Frazil Size and Flow Turbulence

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Frazil ice can have a significant impact on the operation of hydraulic structures in cold regions. It is known that the distribution and size of frazil ice evolves during the supercooling process, but details of this evolution are far from understood. In this paper, a model is advanced to describe the evolution of frazil size in relation to physical characteristics of the flow, in particular frazil buoyancy and turbulence length scales.

1. Introduction

In cold regions, when the water temperature drops to the freezing point, further cooling of water will lead to supercooling state and the subsequent formation of frazil ice. Many ice problems originate from the generation of frazil ice in rivers. Michel (1963) conducted the first experiments on supercooling and frazil ice formation in turbulent waters. Since then, numerous experimental studies on frazil formation have been conducted in a variety of laboratory settings that include flumes (Carstens, 1966; Daly and Colbeck, 1986), turbulent jars (Andres, 1982 and Ettema et al., 1984), and counter-rotating flume (Tsang and Cui, 1994). However, the influences of both the water temperature at nucleation and the turbulence characteristics of a flow on the rate of frazil ice formation and on the variation of frazil ice particle size are still unclear (Ettema et al., 1984).

There are few models describing the various aspects of frazil ice dynamics in river. Daly (1984) presented a dynamic model that can quantitatively describe the process using continuity and heat balance equations. However, these equations are dimensionally incompatible and strongly nonlinear. Mercier (1984) formulated a kinetic model of frazil growth, and simulated frazil formation in channels using a Monte Carlo technique. Svensson and Omstedt (1994) presented a mathematical description of frazil ice dynamics in a river. Hammer and Shen (1995) used a two-dimensional turbulence model to examine the evolution of frazil ice in channels by considering thermal growth, secondary nucleation and flocculation processes. In these modeling efforts, the frazil ice dynamics has generally been treated in a relatively basic way, where only the temperature response due to frazil ice formation has been considered. The time-temperature evolution has been the main concern. The frazil size distribution and its evolution have been assumed a priori. In this paper, a model is presented to describe the frazil size and its distribution in relation to turbulent characteristics of the flow.

2. Turbulence Characteristics in Open Channel

Nezu and Nakagawa (1993) provide a good summary of the characteristics of turbulence in open-channel flow. The three well-known spectral subregions of velocity fluctuations are productive, inertial, and viscous subranges. Turbulence energy is extracted from the mean flow in the production subrange. It is then transferred to small-scale eddies in the inertial subrange, and later dissipated into heat in the viscous subrange. The energy transfer in wall-turbulence fields is analogous to the cascade process through the spectral subrange. From this analogy, the turbulent structure of open-channel flows can be divided into the following three subregions.

1. Wall region [y/h <(0.15-0.2), h is the water depth, and y is a vertical coordinate]. This region corresponds to the 'inner layer' of classical boundary-layer treatments; length and velocity scales are υ / u_* and u_*, respectively. Where υ is the kinematic viscosity of water and u_* is the bed shear velocity.
2. Free-surface region [0.6< y/h <1.0]. In this region, the turbulent structure is controlled by the outer variables, with length and velocity scales specified as the flow depth h and the maximum mainstream velocity $U_{\max}$, respectively.
3. Intermediate region [(0.15-0.2)≤ y/h ≤0.6]. This region is not strongly influenced by either the wall properties or the free surface; instead it corresponds loosely to the inertial subrange of the spectral distribution. The length and velocity scales are y and $\sqrt{\tau / \rho}$, respectively.

According to Zhang et al. (1990) and Ni et al. (1991), fluctuating velocities in the vertical direction (v') can be approximately described by a normal distribution, i.e.,

$$f_1(v') = \frac{1}{\sqrt{2\pi}\sigma_{v'}} \exp\left(-\frac{v'^2}{2\sigma_{v'}^2} \right), \tag{1}$$

where $\sigma_{v'}$ is the root mean square of the vertical fluctuating velocities or the fluctuating intensities (turbulent intensities) in vertical direction. From (1) the following relation for $|v'|$ can be derived

$$f_2(|v'|) = \frac{2}{\sqrt{2\pi}\sigma_{v'}} \exp\left(-\frac{v'^2}{2\sigma_{v'}^2} \right). \tag{2}$$

3. Governing Principle and Frazil Morphology

Hou (1982) suggested that a local isotropic eddy can suspend a corresponding sand grain size in the flow, that is, the eddy size is equal to the size of the sand particle that the flow can deliver. But Ni et al., (1991) questioned this theory asserting that the eddy size should be much larger than the sand because sand is much heavy than water (2.65:1). Herein, we adapt Hou's (1982) theory, which we assume to hold for frazil ice, because the density of frazil ice is much closer to that of water than sand; note, the density difference between frazil ice and water is approximately 0.92:1. A modification for the shape of the frazil ice particles is required. Andreasson et al. (1998) noticed the importance of vertical turbulent velocity and believed that eddies smaller than a certain diameter (D) of frazil crystal will have a negligible influence on the mixing of ice particles of size (D). Based on this, Andreasson et al. (1998) estimated the size of the minimum ice particle present in the surface layer of a flow, the size of a minimum particle floating on the surface, and the maximum particle size at and near the surface.

We propose the following hypothesis regarding frazil evolution in a flow with supercooling conditions: that the vertical fluctuation of the flow plays a central role in determining the frazil size distribution and that the supercooling process progresses in a manner whereby frazil formation reaches the specific suspended-frazil capacity given by the characteristics of the flow. The frazil size distribution in a turbulent flow is derived according to the buoyant force and vertical mixing force.

The description of frazil ice morphology is not simple (Daly, 1984). The various shapes of ice crystals appear to result from a complex interaction between the imposed heat transfer conditions and the intrinsic crystallography of ice. Disk shaped frazil is typically observed in laboratory and field research. For these crystals growing edgewise and thus forming plates, the radius of the face (r) and the edge thickness (D_e) are usually used as the characteristic length for the face and edge, respectively (see figure 1). The assumptions that the diameter-to-thickness ratio is 8:1, and therefore the radius-to-thickness is 4:1, are used in this paper; although this is not inconsistent with the observation from a laboratory flume (Daly and Colbeck, 1986). The volumetric shape factor K_v is defined as (Daly, 1984)

$$\begin{aligned} K_v r^3 &= \pi r^2 D_e \\ &= \pi r^2 \frac{r}{4} \end{aligned} \tag{3}$$

and thus

$$K_v = \frac{\pi}{4} = 0.785 \tag{4}$$

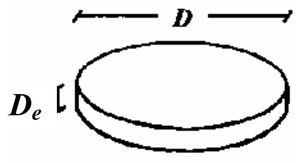

Figure 1. Assumed frazil-ice crystal morphology.

4. Density Distribution of Frazil

It is assumed that frazil particles are suspended in the flow when the summation of the vertical forces is zero; the vertical forces include the self-weight of an ice particle (W), the buoyancy force (F_b), and the drag force (F_d). Suspension is maintained provided

$$F_b = F_d + W , \tag{5}$$

where

$$F_b = \rho g K_v \left(\frac{D}{2}\right)^3 \tag{6}$$

$$W = \rho_i g K_v \left(\frac{D}{2}\right)^3 \tag{7}$$

and

$$F_d = \frac{1}{2}\pi \left(\frac{D}{2}\right)^2 C_d \rho v'^2 . \tag{8}$$

ρ and ρ_i are the densities of water and ice, respectively, C_d is the drag coefficient, and g is the acceleration due to gravity. The drag coefficient of a disk whose major radius is perpendicular to the flow is a well-known function of the Reynolds number and it is available in many texts. However, it should be recognized that real frazil particles rise in an unstable fashion that is not always normal to the direction of flow. It is thought that the oscillations of a rising frazil disc are related to the alternating shedding of vortices. These oscillations significantly increase the effective drag coefficient. Wuebben (1984) conducted a series of experiments using artificial discs and found that effective drag coefficient C_d is around 1.75 at a broad range of disc size.

Replacing (5) with (6), (7), and (8), we get

$$(\rho - \rho_i) g K_v \left(\frac{D}{2}\right)^3 = \frac{1}{2}\pi \left(\frac{D}{2}\right)^2 C_d \rho v'^2 \tag{9}$$

which can be expressed in the form

$$v'^2 = \beta_1 D \tag{10}$$

provided

$$\begin{aligned}\beta_1 &= \frac{\rho-\rho_i}{\rho}\frac{gK_v}{\pi C_d} \\ &= \frac{g'}{2\pi C_d}.\end{aligned} \tag{11}$$

Where g' is the reduced gravity given by

$$g' = 2\frac{\rho-\rho_i}{\rho}gK_v \tag{12}$$

and has a value of 1.294 m/s^2, which is not inconsistent with the value used by Daly (1984) while determining the rise velocity of frazil ice rise when $D = 0.14$ cm. Thus, the value of β_1 is equal to 0.118 m/s^2.

Combining (10) with (2) then the ice particle size density distribution is given by the following

$$\varphi_1(D) = \frac{\sqrt{\beta_1}}{\sqrt{2\pi}\sigma_{v'}\sqrt{D}}\exp\left(-\frac{\beta_1 D}{2\sigma_{v'}^2}\right). \tag{13}$$

Observations of vertical velocity fluctuation show (Zhang et al., 1990; Ni et al., 1991)

$$\sigma_{v'} = (0.45 \text{ to } 1.05)\ u_* . \tag{14}$$

Hui et al. (1997) demonstrated a vertical uniform distribution of $\sigma_{v'}$ in a wide 2-D uniform flow, i.e.,

$$\sigma_{v'} = \alpha u_* , \tag{15}$$

Where α is a constant, and $\alpha \geq 1$ according to their measurements. In our studies, $\alpha = 1.05$ is chosen, thus, $\sigma_{v'} = 1.05 u_*$.

5. Frazil Ice Distribution

If the frazil ice size density distribution follows (13), the expression for the theoretical distribution of frazil ice particles (which is in the form of the ratio of a particle number smaller than a certain diameter to those of all the other particles) is

$$P(D_i) = \int_0^{D_i} \varphi_1(D_i)dD \,. \tag{16}$$

Note, for uniform frazil ice this method yields approximately the same results as those obtained by a weight-percentage method. Substitute (16) with (13), then

$$P(D_i) = \int_0^{D_i} \frac{\sqrt{\beta_1}}{\sqrt{2\pi}\sigma_{v'}} D^{-\frac{1}{2}} \exp\left(-\frac{\beta_1 D}{2\sigma_{v'}^2} \right) dD \,. \tag{17}$$

Assuming $t = \frac{\sqrt{\beta_1 D}}{\sigma_{v'}}$, $D = \frac{\sigma_{v'}^2}{\beta_1} t^2$, $dD = \frac{2\sigma_{v'}^2}{\beta_1} tdt$, then

$$P(D_i) = \frac{2}{\sqrt{2\pi}} \int_0^{T} \exp\left(-\frac{t^2}{2} \right) dt = 2\Phi(T) - 1 \tag{18}$$

where

$$T = \frac{\sqrt{\beta_1 D_i}}{\sigma_{v'}} \tag{19}$$

$$\Phi(T) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{T} \exp\left(-\frac{t^2}{2} \right) dt \,. \tag{20}$$

Here, $\Phi(T)$ is the standard normal probability distribution.

6. Determinations of Minimum and Mean Frazil Size

Theoretically, the smallest frazil particle size might be assumed to be very small or near zero (i.e., comparable to the size of a water molecule). However, the size of a frazil ice particle is restricted by the turbulent eddy. In this study, an inner length (Nezu and Nakagawa, 1993) is assumed to determine the minimum particle size ($D_{\min}$), i.e.,

$$D_{min} = \upsilon / u_* . \tag{21}$$

Using (18), the percentage of (D_{min}) in the frazil ice may be calculated. In the same way, the mean frazil size (D_{50}) can be determined using (18) when $P(D_{50}) = 0.5$.

The shear velocity is determined by (Carter et al., 1963)

$$u_* = U\sqrt{\frac{f}{8}} . \tag{22}$$

where f is a friction factor. Assuming smooth boundaries, the friction factor f can be determined from

$$\frac{1}{\sqrt{f}} = 2\log\left(R_e\sqrt{f}\right) - 0.8 \tag{23}$$

provided $R_e > 10^5$. If $R_e < 10^5$, the Blasius formula

$$f = \frac{0.3164}{R_e^{0.25}} \tag{24}$$

can be used. R_e is the Reynolds number given by $4UR/\upsilon$, where U is the depth averaged flow velocity and R is the hydraulic radius.

7. Modeling Frazil Size

Using the theory developed above and the data (U, B, h) from Michel (1963), Carstens (1966), Osterkamp et al. (1983), and Tsang and Cui (1994) characteristic frazil ice size parameters are predicted, i.e., D_{min} and D_{50} (see table 1). The prediction of these characteristic frazil particle parameters, such as D_{min} and D_{50}, and the associated distribution are an important milestone in the determination of suspended frazil carrying capacity (Ye and Doering, 2001). Table 1 also summarizes the Reynolds number (R_e), friction factor (f), and shear velocity (u_*) computed for the experiments.

Two cases of Carstens (1966) are selected. The first corresponds to Case ‘A’ (figure 6) in Carstens (1966) whereas the second case corresponds to his Figure 7. The size distributions predicted by our model for these two cases are shown in Figure 2. These two cases of Carstens (1966) have been used by previous researchers (Mercier, 1984; Svensson and Ostmedt, 1994; and Hammer and Shen, 1995) to simulate the supercooling process.

Tsang and Cui (1994) conducted a series of experiments on frazil evolution using a counter-rotating flume. Three of these tests are used in the calculations. Michel (1963) carried out over 80 frazil ice growth experiments in an outdoor recirculating flume. He reported experiments with an average flow of 375 gpm (0.025 m^3/s). Using the flume cross-section of 0.24 cm × 0.20 cm yields an average flow velocity of 0.52 m/s.

Table 1. Characteristic frazil diameters of observations

		U [m/s]	B [m]	h [m]	R [m]	R_e	f	u_* [m/s]	D_{min} [m]	D_{50} [m]
Carstens (1966)	Case I	0.5	0.2	0.2	0.067	74488	0.019	0.024	0.00007	0.0024
	Case II	0.33	0.2	0.2	0.067	49162	0.021	0.017	0.00011	0.0012
Tsang and Cui (1994)	Expt. 1	0.18	0.2	0.18	0.064	25427	0.025	0.010	0.00018	0.0004
	Expt. 4	0.26	0.2	0.18	0.064	37781	0.023	0.014	0.00013	0.0008
	Expt. 6	0.18	0.2	0.12	0.055	21940	0.026	0.010	0.00017	0.0004
Michel (1963)		0.52	0.24	0.2	0.075	87291	0.019	0.025	0.00007	0.0028
Osterkamp et al. (1983)		0.6	8	0.4	0.364	487557	0.013	0.024	0.00007	0.0025

Field measurements of the supercooling process and frazil formation in a turbulent stream are difficult to obtain and rare in the literature. An exception is the Goldstream Creek experiment conducted in the interior of Alaska during frazil-ice production in 1971. Osterkamp et al. (1983) reported the time-temperature of the creek; the thermal and hydraulics parameters during these experiments can be found in Gosink and Osterkamp (1983).

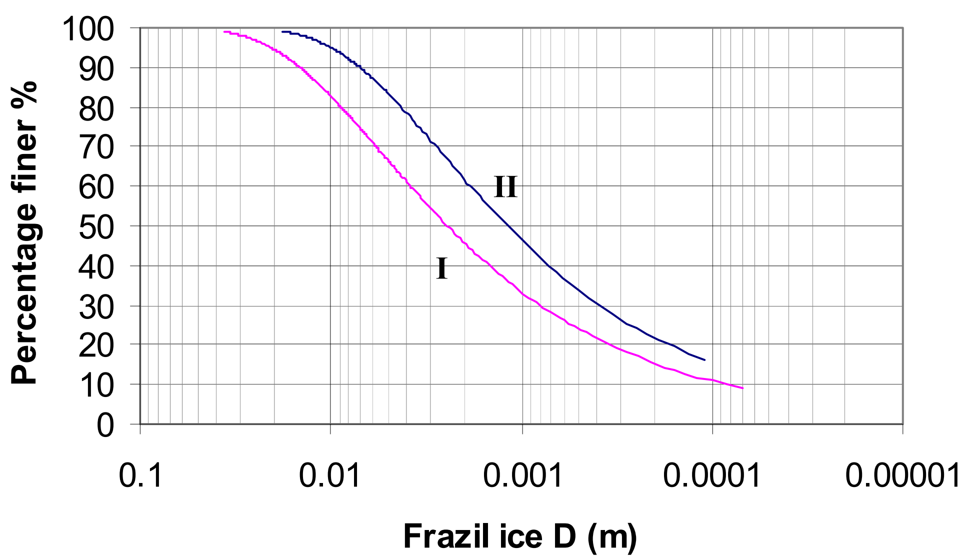

Figure 2. Frazil size distribution for two of Carstens' tests

8. Discussion

The evolution of frazil ice size and concentration are difficult to measure and monitor in nature (laboratory or field experiments). Carstens did not describe frazil size or concentration in his well-known experiments. Mercier (1984) estimated the critical radius of frazil ice should be about $0.4\,\mu m$ for the range of supercooling found in rivers and stream using thermodynamic principles.

Daly and Colbeck (1986) conducted a series of experiments designed to measure the size distribution of frazil ice crystals. The frazil ice diameters generated in their experiments ranged from 35 μm to 0.5 mm. The means of the size distributions were generally about 0.1 mm. It is interesting to note that their observed frazil size distribution could be approximated by a log normal size distribution.

Shen and Wang (1995) reported mean frazil diameters of 7 mm and 11 mm at the Hequ reach of the Yellow River, China during the winters of 1986-87 and 1987-88, respectively. Gosink and Osterkamp (1983) found that the frazil size in Goldstream Creek ranged from 1 mm to 6 mm. The mean frazil of 2.5 mm predicted by our model for the Goldstream data is similar to the measured typical size of 2 mm (Gosink and Osterkamp 1983).

Table 1 shows minimum frazil size predictions ranging from 70 μm to 180 μm, which is bigger than the size predicted by the model of Mercier (1984). The reason for the difference is the philosophy behind our estimations. Our theory is developed based on the principles of turbulence, open-channel flow, and asserts that the frazil size follows a log normal distribution (which is consistent with Daly and Colbeck (1986)) and provides a physical mechanism to estimate both the minimum and mean frazil size in a turbulent flow; such measurements would otherwise be difficult to obtain in practice. The distribution of frazil ice size is the focus of an ongoing research program in the University of Manitoba, Hydraulics Research & Testing Facility.

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