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A model for the vertical distribution of frazil ice

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In cold regions, ice problems originate from the generation of frazil ice in rivers. Frazil ice evolves in number and size during the supercooling process. There is difficulty describing the evolution of frazil ice because of the complicated interactions between the frazil and flow turbulence and hydrothermal conditions. Based on a series of experiments using a counter-rotating flume we described a new approach to the supercooling process that defines the principal period of supercooling as a process whereby the frazil ice reaches the entrained frazil ice generating capacity of the flow. The model relates flow turbulence and frazil size. In this paper we further develop the approach, resulting in a model describing the vertical frazil ice distribution. The model is verified by frazil ice image observations obtained from laboratory experiments.

Introduction

The presence of ice in rivers can have a significant impact on water resources infrastructure. Virtually all of the rivers in Canada experience ice effects each year, and in many cases river ice has produced the most extreme and dangerous flood events on record. River ice causes other problems of particular concern in cold regions, in addition to flooding. On rivers regulated for hydro-power production, river ice can severely limit the power production potential of such facilities (Raban, 1995). Ice can also endanger fish, by encroaching on their over-wintering habitats (Prowse, 2001).

Water cooled at the surface is quickly mixed through the depth by flow turbulence and results in the spontaneous generation of "frazil ice" particles (small discs of ice ranging in size from μm to mm in diameter) throughout the entire water column. These frazil ice particles are highly adhesive and will readily freeze to each other, and to rocks on the river bed resulting in anchor ice growth. As an example, Manitoba Hydro is experiencing losses in generation revenue due to anchor ice growth downstream of the Limestone Generating Station (Girling and Groeneveld, 1999).

Many experiments and observations of frazil ice have been made over the last several decades. These observations include the water temperature response to frazil particle growth (Michel, 1963), the effects of fluid turbulence on the temperature response (Carsten, 1966; Muller, 1978; Ettema et al., 1984), the size distribution of frazil particles (Bukina, 1967; Daly and Colbeck, 1986) and field observations (Osterkamp et al., 1983; Tsang, 1986). Only a single vertical concentration profile obtained from Lachine Rapids on the St. Lawrence River has been reported in the literature. Theoretical models for the vertical distribution of frazil were proposed by Tsang (1988) and Liou and Ferrick (1992). A simple empirical equation of vertical distribution was developed by Tsang and Cui (1994) based on the data obtained from their experiments.

Although great advancements on frazil dynamics have been made, our understanding of these processes is still limited and so is our ability to further develop models of these processes. In the state-of-the-art river ice model, CRISSP, frazil ice properties, such as its size and rise velocity are assumed empirically, or considered as calibration parameters (Shen, 2002).

Priset and Hausser (1961) proposed that frazil ice might be considered as a sediment and that one might use sediment transport equations to describe frazil ice transport in rivers. However, the validity of this analogy is not clear yet (Daly, 1994). The greatest difficulties with this analogy are with respect to frazil size, a description of its variation with time, and its heat transfer properties (Tsang, 1988). Shen and Wang (1995) proposed to consider frazil ice transport under an ice cover as a specific type of sediment transport.

A series of experiments at the University of Manitoba were carried out in the counter-rotating flume (Tsang and Cui, 1994; Clark and Doering 2002) in a cold room at the Hydraulics Research & Testing Facility (HRTF). Based on the findings of these experiments, a new approach for frazil evolution was proposed (Ye, 2002), this work related frazil size, flow turbulence, and air temperature (Ye and Doering, 2003). Herein we further develop this approach to generate a model for the vertical distribution of frazil particles during the supercooling process.

Model Derivation

Ye and Doering (2001) developed a model for the size distribution of frazil ice based on the probability distribution of v'. Herein, a model for the vertical distribution of frazil ice is derived based on similar principles. It is worth noting that a frazil ice particle is treated as a "negatively" buoyant sediment, i.e., its rise velocity, ω, has a negative value. For any given point on a vertical line in two-dimensional flow, the diffusion equation has the form

$$\frac{dC}{dy} = \frac{\omega}{\varepsilon_s} C \,, \tag{1}$$

where ε_s is the diffusion coefficient, C is the point volumetric concentration at y, and y is the vertical distance measured from the bed. For sediment transport, Ni and Wang (1991) proposed

$$\varepsilon_s = \frac{1}{2} l_1 \overline{|v'|} \,, \tag{2}$$

where v' is the vertical fluctuating velocity. The characteristic length for the vertical frazil movement is l_1, or the vertical component of the Lagrangian trajectory of the particle, which is different from the so-called mixing length l_0 (Ni and Wang, 1991). To account for the differences between frazil ice shape and density in comparison to sediment, a factor is proposed and incorporated into (2) for frazil ice-laden flow, i.e.,

$$\varepsilon_s = \frac{1}{2} \frac{\rho_s}{K_v \rho_i} l_1 \overline{|v'|} \,, \tag{3}$$

in which ρ_i and ρ_s are the densities of ice and sediment, typically 920 and 2650 kg/m^3, respectively, and K_v is a volumetric shape factor, 0.785 (Ye and Doering, 2001). Since v' is described by a normal distribution, i.e.,

$$f(v') = \frac{1}{\sqrt{2\pi}\sigma_{v'}} \exp\left[\frac{-v'^2}{2\sigma_{v'}^2}\right] , \tag{4}$$

where

$$\sigma_{v'} = \sqrt{\overline{v'^2}} \,, \tag{5}$$

then

$$\overline{|v'|} = \int_0^{+\infty} f(|v'|)\, v' dv' = \sqrt{\frac{2}{\pi}} \sqrt{\overline{v'^2}} \,. \tag{6}$$

By combining (1), (3), and (6), a diffusion equation is obtained

$$\frac{dC}{dy} = \frac{K_v \frac{\rho_i}{\rho_s} \sqrt{2\pi} \frac{\omega}{u_*} \cdot C}{l_1 \frac{\sqrt{\overline{v'^2}}}{u_*}} \tag{7}$$

Since $\sqrt{\overline{v'^2}} / u_* = F(y/h) = \sigma_{v'} / u_* = \alpha$, a value for α can be computed; a value of 1.05 is used in this study. This approximation is acceptable in most of the flow region according to several measurements by Zhang et al. (1990). Then

$$l = l_1 \frac{\sqrt{\overline{v'^2}}}{u_*} = l_1 \alpha . \tag{8}$$

The following form of the characteristic length

$$l_1 = \frac{\rho_s l}{\rho_i \alpha} = \frac{\rho_s y}{\rho_i \alpha}\left(1 - \frac{y}{H}\right) \tag{9}$$

is chosen, simplified from Ni and Wang (1991) with consideration for the density of ice.

Substituting (9) into (7), the following solution is obtained by the integration of

$$\int_{C_a}^{C} \frac{dC}{C} = \int_{a}^{H} \frac{\alpha K_v \frac{\rho_i^2}{\rho_s^2} \sqrt{2\pi} \frac{\omega}{u_*}}{y\left(1 - \frac{y}{H}\right)} dy \tag{10}$$

Equation (10) can be written in the form

$$\frac{C}{C_a} = \left(\frac{a}{y} \cdot \frac{1 - \frac{y}{H}}{1 - \frac{a}{H}} \right)^{-\alpha K_v \left(\frac{\rho_i}{\rho_s}\right)^2 \sqrt{2\pi} \frac{\omega}{u_*}} = \left[\frac{a}{y} \cdot \frac{1 - \frac{y}{H}}{1 - \frac{a}{H}} \right]^{-z} , \tag{11a}$$

where

$$z = \alpha K_v \left(\frac{\rho_i}{\rho_s} \right)^2 \sqrt{2\pi} \frac{\omega}{u_*} \tag{11b}$$

is called the frazil suspended index, and C_a is the frazil ice concentration by volume at a distance $y = a$ from the bed. Using the stable characteristic of $\sqrt{v'^2}$, a very interesting result is obtained. With the exception of the negative power in (11) and a factor of $\alpha K_v \left(\frac{\rho_i}{\rho_s} \right)^2$, which accounts for the difference between frazil and sediment, the suspension index $\sqrt{2\pi}(\omega / u_*)$ has exactly the same form as the well-known Rouse equation. This occurs because the equality $\sqrt{2\pi} = 1/\kappa \approx 2.5$ is always true for the commonly used value of κ, 0.4. Thus (11) could be called the Rouse equation of frazil ice distribution.

The well-known Rouse vertical distribution has been proven to be qualitatively true for suspended sediment. However, its quantitative examination is difficult because it requires the fall velocity of the sediment and the concentration of the sediment at one point and they are difficult to obtain. The application of the Rouse equation to frazil distribution, of course, will be further handicapped because of the varied rise velocity of frazil ice particles (Tsang, 1988). However, based on our proposed model of the supercooling process and frazil evolution, at any time during the principal supercooling, the frazil ice and frazil distribution in channels could be satisfied with a size distribution, and the corresponding frazil ice concentration determined by flow turbulence and heat balance. At the end of the principal supercooling, the mean frazil diameter (D_{50}) and the entrained frazil generating capacity of a flow (M_v) is known, the vertical frazil ice distribution and the capacity have a relationship as follows

$$M_v = \int_0^H uC dy . \tag{12}$$

The von Karman-Prandtl logarithmic law is

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{y}{H} + const \tag{13a}$$

or

$$\frac{u}{u_*} = \left(\frac{U}{u_*} + \frac{1}{\kappa} \right) + \frac{1}{\kappa} \ln \frac{y}{H} . \tag{13b}$$

Using (12) and (13), letting $\xi = y/H$, following the procedure of Guo and Wood (1995), and considering the negative buoyant velocity of frazil ice, the frazil ice capacity equation (12) can be written as follows

$$M_v = H \int_0^1 uC d\xi \tag{14a}$$

$$M_v = Hu_*C_a\left(\frac{\xi_a}{1-\xi_a}\right)^{-z}\left[\left(\frac{U}{u_*}+\frac{1}{\kappa}\right)J_1+\frac{1}{\kappa}J_2\right]$$
$$= Hu_*C_a\left(\frac{1-\xi_a}{\xi_a}\right)^{z}\left[\left(\frac{U}{u_*}+\frac{1}{\kappa}\right)J_1+\frac{1}{\kappa}J_2\right], \tag{14b}$$

in which

$$J_1 = \int_0^1\left(\frac{1-\xi}{\xi}\right)^{-z}d\xi = \int_0^1\left(\frac{\xi}{1-\xi}\right)^{z}d\xi = \frac{z\pi}{\sin z\pi}, \tag{15}$$

$$J_2 = \int_0^1\left(\frac{1-\xi}{\xi}\right)^{-z}\ln\xi d\xi = \int_0^1\left(\frac{\xi}{1-\xi}\right)^{z}\ln\xi d\xi = \frac{z\pi}{\sin z\pi}\left[f(z)-1\right], \tag{16}$$

and

$$f(z) = z\sum_{n=0}^{500}\left[(n+1)(z+n+1)\right]^{-1}. \tag{17}$$

Then

$$M_v = Hu_*C_a\left(\frac{1-a}{a}\right)^{z}\left[\frac{U}{u_*}+\frac{f(z)}{\kappa}\right]\frac{z\pi}{\sin z\pi}. \tag{18}$$

Alternatively, integrating (11) can give the mean concentration $\overline{C}$ as

$$\frac{\overline{C}}{C_a} = \left(\frac{1-\xi_a}{\xi_a}\right)^{z}\frac{z\pi}{\sin z\pi}. \tag{19}$$

The corresponding position $\overline{y}$ or $\overline{\xi}$ for $\overline{C}$ is

$$\overline{y} = H\overline{\xi} = H\left[1+\left(\frac{z\pi}{\sin z\pi}\right)^{-1/z}\right]^{-1}. \tag{20}$$

If (20) is substituted in (18), the capacity of the flow is

$$M_v = Hu_*\overline{C}\left[\frac{U}{u_*}+\frac{f(z)}{\kappa}\right]. \tag{21}$$

If the mean concentration and its location are chosen as the reference point, then (11) becomes

$$\frac{C}{\overline{C}} = \left[\frac{\overline{\xi}}{1-\overline{\xi}} \cdot \frac{1-\xi}{\xi} \right]^{-z} . \tag{22}$$

Model verification and discussion

The only published field frazil ice measurement is from Lachine Rapids (Tsang, 1986). However, no detailed hydrothermal data accompany these measurements to verify this model.

The vertical distribution model can be verified by frazil observations using a Digital Image Processing System (Doering and Morris, 2003) in the studies of Ye (2002). These observations contain information about the vertical distribution of frazil, but are subject to limitations of the DIPS, namely grabbing and recognizing a frazil ice particle smaller than a certain size (2 mm). However, the system can still provide important quantitative data regarding the vertical distribution of frazil ice.

In order to compare the modeled vertical distributions of frazil ice, the capacity of entrained frazil ice (M_v) is replaced by an instantaneous mass of frazil ice, which is computed from the sum of all frazil ice particles in an image; the mean frazil ice diameter is determined by hydraulic variables (Ye and Doering, 2001). Therefore, the modeled vertical distribution is a relative profile, showing only the tendency or shape of the profile; it is not an absolute value. An absolute profile could be obtained if all the frazil particles/mass existing in the flow could be measured. However, the DIPS has not yet been developed to provide this information.

Table 1. Experiments for examining the vertical distribution of frazil ice.

Exp.	U [m/s]	H [m]	M_* [%]	u_* [m/s]	D_{50} [cm]	ω [cm/s]	z
60	0.424	0.15	0.133	0.0309	0.4	1.9	0.152
63	0.612	0.15	0.106	0.0444	0.84	6.66	0.373
69	0.641	0.15	0.104	0.0466	0.92	7.83	0.417

To consider the impact of flow velocities, 3 tests are selected to verify the model (Table 1). All of the experiments selected are conduced at an air temperature of -10°C. A regressed equation from field measurements (Gosink and Osterkamp, 1983) is applied to determine the rise velocity

$$\omega = 226 D_{50}^{1.7} . \tag{23}$$

Note, the rise velocity and diameter of frazil particles are in units of m/s and m, respectively.

Figures 1 to 2 show the vertical distribution of frazil at the end of the principal supercooling process, that is, at the observed peak frazil area during the supercooling period for these experiments. All the figures show that the observed and modeled distributions are quite comparable. These distributions also show that higher velocities yield a less uniform profile due to larger frazil particles and the associated higher rise velocities. Consequently, this yields a higher suspension index, z .

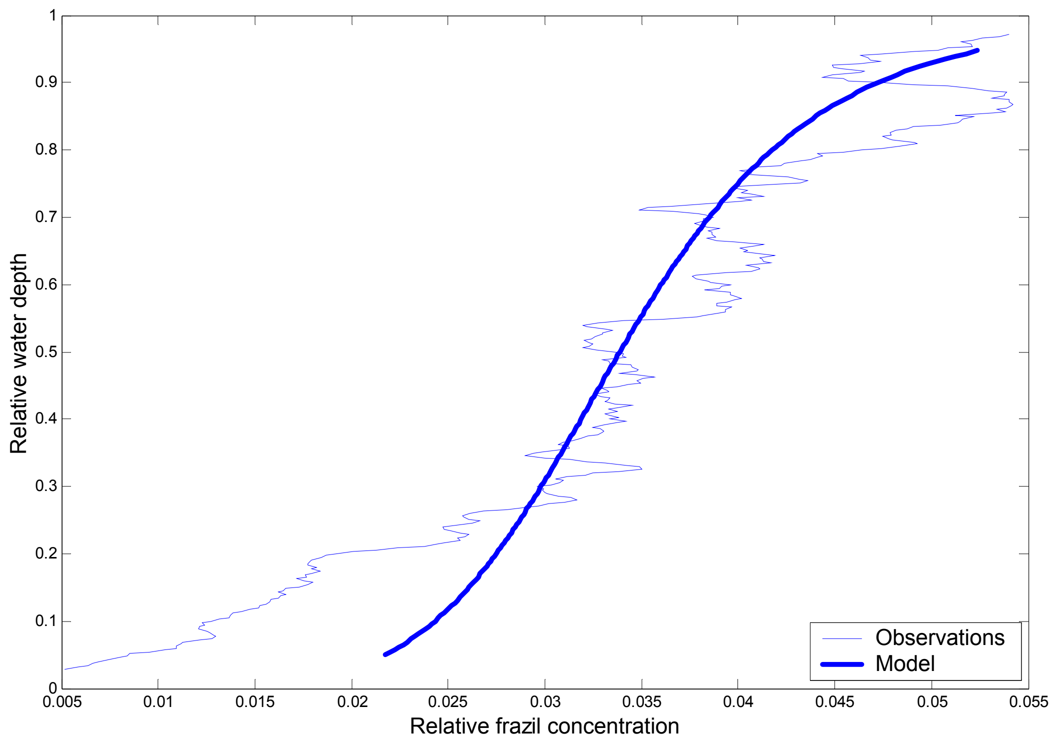

Figure 1. Vertical profile of frazil concentration for Exp. 60 at t = 27 min.

Figure 3 shows the vertical frazil distributions at other times during the supercooling process for Exp. 63. The four distributions are similar except for the absolute values. All of the distributions can be described using the same mean diameter of frazil size and the same rise velocity as at the saturated state of the entrained frazil ice formation, *i.e.*, at the end of the principal supercooling process. This finding further supports the approach that the frazil size distribution is governed by the flow turbulence. It can be considered the same during the principal supercooling process, unless the hydraulic conditions change due to the formation of a surface frazil ice layer.

Summary

A model for the vertical distribution of frazil ice was proposed and tested. The model provides an opportunity to describe frazil ice characteristics and its vertical distribution during the principal supercooling period for practical engineering applications. However, further verification of this model would be possible when field measurements are available. In addition, more experiments are needed to assess the role of flow turbulence and air temperature on frazil distributions during the residual supercooling process, which is more significant in practical engineering and river ice modelling.

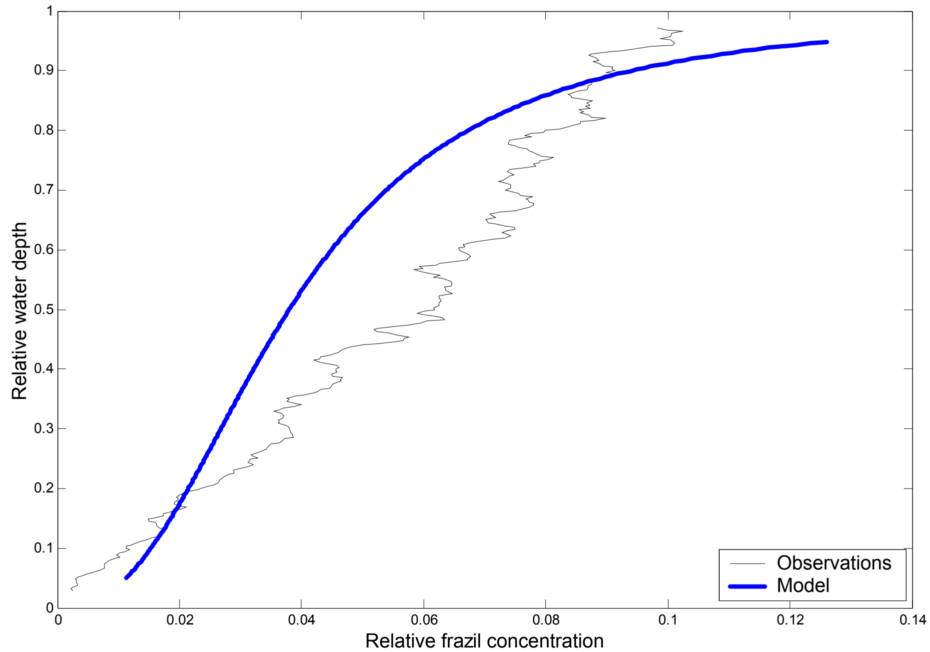

Figure 2. Vertical profile of frazil concentration for Exp. 69 at 26 min.

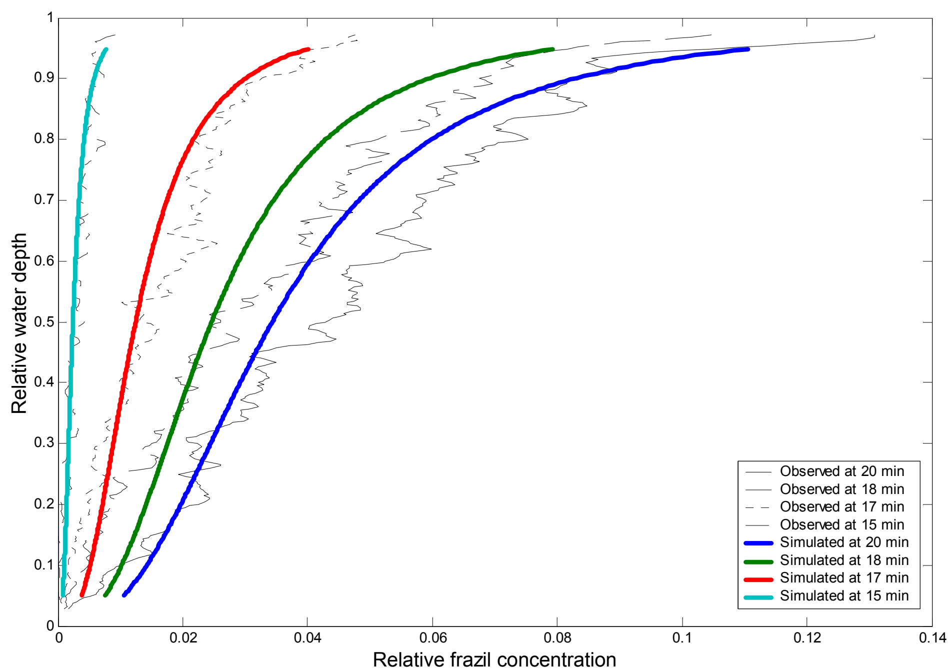

Figure 3. Variation of the vertical profile of frazil at various times during supercooling for Exp. 63.

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