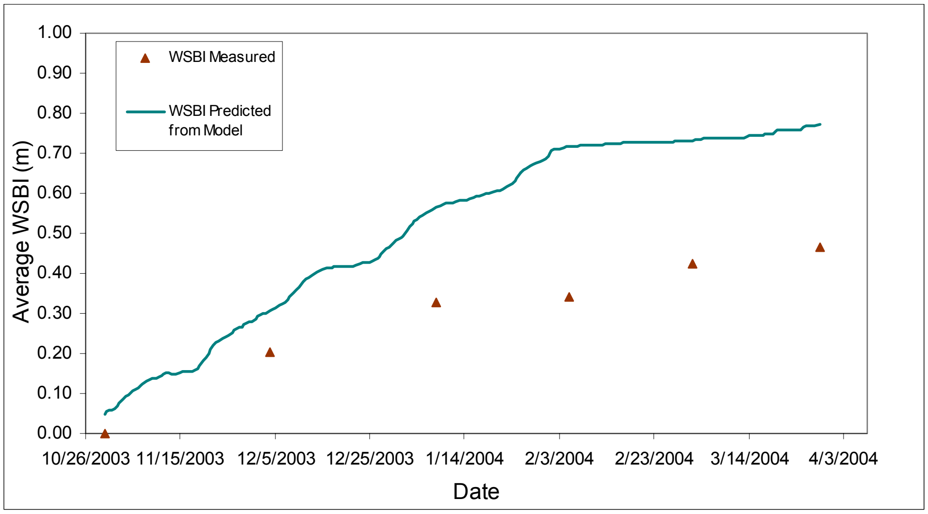

Figure 6.17 - Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 07BC002 During the Winter of 2003-2004

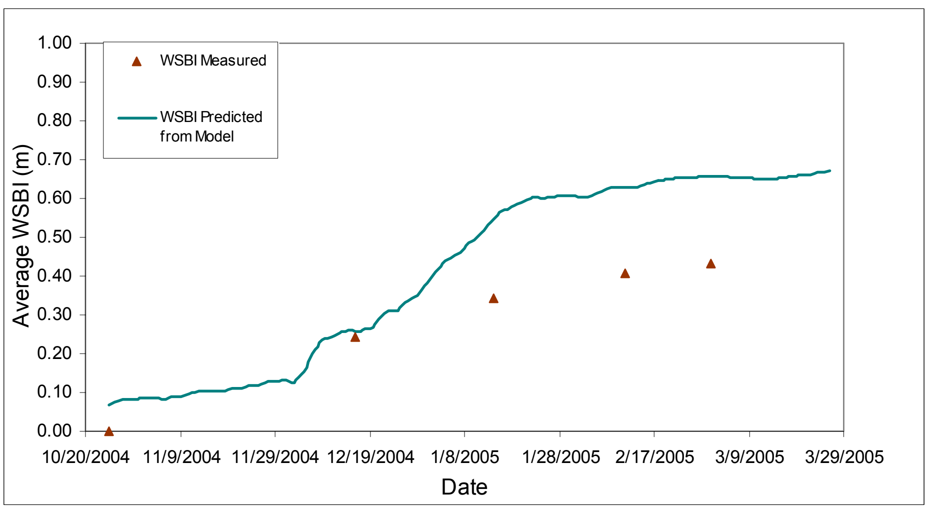

Figure 6.18 - Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 07BC002 During the Winter of 2004-2005

i) 07AF002 Ice Growth Model

Based on 30 average WSBI data records and climate data from the winters of 1997-1998, 1998-1999, 1999-2000, 2000-2001, 2001-2002, and 2002-2003, the following ice thickness model was developed for hydrometric station 07AF002 (McLeod River above Embarras River).

$$t_{avg} = 0.0245 + 0.00103AFDD - 0.000000236AFDD^2 - 0.00035CumSnow \quad [18]$$

Figures 6.19 and 6.20 demonstrate that the model predicts WSBI within 5 cm for the winter of 2003-2004 and within 15 cm for the winter of 2004-2005.

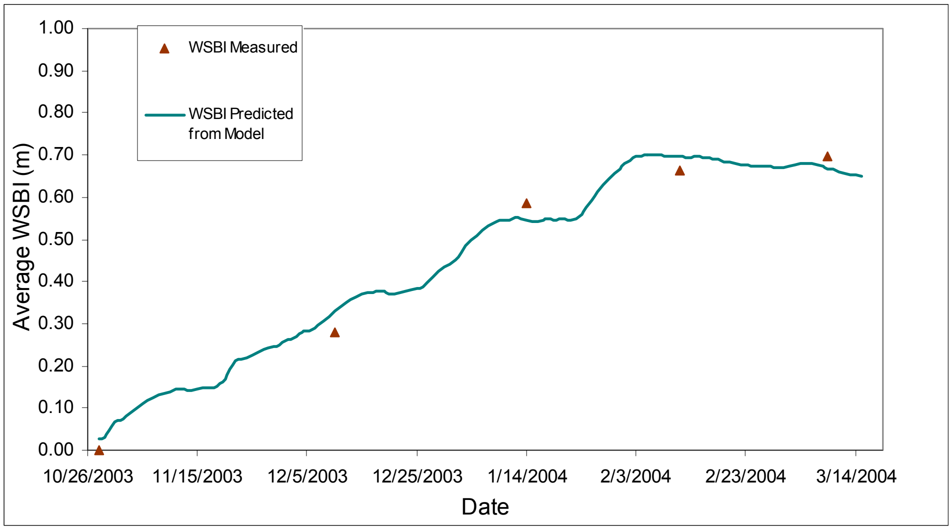

Figure 6.19 - Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 07AF002 During the Winter of 2003-2004

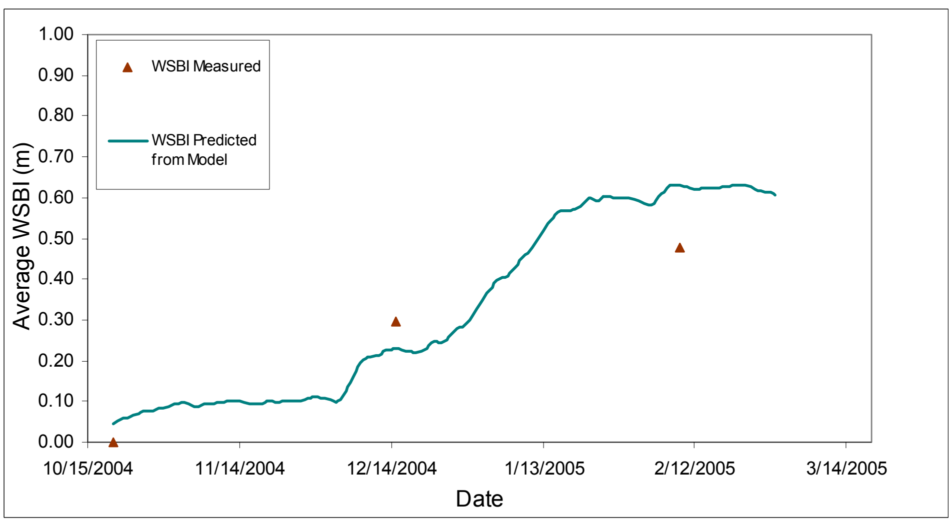

Figure 6.20 – Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 07AF002 during the Winter of 2004-2005

j) 05AJ001 Ice Growth Model

Based on 18 average WSBI data records and climate data from the winters of 1996-1997, 1998-1999, 2001-2002, and 2002-2003, the following ice growth model was developed for hydrometric station 05AJ001 (South Saskatchewan River at Medicine Hat).

$$t_{avg} = -0.0021 + 0.00141 AFDD - 0.000000811 AFDD^2 \qquad [19]$$

Figures 6.21 and 6.22 demonstrate that the model predicts WSBI within 13 cm for the winter of 2003-2004. The model predicts poorly the WSBI for the winter of 2003-2004. This ice growth model is not an accurate model for this site; this is primarily due to the small amount of data used to create the model (only 18 data records). The ice growth period at this site appears to be very short; therefore there is a lack of WSBI measurements taken during the ice growth period used for model construction and validation. The use of more data would improve the accuracy of the model.

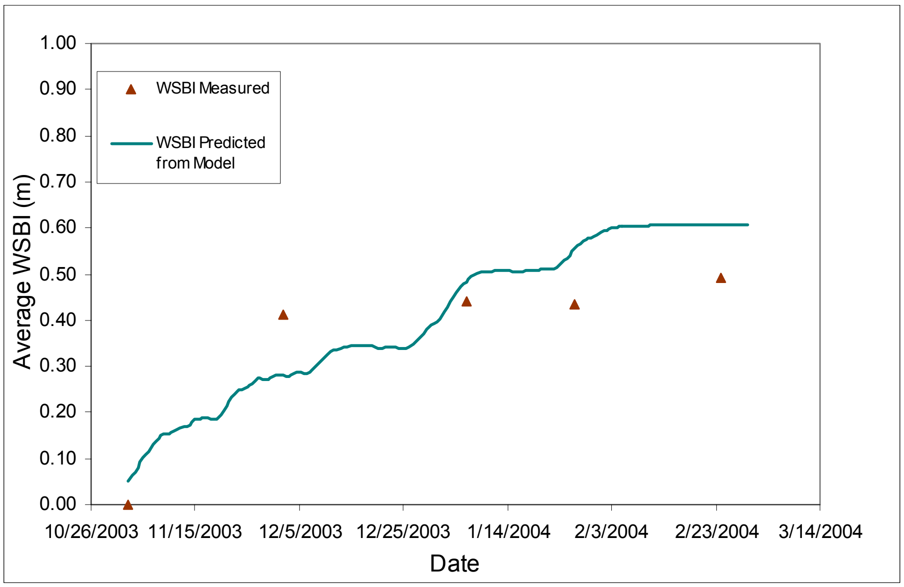

Figure 6.21 – Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 05AJ001 during the Winter of 2003-2004

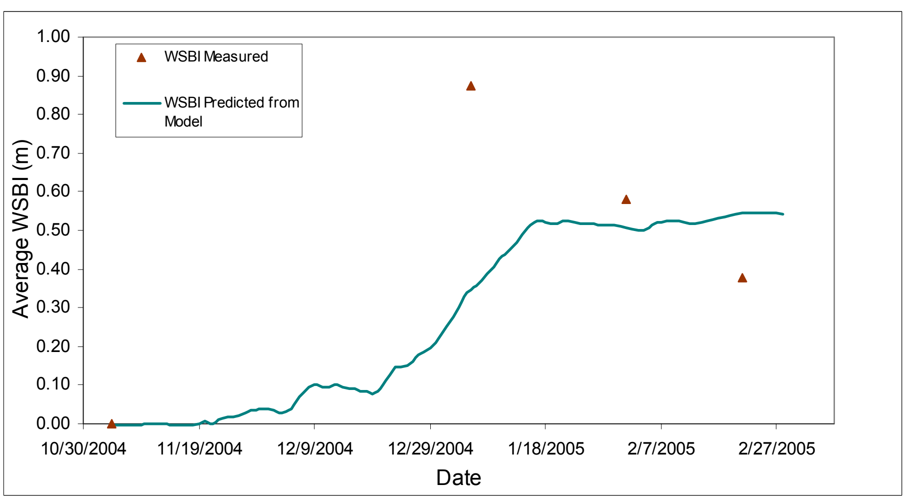

Figure 6.22 - Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 05AJ001 during the Winter of 2004-2005

k) **05DA007 Ice Growth Model**

Based on 20 average WSBI data records and climate data from the winters of 1997-1998, 1998-1999, 1999-2000, 2000-2001, 2001-2002, and 2002-2003, the following ice thickness model was developed for hydrometric station 05DA007 (Mistoya River near Saskatchewan Crossing).

$$t_{avg} = 0.034 + 0.00169AFDD - 0.000000853AFDD^2 - 0.00174CumSnow \quad [20]$$

Figures 6.23 and 6.24 demonstrate that the model overestimates WSBI by a maximum of 21 cm for the winter of 2003-2004. The model predicts poorly the WSBI for the winter of 2004-2005: the model overestimates WSBI by a maximum of 28 cm. This site also appears to have a short ice growth period, with the ice thickness beginning to decrease as early as the beginning of February. As a result, only 20 records were used to develop this model. If more measurements could be obtained during the ice growth period, a more accurate model could likely be developed.

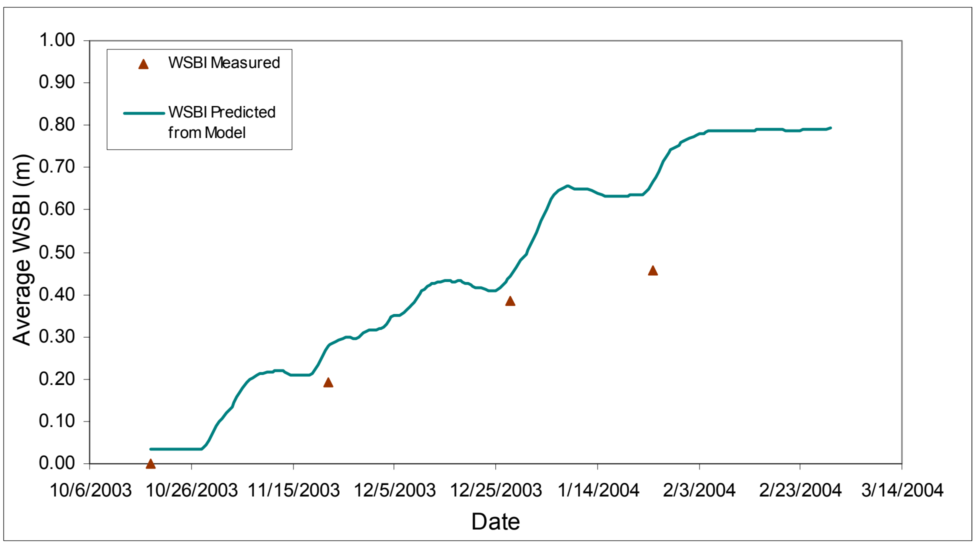

Figure 6.23 - Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 05DA007 during the Winter of 2003-2004

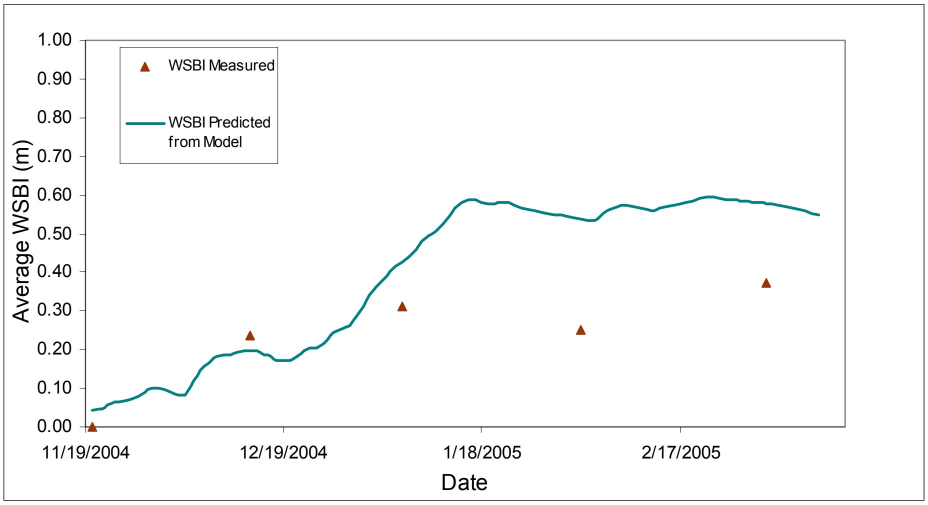

Figure 6.24 - Comparison Between the Model Predictions of WSBI and Measured Values of WSBI for Hydrometric Station 05DA007 during the Winter of 2004-2005

7. Sources of Error in the Models

a) Climate Data

The large distance between the climate stations and the hydrometric stations may contribute to errors in the accuracy of the climate data and therefore errors in the ice growth model. Also there was some climate data that was missing from some of the climate stations. Missing values were estimated from other available climate data or from climate data found at other climate stations. This may also contribute to errors in the models.

b) Location of Measurements

Another source of error in the measured data is the variation in the location of the ice measurements. Although all measurements were done at the same hydrometric stations, it has been observed that there is some variation in the cross-sectional profiles, which indicates that the locations of the measurements were not always consistent at a given site. It has been observed that natural spatial variations in ice cover thickness can be in the order of 20 % of the overall thickness (Hicks, 2002).

Spatial variations in ice cover thickness at nearby locations that experience the same climate conditions are likely due to hydraulic factors such as the velocity and depth of water beneath the ice (Huntington, 2003). Flow velocities are high (estimated at about 3 ft/s) through leads (areas of open water during the winter) (Ettema, 2001). This indicates that velocity is likely a contributing factor to the thickness of the ice. Sections with larger velocity would possess greater kinetic energy which may decrease the ice growth at these locations. The effect of velocity on the resultant thickness of the ice has not been taken into account in these ice growth models and would likely explain some of the variation between the model results and the measured data.

c) Insufficient Data

Some of the sites located in Southern Alberta had very short ice growth periods and therefore there was less ice growth data available to use in the development of the ice growth models. For example there were only approximately 20 records used to develop the ice growth models for hydrometric stations 05AJ001 and 05DA007; this may explain why the models developed at these sites were inadequate.

d) Average WSBI may not be an Accurate Representation of Panel WSBI

As discussed in section 3, average panel WSBI across the entire cross section was found to be close to the panel WSBI at the maximum discharge for hydrometric station 06AD006 for early and mid-winter (the differences were less than 8 cm). However, there were some large variations in panel WSBI observed at some stations that may contribute to some error with the ice growth models.

8. Conclusions

As a result of developing statistical ice growth models for these hydrometric stations, it was found that AFDD is the most significant parameter affecting the thickness of river ice. The fact that all of the ice growth models developed were quadratic functions of AFDD, reveals that as AFDD increases and the thickness of ice increases, the ice becomes self-insulating and the ice growth slows. At about half the stations it was found that Σ (Snow Depth x Snow Density) was also a significant parameter in the ice growth model (although not as significant as AFDD). Ice growth on top of the ice caused by the weight of the snow cover pushing downwards on the ice and water seeping through cracks in the surface forming snow ice, was also predicted by the ice growth model. Typical top ice growth throughout the winter was predicted to be less than 8 cm. Cumulative Radiation values or Σ (Sunshine Hours x Solar Altitude) were not found to be significant parameters in the ice growth models as they were negatively correlated with AFDD.

Models at the following hydrometric stations were found to adequately represent the river ice thickness after validation with 2004-2005 data:

- 06AD006, maximum error -10 cm
- 07AF002, maximum error +15 cm
- 05CK004, maximum error +/- 6 cm
- 05DC006, maximum error -14 cm
- 05AA023, maximum error +/- 5 cm
- 10LC014, maximum error +22 cm (but this error represents only about 20 % error because the ice thickness at this station grows to over 1 m thick)

Models at the following hydrometric stations were found to be inadequate at representing the river ice thickness either because of large discrepancies when the model was validated, or the statistical analysis revealed that the model was not adequate (i.e. low R^2 value):

- 05CB004, poor validation

- 07GD004, low R^2 value due to inconsistent measurement locations
- 07BC002, poor validation
- 05AJ001, low R^2 value, insufficient data available for model
- 05DA007, insufficient data available for model

Although some of the models were found to be inadequate at predicting WSBI, there are quite a few models that do adequately predict WSBI. As a result, it can be concluded that with the elimination of some of the sources of error (i.e. obtaining more accurate climate data, more WSBI data, and ensuring that WSBI measurements are carried out consistently at the same location), adequate river ice thickness models can be developed for hydrometric stations using this technique.

It should also be noted that this modeling technique does not take into account hydraulic parameters such as velocity which likely contributes to spatial variations in river ice thicknesses. The absence of this parameter in the model may account for some of the error in the ice growth models.

9. Recommendations

- Install temperature and precipitation sensors at hydrometric stations that are located more than 20 km from the nearest Climate station to obtain more accurate climate data.
- Require the hydrometric technicians to record the location of each WSBI measurement (i.e. distance upstream or downstream of gauge).
- Require the hydrometric technicians to record the measurement of the water surface to the top of the ice. This will reveal information on whether or not the ice cover is floating. This will also eliminate errors associated with the estimation that WSBI is 92% of ice thickness.
- At stations with shorter ice growth periods, WSBI measurements should be done more frequently during the ice growth period.
- The effect of velocity on river ice thicknesses should be examined. It would be beneficial to study the relationship between ice thickness and velocity by monitoring the ice thicknesses and corresponding velocities at different locations near a hydrometric station (i.e. such that the climate effects would remain constant).

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