Bearing Capacity of Floating Ice Covers: Theory versus Fact

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Various criteria have been advanced to describe and predict the conditions under which a load will break through a floating ice cover. As a rule, the experimental basis for each of these criteria has been limited, and a comprehensive performance evaluation has yet to be carried out. Criteria based on stress, strain, deflection, and strain energy are discussed first, in conjunction with the viscoelastic nature of ice and the mode of failure of an ice sheet. Five data sets are then described and utilized to test and compare the performance of failure criteria that have been proposed in the literature. Stress-based criteria, though simple to use, are only useful for very brief loading because they do not account for loading time and history. Strain- and deflection- based criteria are also subject to time and history effects. On the other hand, strain-energy criteria are found to apply equally well to short- and long-term loads of different histories. This feature is attributed to consideration of time and load-history effects through integration of the deflection–load variation.

1. Introduction

Ice on Canadian lakes and rivers is an integral part of winter transportation routes, and often used as a construction platform. Floating ice sheets have also provided access for offshore oil exploration in the Canadian Arctic. The load bearing capacity of floating ice covers is a central design consideration in such activities, and likely to attract increasing attention as a result of climate change impacts.

This problem would present no particular difficulty if ice were an ordinary material of the type commonly encountered in engineering applications. Despite its association with what humans perceive as cold weather and cold regions, ice is a "hot" material, not much cooler than its melting point (e.g. see Sinha and Cai, 1996). This condition entails large creep effects and suggests that bearing capacity depends not only on the applied load but also on the loading time and the loading history. Therefore, the bearing capacity cannot be fully predicted unless the following information is available:

(a) a failure criterion, expressed in terms of an observable quantity, such as a critical load or deflection; and

(b) a method to predict the time-dependent response of the ice cover to a given loading history or load-time variation.

It is the former question that is considered herein, using five comprehensive data sets on lake and sea ice that span a thickness range of under 0.1 m to over 2.0 m.

The main objective is to test the efficacy of the various criteria that have been proposed in the literature, some originating in 19th century work (Kerr, 1975). No attempt is made to assess theoretical aspects of such criteria, beyond briefly outlining the applicable physical concepts. The focus is on the collapse and failure of an ice sheet under load, which is a related, but different question from what constitutes a safe load. Moreover, the present discussion is limited to *distributed* loads, that is, loads whose radial extent is more than about two ice thicknesses (Sodhi, 1995). Otherwise, the load is *concentrated* and breaks through by simply punching a hole in the ice cover (shear failure), a mechanism that is well understood and quantified. Finally, it is stressed that the present discussion exclusively refers to floating ice covers. Without the water reaction forces that are created by ice displacements from the free-floatation level, the bearing capacity may be greatly reduced because ice in itself is a relatively weak material, unable to support its own weight over significant spans.

2. Background Information

2.1. Mode of ice plate collapse under a distributed load

Field tests have shown that the fracture of an ice sheet under a distributed load begins with radial cracks that start under the load and propagate outward for a considerable distance (Fig. 1). This condition is not critical because the combined resistance of the ice "wedges" can still support the load. As the load continues to increase, however, circumferential cracks begin to appear, with consecutively decreasing radii. Breakthrough occurs along the innermost circumferential crack, typically located several ice thicknesses away from the edge of the load (Frankenstein, 1963, 1966; Lichtenberger et al, 1974, 1975; Beltaos, 1974-1976, unpublished). In long-term loading tests, water may seep through the cracks and flood a portion of the deflection bowl. If flooding does occur, it is usually noticed near the end of the test.

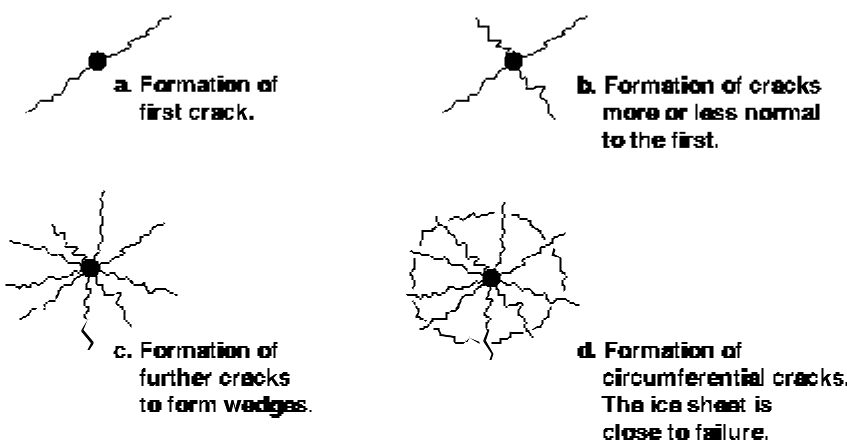

Fig. 1. Schematic illustration of crack development (after Ashton, 1986, with changes)

2.2. Basic theory: instantaneous loading

For very brief loading times (instantaneous loading, or IL for short), ice responds elastically. A floating ice cover can thus be considered an elastic plate resting on an elastic foundation, a problem that is mathematically tractable and has received the attention of many theoreticians (e.g. see review by Kerr, 1975). The elastic analysis does predict radial cracking as a result of large tangential stresses that develop under the load; and circumferential cracking when the resulting ice wedges can no longer support the increasing load (Fig. 1). Fracture is due to tensile stresses, applied on the bottom and top halves of the plate for radial and circumferential cracking, respectively. In some variations of the theory (e.g. Meyerhof, 1962), ultimate collapse is assumed to occur under full utilization of the plastic bending moment per unit length of the circumferential crack. In his comprehensive review, Kerr (1975) showed that the results of all such analyses could be approximated by the following equation:

$$P_{fo} = \sigma_N h^2 (1 + k \frac{R}{L}) \qquad (1)$$

in which P_{fo} = instantaneous breakthrough load; σ_N = *nominal* ice strength, generally equal to a constant times the flexural strength or the tensile yield strength of the ice, depending on the particular theory being used; h = ice cover thickness; k = dimensionless coefficient; R = load radius; and L = *characteristic length* of the ice cover, defined by

$$L = \sqrt[4]{\frac{Eh^3}{12(1-\mu^2)\gamma}} \qquad (2)$$

in which E = elastic modulus; μ = Poisson ratio ≈ 1/3; and γ = unit weight of water. For good-quality, *freshwater* ice, the modulus of elasticity, E, is about 7 GPa, so that Eq. 2 gives (Gold, 1971; both L and h in metres)

$$L = 16 h^{3/4} \qquad (3)$$

An important departure from the bending collapse mechanism is Sodhi's theory (1998). It postulates that the final ice collapse results from an excess of *compressive* stress that develops within the lower portion of the ice thickness along the circumference of the final crack. This residual resistance to collapse is provided by the outer portion of the ice sheet, which restricts rotation and breakthrough of the inner wedges at the crack surface. Sodhi (1998) corroborated the theory with specially-designed laboratory tests, and showed that it can also be described by Eq. 1, with the exceptions that (a) the influence of load distribution is negligible, which is equivalent to setting k = 0 in Eq. 1; and (b) that the nominal strength, σ_N, is now equal to a constant times the *compressive* strength of the ice.

The bending-failure breakthrough theories capture some features of the collapse mechanism, but fall short in one important respect: they do not take into account, and thence do not predict, what is actually observed, i.e., the formation of not one, but several circumferential cracks prior to breakthrough. On the other hand, the compressive-failure theory focuses on the final, innermost circumferential crack but does not take into account the presence of several other cracks of larger radii that have already formed by the time of breakthrough.

In addition to the collapse load, the deflection, δ, of the ice cover is an important parameter that can also be predicted by theoretical means. The deflection *bowl* is defined by the radial variation of δ:

$$\frac{d}{d_m} = f(\frac{r}{L}) \tag{4}$$

in which r = radial distance from the centre of the load and δ_m = maximum deflection at r = 0. For an uncracked ice sheet, the deflection decreases with increasing r, becoming equal to zero at r = 4L. Beyond this distance, δ becomes slightly negative (upward deflection) and attains a minimum of about $-0.02\delta_m$ at r = 5L, before practically vanishing at r = 7L (e.g. see Nevel, 1968). For any load, P, applied on an uncracked sheet, the value of δ_m is given by the well-known formula (e.g. Kerr, 1975):

$$d_m = \frac{P}{8gL^2} \tag{5}$$

Where the ice cover has already developed cracks, equations similar to 4 and 5 will apply, however, the form of the function f in Eq. 4, and the numerical coefficient in Eq. 5 may change.

2.3. Creep effects

In bearing capacity literature, the description “short-term” load is often used to denote virtually elastic response, but is rarely quantified. From a practical viewpoint, a short-term load would have to be associated with a duration of at least several minutes, since it would take a finite time for a stationary load to perform its intended task. Even with moving loads, the operator would need some time to deal with a minor breakdown, or to determine that a breakdown is major and seek assistance. However, a loading time as brief as minutes still produces significant creep effects, as discussed in the next paragraph. To avoid the resulting ambiguity in terminology, the adjective “instantaneous” is consistently used herein to describe loading conditions that are free of creep effects. The description “short-term” is reserved for brief loads of specified duration, without precluding the occurrence of significant creep.

Numerous experiments on small specimens and prototype field tests indicate that ice begins to creep as soon as a load is applied. In fact, the initial creep rate under a constant load is the maximum for the entire test, with the possible exception of rates that occur just prior to failure. This feature is illustrated in Fig. 2, which depicts the temporal variation of ice deflection under a constant load. Typically, the rate of deformation decreases as time goes on (primary creep), and approaches a constant value (secondary creep) before it increases again in precipitous fashion (tertiary creep) that leads to collapse. Thus, the IL condition must be viewed as a limiting behaviour that only occurs when $t \rightarrow 0$. What time interval is small enough to ensure negligible viscous effects, is not known with certainty because it may depend on ice properties and

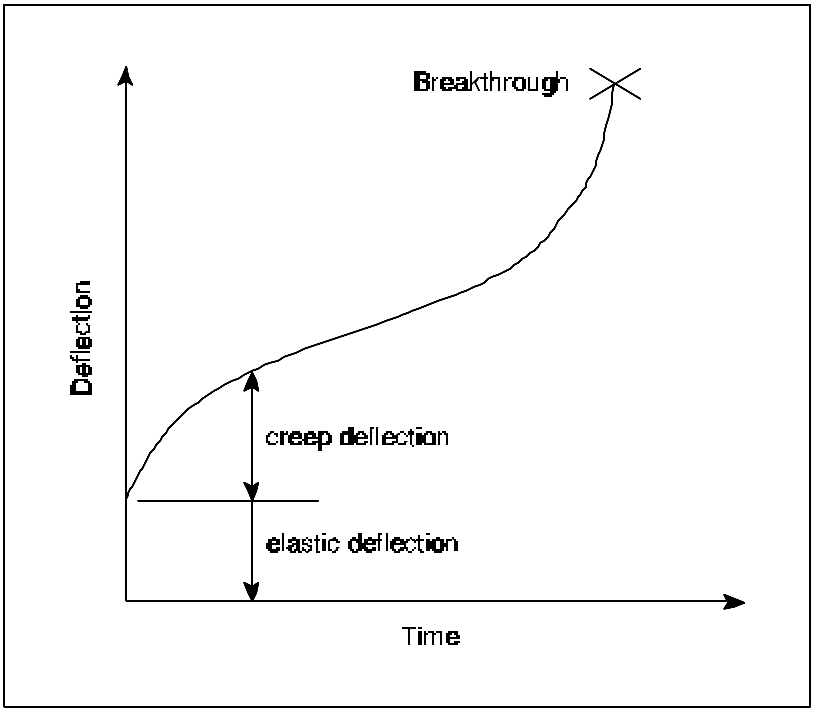

Fig. 2. Schematic illustration of ice deflection – time variation under a constant load.

temperature distribution within the ice cover, as well as on the magnitude of the load. Using Sinha and Cai's (1996) approach and assuming a commonly occurring grain size of about 10 mm, we can estimate that the viscous strain will be 10% of the elastic value at t = 5 s, and increase to 40% of elastic strain at t = 9 min. This is in accord with experience, which has shown that significant creep of ice covers is likely to occur within a few minutes (Beltaos, 1978).

Various viscoelastic models have been developed to describe the creep of ice covers, but none has proved satisfactory, because calibrated viscoelastic constants vary from one test to another (e.g. see Frankenstein, 1966). The models do show, however, that stress decreases with time under a constant load (Nevel, 1966, 1976, Hui, 1986). Consequently, when an ice cover fails after prolonged creep, the maximum stress is much less than the strength of the ice. Another interesting result of viscoelastic analysis, and one that has been approximately confirmed by observation, is that Eqs. 4 and 5 can still be used, with the understanding that the value of L is now reduced, to account for a typical "narrowing" of the deflection bowl (e.g. see Beltaos, 1978, Rose et al, 1975). Thus, under creep conditions, the effective characteristic length is a function of time, and can be simply calculated from

$$L(t) = \sqrt{P(t)/8\gamma\delta_m} = L\sqrt{\delta_{mo}/\delta_m} \qquad (6)$$

in which L(t) = time-dependent, apparent, characteristic length; L = true characteristic length, as defined in Eq. 2; and δ_{mo} = value of δ_m under an instantaneous load that is equal to P(t).

3. Failure Criteria

3.1. Stress criteria

Various criteria have been proposed for predicting the collapse of an ice sheet. The most common is the stress criterion, which can be quantified by the elastic analysis for the IL case, and culminates in Eq. 1. For creep conditions, some authors have proposed the use of empirically reduced values of σ_N (e.g. Meyerhof, 1962) or by using empirically derived relationships of the form (Assur, 1961; Panfilov, 1961, as quoted in Kerr, 1975)

$$\frac{P_f}{P_{fo}} = g(t_f) \qquad (7)$$

in which P_{fo} = instantaneous breakthrough load; t_f = required time of application of a load P_f (< P_{fo}) in order to attain breakthrough. The function g is equal to 1 at t_f = 0, and decreases continuously with increasing time. Such equations were derived for constant loads, and cannot be expected to account for the effects of more complex loading histories, e.g. step loads, ramp loads, cyclic loads, etc.

3.2. Deflection and strain criteria

To avoid the obvious problems associated with stress criteria, other authors have proposed that breakthrough occurs upon attainment of a critical deflection, regardless of whether failure is preceded by creep. For instance, Kerr (1975) quotes Russian investigators who proposed this concept as early as 1942. More recently, Frederking and Gold (1976) proposed that the maximum deflection at failure, δ_{mf}, is approximately equal to 0.9h, based on the distributed load tests by Frankenstein (1963). On the other hand, Sodhi (1995) concluded that δ_{mf} should be ≈ 0.50h on the basis of his postulated collapse mechanism.

Deflection is an important, though not the only, factor in the magnitude of strain that occurs within the ice sheet. The concept of critical strain is one of the most common in the literature but the maximum strain within an ice sheet is not easy to measure. An estimate can be obtained by noting that strain is approximately equal to $0.5h(\partial^2\delta/\partial r^2)$, and using Eq. 4 to derive:

$$\varepsilon_m = \frac{|f''_m| \delta_m}{2\{L(t)\}^2} \qquad (8)$$

in which ε_m = maximum strain; and f''_m = maximum value of the second derivative of the function f with respect to r/L. Substituting the value of L(t) from Eq. 6, and applying the result to breakthrough conditions, gives:

$$\varepsilon_{mf} = const\left(\frac{\gamma h \delta_m{}^2}{P}\right)_f \qquad (9)$$

If breakthrough occurs at a fixed value of strain, the quantity within the parentheses on the RHS of Eq. 9, which can be readily evaluated from prototype test results, should be a constant.

3.3. Strain-energy criteria

The concept of a critical strain-energy per unit volume has also been used extensively in the strength-of-materials literature. Using this concept, and based on empirical evidence, Beltaos (1978) found that the work done by the load at the time of collapse, W_f, varies in proportion to $h^{2.5}$:

$$W_f \equiv \int_S^B P d d_m \approx C h^{2.5} \qquad (10)$$

in which the limits of integration (S and B) denote “start of loading” and “breakthrough”; and C is a constant with dimensions of $Nm^{-1.5}$ in the metric system of units. The advantage of the strain-energy criterion is that, unlike other criteria, it takes into account the loading and strain histories via the integration indicated in Eq. 10.

3.4. The size-effect controversy

Bazant and Planas (1998) review a number of publications in which a size effect on the bearing capacity of ice covers is predicted, based on the theory of fracture mechanics. The size effect is manifested by a decrease in the nominal strength of a structure, or of a specimen, as its size increases. There is strong direct and indirect evidence in support of a size effect in ice, but it pertains to much simpler failure mechanisms than the collapse of an ice cover. Bazant and Kim (1998) revised a previously held view, and suggested that the relevant dimension to the bearing capacity of floating ice covers should be the thickness, h, rather than the characteristic length, L. As h increases, the nominal strength (σ_N) decreases, and approaches an inverse square-root variation at relatively high h-values ($\sigma_N \propto 1/\sqrt{h}$).

Sodhi (1995, 1998) reasoned that the failure mechanism of an ice cover subjected to a vertical load does not admit a size effect because the collapse is governed by ductile, rather than brittle behaviour that is typically associated with fracture. As evidence he cited the fact that log-log plots of P_f vs. h from prototype tests define, albeit with considerable scatter, a straight line of slope 2:1. This assertion was disputed by Bazant and Kim (1998) who detected a decreasing trend in plots of P/h^2 versus h. Criticisms of the latter article in discussions by Dempsey (2000) and Sodhi (2000) did not seem to convince the authors (reply by Bazant and Kim, 2000). Two major points of dispute are (a) whether radial cracks, which initiate at the bottom of the cover, extend right through to the top; and (b) whether the size effect can occur under conditions of significant creep.

4. Prototype Test Data

Five sets of tests are used herein to examine the performance of the various failure criteria. They include tests on lake ice (Frankenstein, 1963, 1966; Beltaos, 1978); and tests on much thicker sea ice at Resolute Bay, Canada (Lichtenberger et al, 1974, 1975). Many of the tests did not achieve breakthrough, either by accident or by design. Though such tests are very useful in studying creep response, they are not considered herein. Brief descriptions of the five sets of breakthrough tests follow.

ARC tests (Beltaos, 1978): Numerous tests were carried out by the Surface Water Engineering section of Alberta Research Council (ARC) during the period December 1974 to March 1976 at

Joseph Lake, Alberta. Different size-tanks were deployed to test the response of ice covers ranging from 0.06 to 0.47 m in thickness. The majority of the breakthrough tests were of two types, either ramp-load to failure, or ramp-load followed by constant load. There were also a few tests with stepped load application, comprising more than one ramp or constant-load levels. Ice temperature profiles were measured with thermistors embedded in the ice at different depths, but the results were not always reliable due to drift in the read-out calibration. Most of the measured profiles could be approximated by straight lines through 0°C at the bottom. Average ice temperatures varied between -7.9 and -2 °C. The time to failure ranged from 6 min to over 6 hours, with the exception of one test in which it was 24 hours.

CRREL-1 (Frankenstein, 1963): Seven distributed – load tests were conducted by the US Army Cold Regions Research and Engineering Laboratory (CRREL) during January – March, 1956, on Portage Lake, Michigan, using a 3.7 m diameter tank. Water from the lake was pumped into the tank until the ice cover collapsed or until the tank was filled. In the latter case, the full-tank load was left on the ice until collapse occurred some time later. Breakthrough was not achieved in one of the tests. Ice thickness was between 0.26 and 0.50 m, while the ice temperature, taken at a depth of one-third of the ice thickness, ranged from –3.0 to –0.2 °C. The time to failure varied from 40 to 204 min.

CRREL–2 (Frankenstein, 1966): These tests were preformed on an "Arctic Lake" during October and November 1961. In all six but one of the distributed tests, the load increased steadily by pumping water into suitable tanks until breakthrough occurred. In the remaining test, the maximum load was reached after 17 minutes of pumping, and subsequently allowed to stay on the ice until breakthrough, some 13 minutes later. Failure times ranged from 11.7 to 30.9 min. Ice thickness ranged from 0.16 to 0.40 m, while the ice temperature, taken at a depth of 0.15-0.21 m below the ice surface, was between –8.3 and –1.2°C.

SO – 1 (Lichtenberger et al, 1974): Fourteen tests were carried out by Sun Oil Company (SO) on sea ice in Resolute Bay, Canada, during Dec. 1973 – May 1974, using 6.1 m and 9.1 m diameter tanks. Ice thickness ranged from 0.71 to 2.08 m. Most of the tests that achieved breakthrough were of the ramp type, though there were also a few that combined a ramp-load period with a constant-load period. Ice temperature profile measurement was attempted but the results were unreliable. The authors suggested that halving the surface temperature (expressed in degrees C) would provide a better estimate of the average temperature of the ice. The later value was between –19.4 and –6.4 °C, and generally colder than –13°C. Ice density varied between 0.95 and 0.99 while the salinity was between 4 and 4.8 ppt. In some of the tests, the ice was reinforced with wire mesh in the hope of increasing the bearing capacity. Such tests are excluded from the present study, except where the reinforcement proved to be ineffective. The time to failure ranged from 17 min to over 3 hours.

SO – 2 (Lichtenberger et al, 1975): A similar series of tests were again carried out at Resolute Bay, during Nov. 1974 – Jan. 1975. Ice thickness ranged from 0.51 to 2.16 m, and included various combinations of natural and artificial ice. Salinity measurements indicated that the brine from the artificial ice layer migrated into and probably through the natural ice. In general, the salinity of the ice was in the range of 6-15 pp. The limited available evidence shows no obvious difference in the bearing capacity of fully natural and partly artificial ice sheets. Similarly, brazil strength tests on natural and artificial ice showed no significant difference. The time to failure ranged from 2.5 to 11 hours.

Assumptions and approximations: For all tests, it is necessary to determine the load distribution factor R/L, which is used in stress-related criteria. The load radius R is, on some occasions, a nominal value, calculated so as to match a non-circular load area. This approximation was only used for some of the SO and ARC series, since the CRREL tests involved exclusively circular loads. For all freshwater tests, the characteristic length of the ice plate, L, was determined from Eq. 5, where fixed values of E and γ are assumed for lake ice – lake water combinations. For the sea-ice tests (SO-1 and 2), the L-values reported by Lichtenberger et al (1974, 1975) were used. These were based on estimated sea-ice elasticity and seawater density. Normally, the work done by the applied load at breakthrough was calculated by plotting load versus deflection and integrating, as indicated in Eq. 10. However, the SO reports that the author was able to obtain did not include detailed deflection-time plots, but simply quoted the deflection at breakthrough. Because most of the SO tests were of the ramp-load type, an empirically noticed similarity between ramp load - deflection curves in the CRREL and ARC tests was employed to: (a) determine that the integral is approximately equal to $0.71P\delta$ (±10%); and (b) use this equation to compute the work at breakthrough for the SO tests. For tests that involved constant-load steps in addition to ramps, the similarity property was used in conjunction with estimated intermediate deflections.

5. Comparison of Theory with Test Data

5.1. Stress - related criteria

The first step in examining stress – related criteria is to consider the effect of creep on the breakthrough load. This is illustrated in Fig. 3, where a strong decreasing trend with increasing

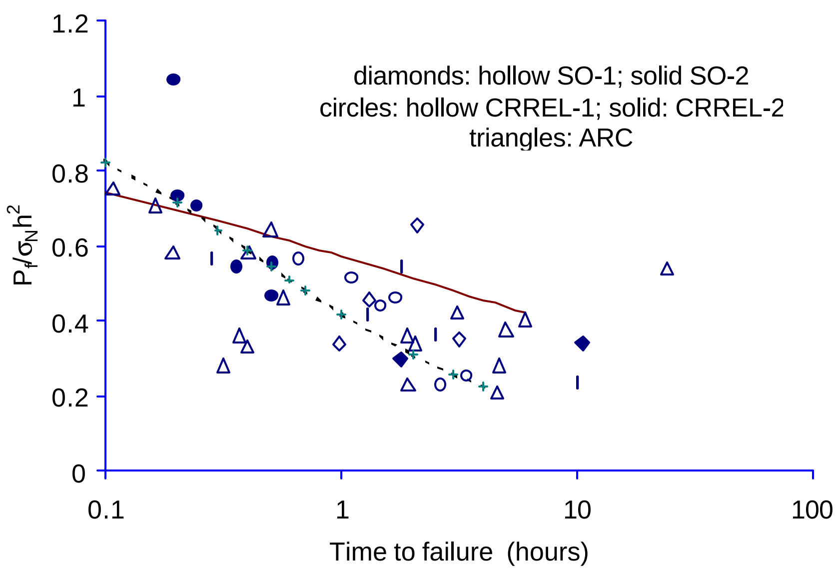

Fig. 3. Effect of time on breakthrough load. Continuous and dashed lines are average curves through data reported by Panfilov (1961; as quoted by Kerr, 1975) and by Assur (1961). The value of σ_N is equal to 4000 kPa

time is evident. For reasons that will be explained later, all P_f/h^2 values have been normalized with σ_N = 4000 kPa.

Clearly, there are other factors at work, in addition to the loading time. The manner in which the load is applied, i.e. the loading history, is also very important, though difficult to quantify at present. This is illustrated by the relatively high data point on the right side of the graph: here, a constant load produced moderate deflections during 24 hours or so. The load was then increased rapidly in ramp fashion, achieving breakthrough within minutes. Though the total loading time was relatively long, the "damage" inflicted on the ice cover remained small until the last few minutes of the test.

A parameter that may also be relevant is the load distribution factor α (=R/L), as suggested by Eq. 1. A plot of P_f/h^2 vs. α, without regard to simultaneous creep effects, is presented in Fig. 4, which appears to suggest an increasing trend. Upon reflection, however, this trend was deemed spurious, owing to testing limitations. As ice thickness grows during a testing program, it is becoming more and more difficult to attain loads that are large enough to break the ice as rapidly as before. Relatively lesser loads (i.e. with lesser P/h^2 values) are thus applied for longer times to achieve breakthrough. Plots of t_f versus h for the five data sets confirmed the existence of this bias by exhibiting positive correlation. Since P_f/h^2 decreases with increasing t_f (Fig. 3) one would also expect to find a decreasing trend between P_f/h^2 and h. This implies further that P_f/h^2 would appear to increase with increasing R/L, even if there were no dependence on the load radius, R (by Eq. 5, $L \propto h^{0.75}$).

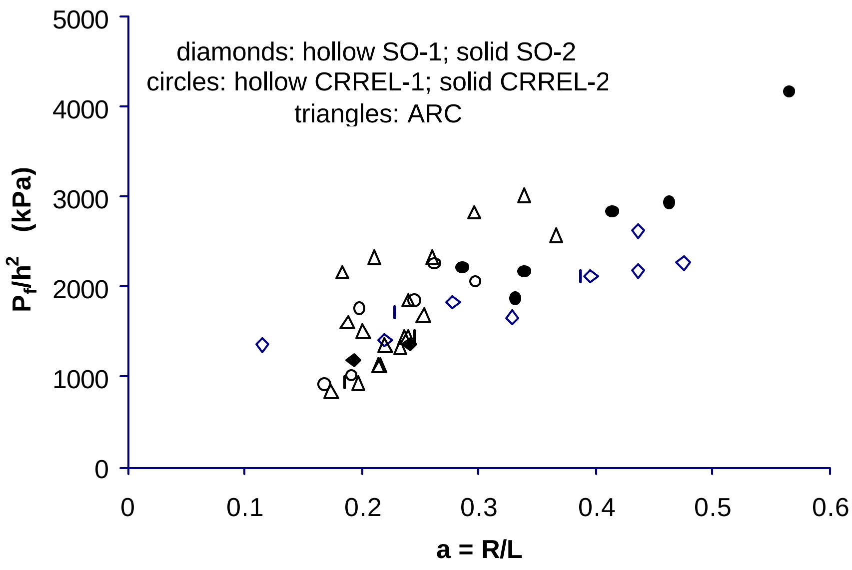

Fig. 4. Possible effect of distribution factor on breakthrough load. All test data included, regardless of creep effects.

This question can only be resolved by referring to the creep-free (IL) case. Since this is an idealized, practically unattainable condition, the present data were screened and ramp-load tests with t_f < 1 hour were selected. This is a compromise between the need to approach an IL condition and to have at least a few data points to work with, while matching test types. The creep effect is illustrated in Fig. 5, where the average trend is described by the line drawn through the data points. The latter can be used to estimate instantaneous breakthrough loads $[P_{fo}/h^2 \approx (P_f/h^2)(1+2t_f)]$ under identical test conditions. When these are plotted against the distribution factor, no trend can be discerned (Fig. 6).

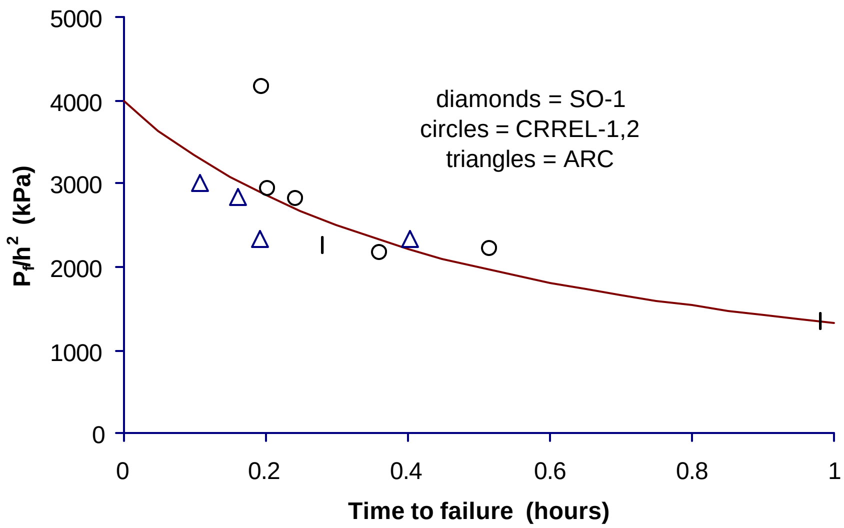

Fig. 5. Creep effect on ramp-load tests lasting under 1 hour. The curve drawn through the data points has the equation: $P_f/h^2 = 4000/(1+2t_f)$

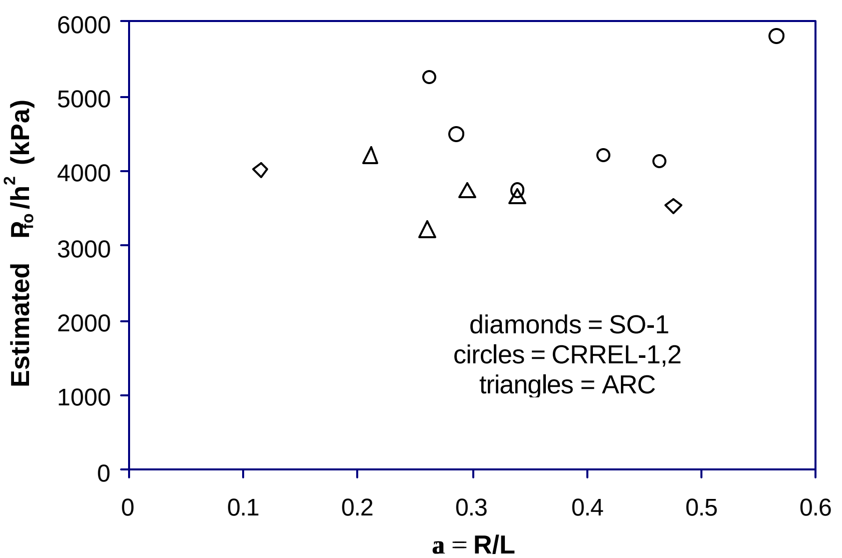

Fig. 6. Effect of distribution factor on estimated instantaneous breakthrough load. No trend is evident.

Consequently, the following relationship is deduced, as an average for the data points in Fig. 6:

$$P_{fo} \approx 4000h^2 \qquad (11)$$

in which the load is in kN and the thickness in metres, while it is implied that σ_N is equal to 4000 kPa. The finding expressed by Eq. 11 does not quite agree with the predictions of traditional theories of bending failure along a circumferential crack (Eq. 1; k>0), but is consistent with Sodhi's (1998) theory of failure-in-compression (Eq. 1; k=0).

The same data set indicates a lack of correlation between P_{fo}/h^2 and h in Fig. 7, which implies that there is no size effect (Eq. 11) within the range of the data points (h<1.2 m). The effect of ice temperature is explored in Fig. 8, where it is seen to be minimal, if any. It is possible that a reduction in strength due to salinity is compensated by an increase due to the lower temperatures associated with the sea-ice tests, represented by the diamonds in Fig. 8.

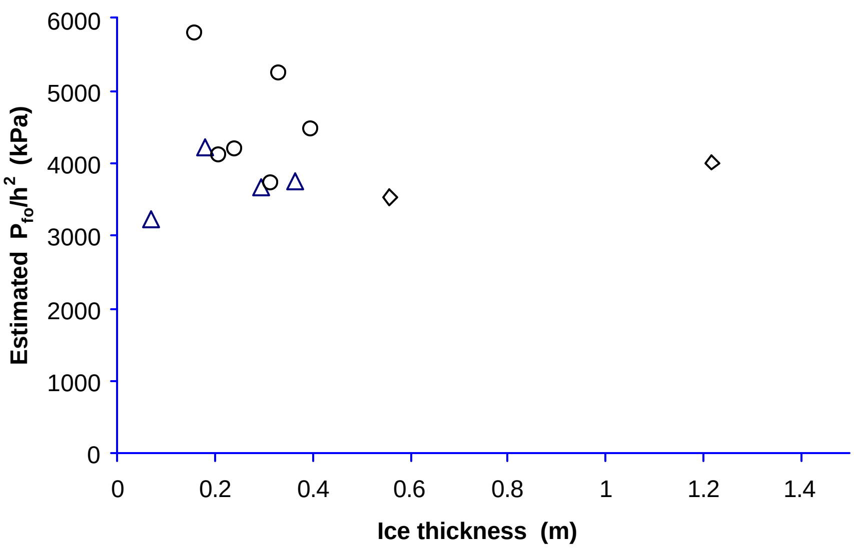

Fig. 7. Testing for size effect: estimated instantaneous breakthrough load versus ice thickness. No trend is evident. Legend same as in Fig. 6.

5.2. Deflection and strain criteria

As already discussed, some authors have suggested that breakthrough occurs when the maximum deflection attains a certain value that depends on ice thickness. This postulate is tested first by plotting the maximum deflection at failure, δ_{mf}, against the ice thickness, h, as shown in Fig. 9. The data points show a strong relationship, though one that is more of the square-root form rather than linear. The former is the type of dependence that describes the IL condition: from Eqs. 1 and 4, it can be deduced that $\delta_m \propto (h^2/L^2)$; since $L \propto h^{3/4}$ (Eq. 2), it follows that $\delta_m \propto h^{1/2}$.

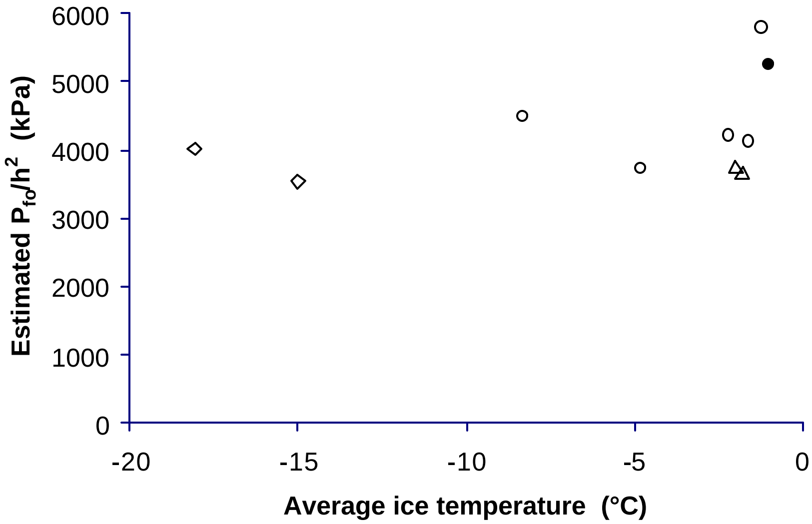

Fig. 8. Testing for ice temperature effect: estimated instantaneous breakthrough load versus ice thickness. Legend same as in Fig. 4.

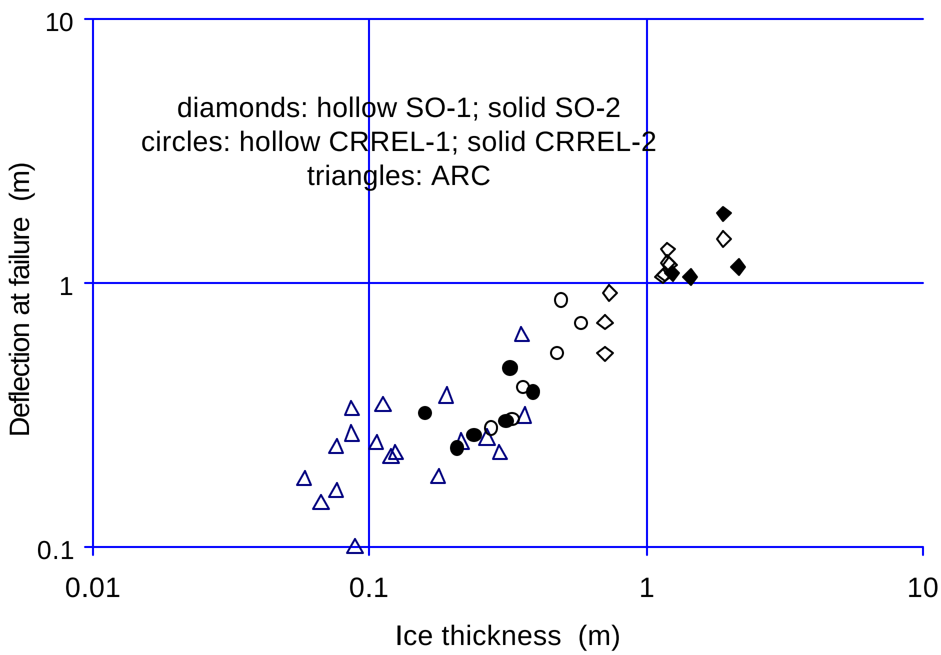

Fig. 9. Breakthrough deflection versus ice thickness

At this point, one might be tempted to draw a lower bound to the data points and use it as a "safe" deflection. As shown by Beltaos (1978), however, this approach is erroneous and can lead to unsafe conditions because the deflection at failure also depends on the loading history and time. For instance, supposing a deflection of 0.6 m to be "safe" for an ice cover 1 m thick (lower bound of data points at h = 1 m in Fig. 9) and using Eqs. 2 and 4, the corresponding safe instantaneous load is estimated as 12,000 kN. This corresponds to P_{fo}/h^2 = 12,000 kPa, which is three times greater than the instantaneous breakthrough load given by Eq. 11.

The time dependence of the failure deflection is explored in Fig. 10, where the quantity $\delta_{mf}/h^{1/2}$ is plotted versus the time to failure, t_f. The increasing trend suggests that an ice cover can sustain greater deflections under prolonged, moderate loads than under large but rapidly applied ones. From a physical point of view, this finding points toward energy – based criteria, to be discussed in the next section. The effect of load distribution on the failure deflection was also examined by plotting $\delta_{mf}/h^{1/2}$ versus R/L (not shown herein) but no trend could be discerned.

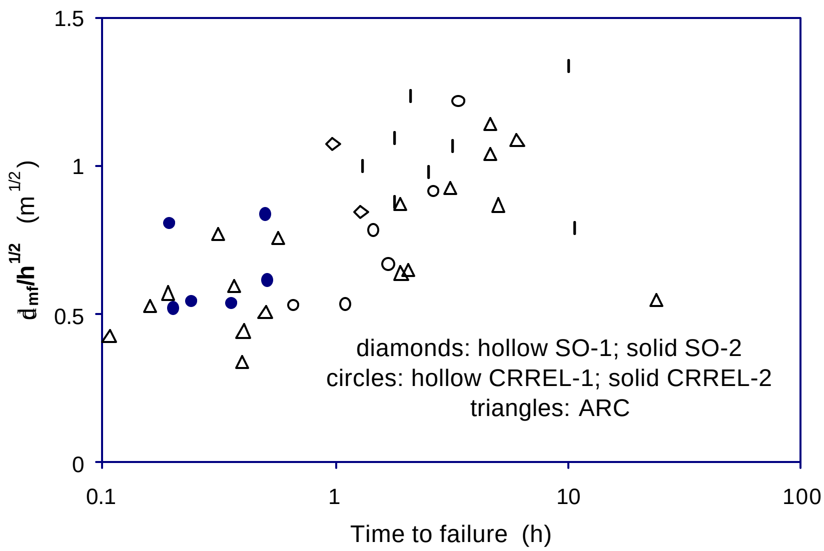

Fig. 10. Breakthrough deflection, normalized with square-root of ice thickness, versus loading time.

Together, the trends illustrated in Figs 3 and 10 imply that a strain criterion does not apply: by Eq. 9, the strain at failure is proportional to $(\delta_m^2/P)_f$; the nominator of this fraction increases with loading time, while the denominator decreases; consequently, the failure strain will increase with the time of loading. This was confirmed by plotting all available data in the form suggested by Eq. 9 (that is, $\gamma h(\delta_m^2/P)_f$ versus t_f, as illustrated in Fig. 11.

5.3. Strain-energy criteria

Using the ARC and CRREL test series, Beltaos (1978) showed that the work done by the applied load at failure, W_f, varies in proportion to $h^{2.5}$, in agreement with the concept of a critical strain energy (Eq. 10). This finding is further confirmed in Fig. 12, where the SO data sets extend the previous range of ice thickness to 2.2 m. The line plotted through the data points has a slope of 2.5:1 and adequately describes the general trend over nearly two log cycles of ice thickness. The scatter in Fig. 12 cannot be attributed to the load distribution factor, to ice temperature, or to the time of loading (Fig. 13).

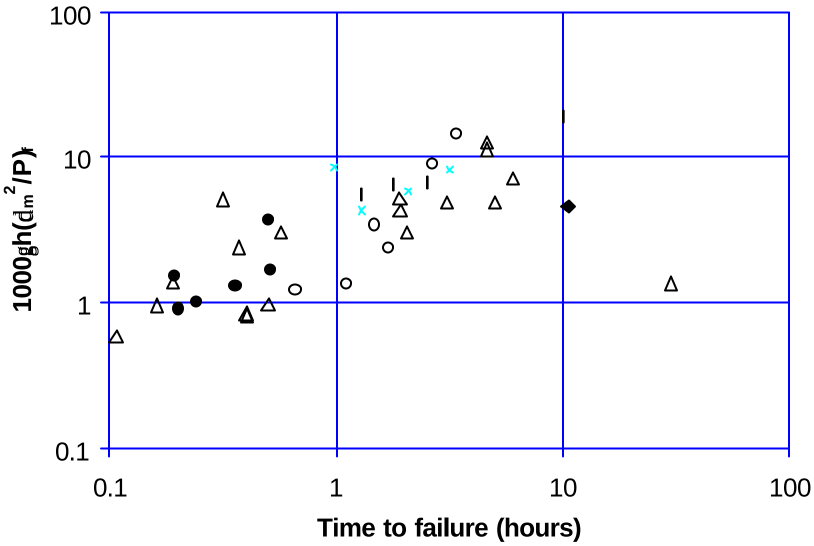

Fig. 11. Test of strain criterion (see also Eq. 9). Legend same as in Fig. 10.

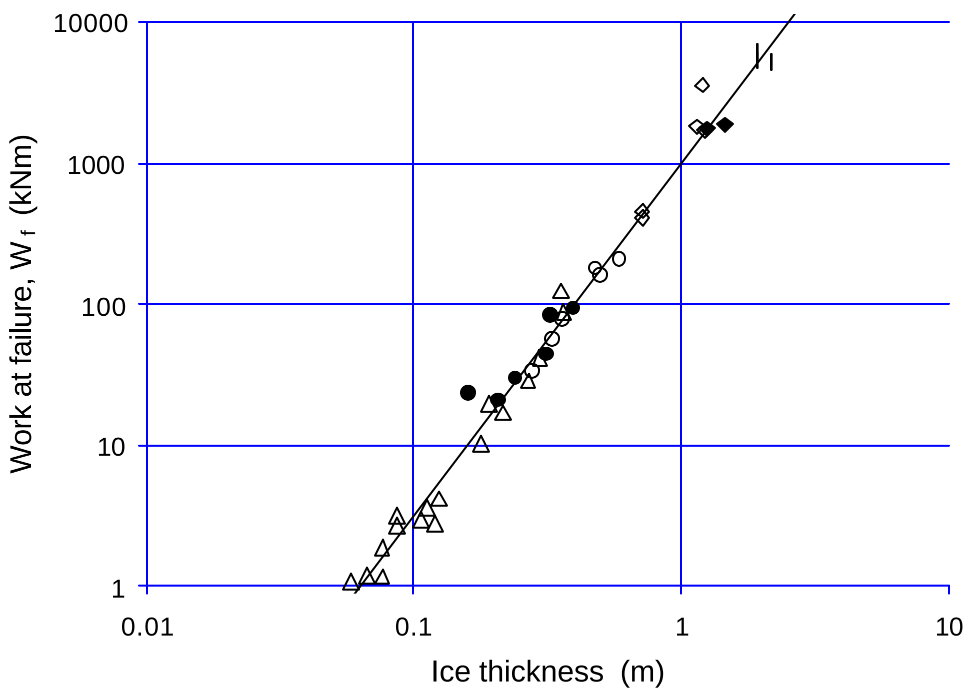

Fig. 12. Work done by the applied load at failure versus ice thickness. Legend same as in Fig. 10.

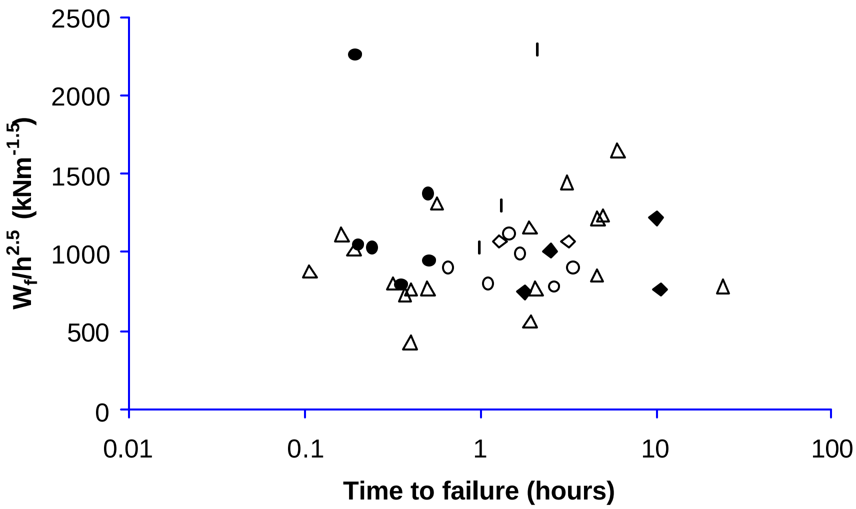

Fig. 13. Work at failure, normalized with $h^{2.5}$, versus time to failure. Legend is same as in Fig. 10.

The lack of time influence in Fig. 13 suggests that the strain-energy criterion adequately accounts for creep effects by integrating the load-deflection variation. With few exceptions, the data points in Fig. 13 fall within a 2-fold band (700 – 1400 kNm$^{-1/2}$), and can be described by the following equation, as an average:

$$W_f \approx 1000\, h^{2.5} \qquad (12)$$

in which W_f is in kN and h in metres. Unlike Eq. 11, which is limited to IL conditions, Eq. 12 applies to all loading conditions, at least within the range of the five test series examined herein.

Since the strain-energy criterion is independent of time, it should also apply to the IL case. Consequently, Eqs. 5, 11 and 12 can be combined to determine the corresponding value of the maximum deflection at failure [both $(\delta_{mf})_o$ and h in metres]:

$$(d_{mf})_o = 0.25\sqrt{h} \qquad (13)$$

which appears to furnish a plausible zero-time intercept for Fig. 10. It is also consistent with small-scale experiments by Panfilov (1961; as quoted by Kerr, 1975) on ice plates, 1 cm – 6 cm thick, that resulted in very similar failure deflections (coefficient = 0.22 m$^{1/2}$). The ice temperature in these tests was between –8.5 and –3 °C.

6. Discussion

The preceding sections have shown that only the strain – energy criterion takes into account the loading history and creep that may precede failure, while at the same time, providing plausible results when applied to the limiting case of zero creep (IL). Even so, the test data indicate considerable scatter that cannot be explained by such predictable factors as load distribution or ice temperature. Another important factor could be variability in the viscous properties of the ice cover. This question is not well understood at present, because no generally applicable formulation of the time–dependent ice response to load exists, and thus viscous properties of the ice cover have yet to be quantified. Nevertheless, it may be expected that the temperature variation within the ice cover, which responds to air temperature changes, and thermal cracking activity would be relevant in this context. Based on his own testing experience, the writer further suspects that there is a random element in the load bearing capacity of floating ice covers, which is manifested in, and perhaps caused by, a similar randomness in cracking patterns. Though all breakthrough tests involve both radial and circumferential cracks, the number and spacing of these cracks vary, even where other test conditions appear to be identical.

The present findings with respect to the IL case, are consistent with the compressive-stress theory of Sodhi (1998) but do not support the size effect theory. It must be cautioned, however, that the evidence is indirect, because it is based on estimates of the instantaneous breakthrough load, extrapolated from the results of relatively brief ramp loadings. Direct evidence can only come from tests that are of much shorter duration than the ones examined herein. A related finding is the bias toward longer test times with increasing ice thickness, which arises from practical limitations on load magnitude and load application rates.

It is important to remember that the results obtained herein pertain to *failure* conditions and should not be used as indications of *safe* load, deflection, or work done by the load. As pointed out by Beltaos (1978), safety limits should be based on the condition known as the *onset of failure*, which occurs when the deflection begins to increase much more rapidly than a previously established pattern, and leads to breakthrough. The work done by the load at the onset of failure, W_{OF}, has also been found to increase in proportion to $h^{2.5}$, but is much less than W_f, i.e.:

$$W_{OF} \approx 300h^{2.5} \qquad (14)$$

With allowances for scatter and 10-minute creep, Beltaos (1978) calculated that the corresponding *safe* "short-term" load is about $1790h^2$, much as has been found by experience on good-quality ice covers. For long-term loads, the deflection must be known as a function of time before the work can be calculated. Prediction of time-varying deflections is not an easy task however, and should be based on in situ monitoring.

7. Conclusions

The load bearing capacity of floating ice covers is complicated by creep effects that severely limit the applicability of traditional, stress-based theories of failure. The nominal ice strength depends on loading history and generally decreases with time of loading. Ice deflection at failure has been proposed as a more promising parameter, but the test results show that it also exhibits strong dependence on loading history. Other things being equal, the failure deflection increases with increasing loading time, which implies that ice can sustain higher deflections under gradually applied, moderate loads than under rapid, large ones. This property is further manifested in the relative success of strain – energy criteria, which can be referred to the work performed by the load during the deformation process. The work at failure is shown to increase as the 2.5 – power of ice thickness and accounts for creep effects by integrating the load – deflection variation. Implications of the present findings to safety limits are discussed.

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