Is ϕ a Constant for Broken Ice Rubble?

G.W. Timco and A.M. Cornett

Canadian Hydraulics Centre
National Research Council
Ottawa, Ontario K1A 0R6
garry.timco@nrc.ca

Numerical models of river ice jams require correct properties of the ice rubble for reliable results. The rubble is usually assumed to behave as a Mohr-Coloumb material with a cohesion (c) and an angle of internal friction (ϕ). Previous work on the value for ϕ has shown a wide scatter. Recent tests using a controlled experimental apparatus indicate that the scatter may be largely a result of inappropriate test technique. This paper discusses these tests and presents the results of tests carried out with a large bi-axial compression tester for ice rubble. The paper shows that ϕ behaves in a systematic manner and can vary over a wide range depending upon the confinement conditions and stress history.

1. Introduction

Ice rubble is important in many applications in river and lake engineering. Accumulations of broken ice pieces in rivers cause ice jams which can significantly influence the hydraulic flow in a river, and in some cases, transportation. Recent numerical models of river ice movement and ice jamming provide a powerful new tool for understanding and predicting this river ice behavior. These models couple a hydrodynamic model with the appropriate physical and mechanical properties of river ice, and this is applied to a specific river geometry. For valid results, it is important to ensure that the input parameters are correct. With respect to the mechanical properties of the ice, the most relevant properties relate to describing the overall behavior of broken ice. These are usually described in terms of a Mohr-Coloumb material with a cohesion (c) and an angle of internal friction (ϕ). In using the numerical models, researchers usually pick single values for c and ϕ and use these values throughout the analysis process. This paper questions the validity of assuming a constant value for ϕ during these analyses.

The choice of ϕ is usually based on experimental measurements made using "shear boxes". Results of shear box measurements show a wide range in the values for ϕ, with no systematic dependence on any parameters. Therefore, almost any value can be chosen for ϕ and "justified" from experimental data. It is known, however, that the behaviour of broken ice is influenced by many factors including the mechanical properties of the ice pieces, the mechanism of formation, the stress history and the local thermodynamics.

This paper briefly reviews the past work done to measure the properties of ice rubble. In this review, the factors that affect the ice rubble are probed in an attempt to understand the behavior of ice rubble and the factors that affect its behavior. Published experimental results on the value of ϕ for ice rubble are examined and discussed. The paper will show that properly conducted experiments to measure ϕ indicate that it is not a constant; rather its value can vary in a systematic manner over a wide range, depending upon the stress history of the rubble and the loading scenario.

2. Measurements of Ice Rubble Behaviour

Ice rubble is usually assumed to behave as a linear Mohr-Coulomb material for which the shear stress τ and the normal stress σ_n on a failure plane are related by

$$\tau = c + \sigma_n \tan\phi \tag{1}$$

where c is the apparent cohesion and ϕ is the effective angle of internal friction (see Figure 1).

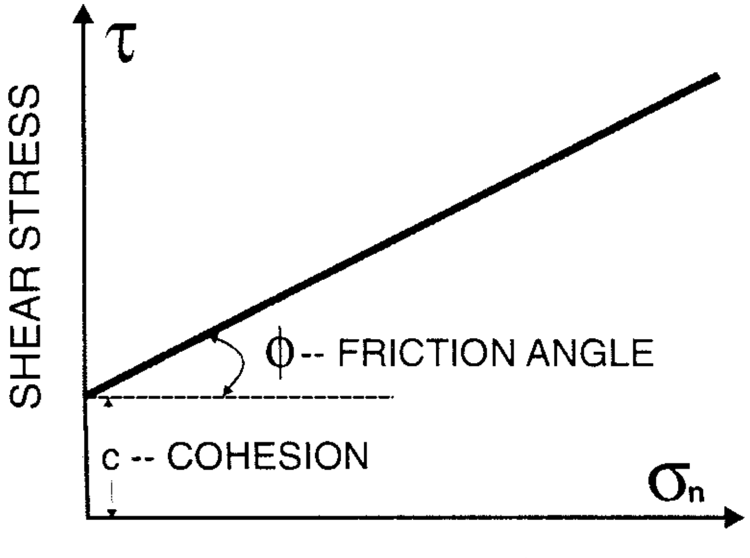

Figure 1. Illustration of the definition of c and ϕ for ice rubble.

The linear Mohr-Coulomb model was originally applied to describe the behaviour of ice rubble on the basis of analogies to soils, since both of these materials are comprised of an arrangement of irregularly shaped discrete particles. Some early tests on small samples of ice blocks showed that normal and shear stresses were proportional along a plane of failure. However, more recent research indicates that this behavior is not linear over a wide range. The major difference between the behaviour of ice rubble and of soils is that the strength of the individual ice blocks is much lower than the strength of individual soil grains (e.g. sand). Because of its relatively low strength, the deformation of bulk ice rubble usually results from re-arrangement and breakage of the blocks. Unlike soils, considerable deformation, crushing and breakage of individual ice blocks may take place in ice rubble.

Several different techniques have been used to measure the properties of freshwater ice rubble. Each of these methods is briefly reviewed in the following sections.

3. Shear Box Tests

Most early experiments used the direct shear approach because of its simplicity. In these tests, a large 2-part cavity is filled with broken ice pieces. The test apparatus is designed such that both a normal and shear stress can be applied to the ice rubble (see Figure 2).

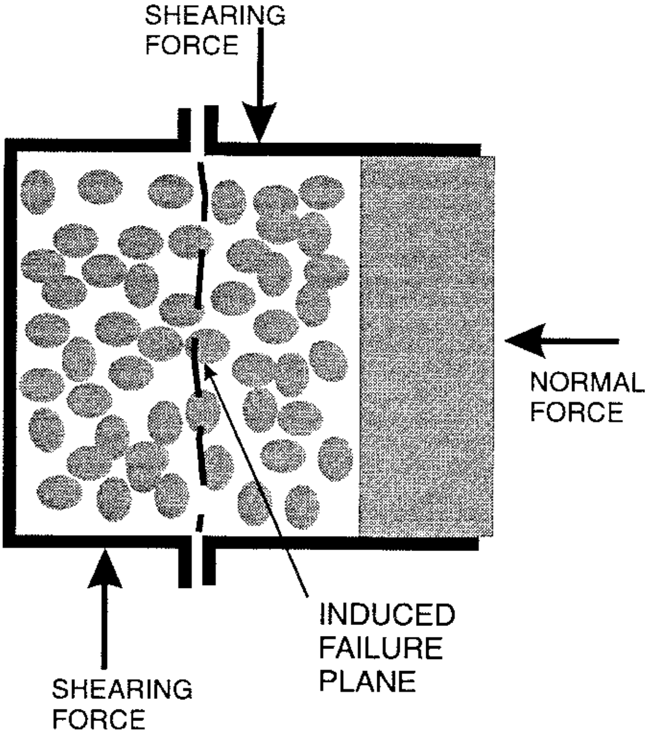

Figure 2. Illustration of the technique used for direct shear tests of ice rubble.

Ettema and Urroz (1991) and Chao (1993) have reviewed these tests and report that values of ϕ measured with this type of apparatus range from 8° to over 50°. There was no systematic dependence observed for ϕ on any parameters in these tests. The results of these tests for ice rubble must be viewed with caution. This type of apparatus produces non-uniform stress and strain distributions in the sample and "forces" the rubble to fail along a specified failure plane. The results, therefore, depend directly on the characteristics of the individual test apparatus and the test procedures. In consequence, the measured material properties are not an accurate description of the true material properties of ice rubble. No reliable quantitative results on bulk material properties for ice can be obtained from tests of this type.

4. Simple Shear

An alternate approach for measuring the properties of ice rubble is to use a simple shear box as shown in Figure 3. In this approach, one wall of the apparatus is attached to the sidewall of an ice tank, while the other wall is attached to a motor that can shear the box at a specified rate. The apparatus is filled to various depths with floating pieces of broken ice that are unconstrained vertically. Then, using the motor, the box is deformed in simple shear.

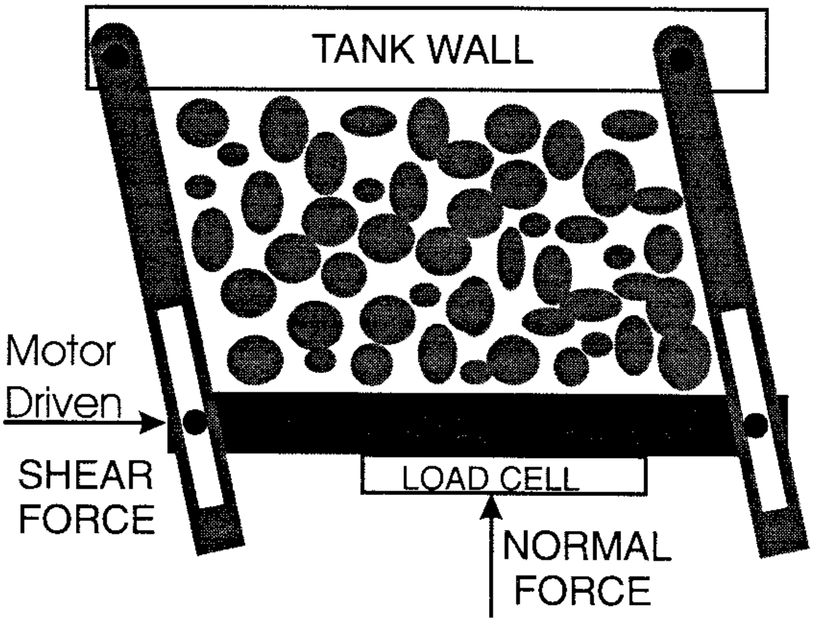

Figure 3. Illustration of the simple shear approach for measuring ice rubble behaviour.

Urroz and Ettema (1987) developed and used this apparatus and found it gave reliable results for simple shear of floating ice rubble. Measurements indicated that the shear strength depended on the rubble thickness, layer porosity and shear rate. Although this type of apparatus can yield good results, the range of confinement conditions is limited since there was no means to increase the normal (vertical) force on the ice rubble.

5. Bi-Axial Compression Apparatus

The Canadian Hydraulics Centre (CHC), a part of Canada's National Research Council in Ottawa, has developed a unique apparatus, the *Bi-axial Compression Chamber,* that provides direct measurements of the properties of broken ice under strictly-controlled boundary conditions. This apparatus produces uniform stresses and strains within a large sample of ice rubble. Failure of the rubble is not "forced" on any specific failure plane or area, as is done using the shear box approach discussed above. Also, the apparatus allows testing over a wide range of confinement conditions and loading rates. In theory, the approach is simple, but the apparatus required to perform the tests is quite complex.

Figure 4 shows a schematic illustration outlining the approach. This figure shows a plan view of the shape of the apparatus before and after a test. During a test, the shape of the test box changes as shown in the figure and the rubble inside the box must deform to follow this change in shape.

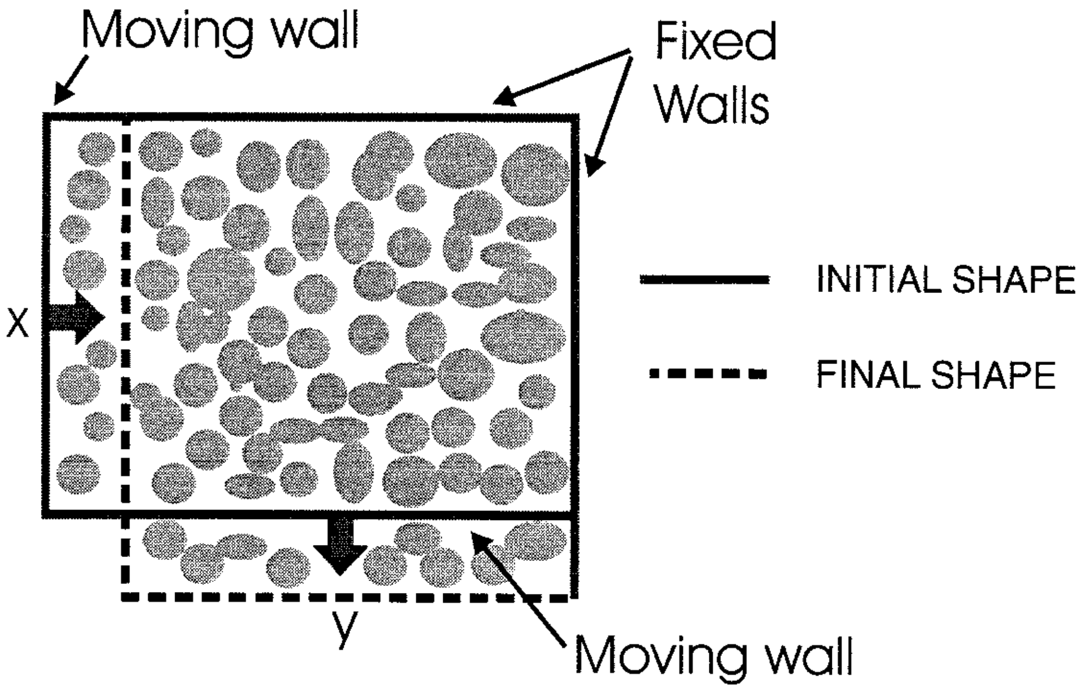

Figure 4. Plan view illustration of the bi-axial compression approach for measuring the properties of ice rubble.

The mechanical characteristics of the *Bi-axial Compression Chamber* have been described in detail by Timco et al. (1992). The apparatus is essentially a rectangular stainless steel container with moveable walls on two adjacent sides (see Figure 5 and Figure 6). When fully expanded, the container dimensions are 1 m by 1 m by 0.5 m (a volume of 0.5 m^3). The fully contracted dimensions are 0.8 m by 0.8 m by 0.5 m (a volume of 0.4 m^3). The moveable walls are powered by electric motors that can generate up to 40 kN of force per axis. Instruments are deployed to record the displacements of the moveable sidewalls, and the forces acting on the boundary of the sample in three orthogonal (x, y, z) directions. Tests can be conducted either with the lid on (plane-strain condition) or off (plane-stress condition). The entire apparatus is housed inside an environmental chamber, where the temperature can be varied over the range from −15 to +20°C.

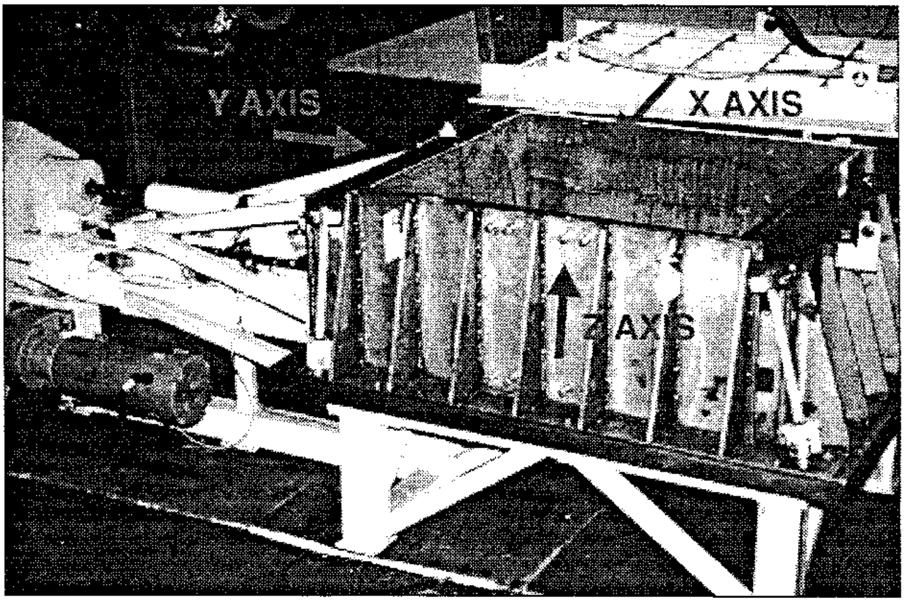

Figure 5. Photograph of the NRC Bi-axial Compression Chamber.

The apparatus is equipped with a digital controller that makes it possible to specify the type of control to be used for each axis as well as to set the values of key parameters. Different control protocols can be used to determine the behavior of ice rubble over a wide range of conditions. There are 3 types of axis control available:

1. *Velocity Control* – In this case, the velocities of the wall movement along the x- and y-axis are specified and maintained constant throughout the test run. For example, as shown in Figure 4, movement along the x-axis can be programmed to advance at a constant rate v_x, and the y-axis can be programmed to retreat at a constant rate of v_y. The ratio between these 2 rates defines the confinement conditions for the ice rubble under test.

2. *Pressure Control* - Three types of pressure control are available: *set-point* pressure, *maximum* pressure, and *proportional pressure* control. In set-point pressure control, the pressure exerted on the sidewall is controlled to track the set-point value. In maximum pressure control, the position of the sidewall is adjusted to ensure that the pressure on the side wall does not exceed the maximum value specified. In the

proportional pressure control mode, the pressure in one direction is controlled to be directly proportional to the pressure measured in the other direction; and

3. *Programmable Path Control* – A recent addition to the controller allows the axis to follow pre-defined stress paths in both active directions during a test. Figure 7 shows an example of the complex stress paths that can be followed during a test.

The apparatus offers a number of significant advantages over the direct shear boxes that have previously been used to explore the mechanical behaviour of bulk ice rubble:

- The sample volume is large (up to 0.5 m^3), so large ice pieces can be used.
- The stresses and strains within the sample are uniform.
- Tests can be conducted using any number of controlled stress or strain paths.
- Tests can be carried out with either plane-stress or plane-strain boundary conditions.
- Both dry and submerged (wet) samples can be tested.
- Both freshwater and saline ice rubble can be tested.
- A pre-stress can be applied to the sample prior to testing.
- The temperature of the sample can be controlled prior to testing.

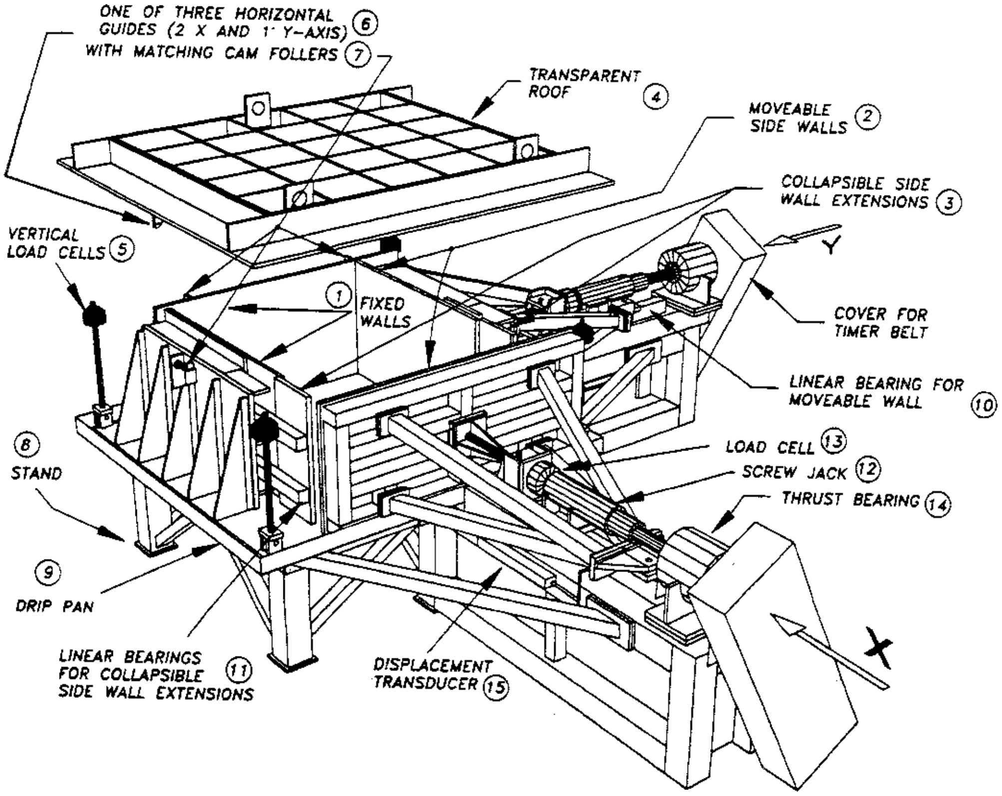

Figure 6. Schematic of the NRC Bi-axial Compression Chamber.

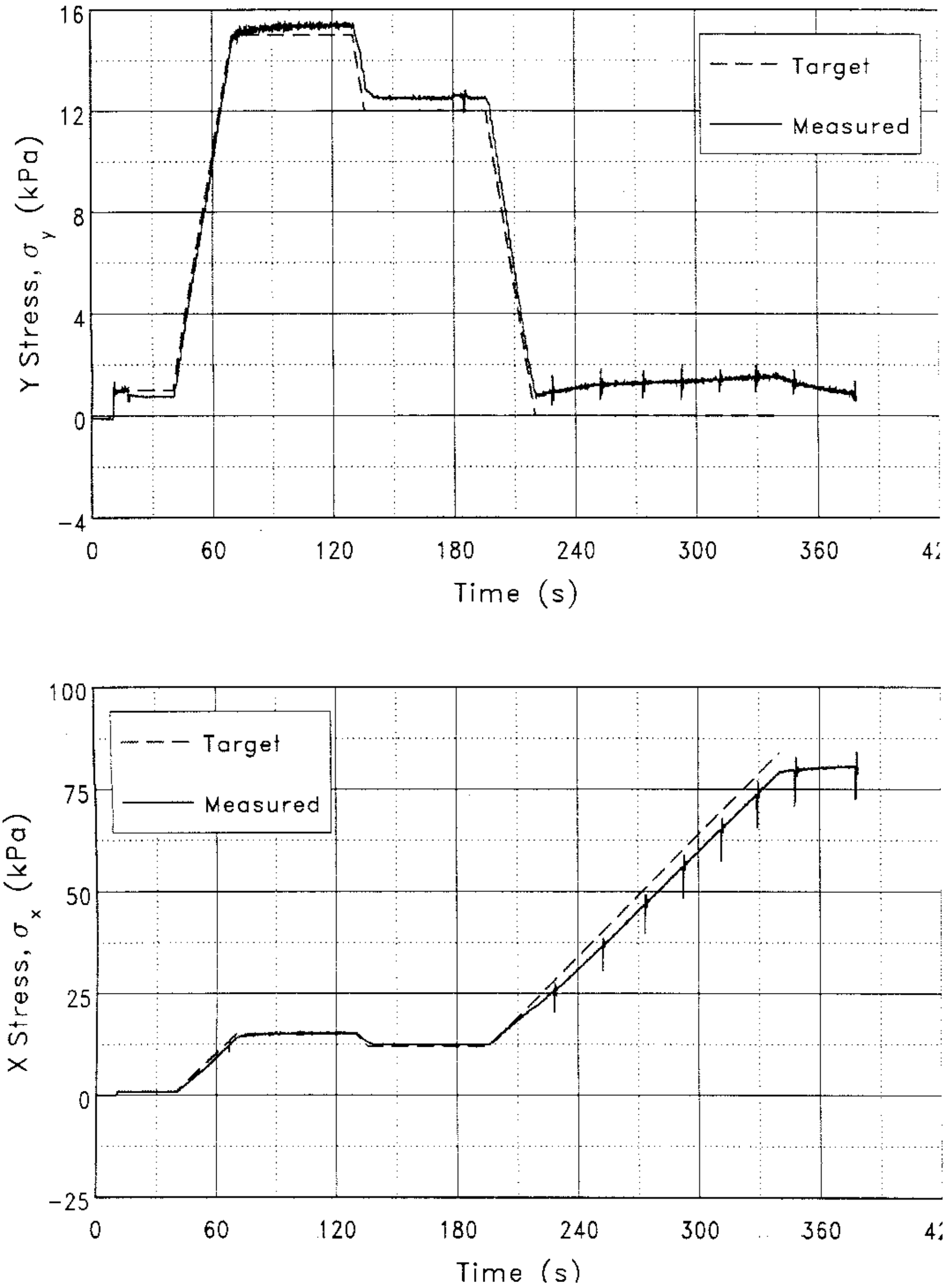

Figure 7. Comparison of target pressure and measured stress in the x- and y-directions during a test with distinct compaction and loading phases.

In the past, the apparatus has been used to investigate the properties of various types of broken ice, including freshwater ice (Loset & Sayed, 1993), saline ice (Cornett & Timco, 1995, 1996) and model ice (Sayed et al., 1992). These tests have examined, to a limited extent, the influence of piece size, piece shape, and the whether the ice rubble is dry or wet (ie. submerged under water).

6. Typical Stress-Strain Analysis

In a test in the bi-axial compression apparatus, the sample is contained within a rectangular volume with initial dimensions in the x and y directions of L_{x0} and L_{y0}. Stresses are developed within the sample by imposing displacements to the boundaries of the sample in both the x and y directions (by moving a vertical sidewall in each direction). The sample may be confined to prevent displacements in the vertical direction with a rigid bottom and top cover (plane-strain condition), or unconfined in the vertical direction (plane-stress condition).

The test apparatus is equipped with load cells to measure forces in the 2 loading directions (x- and y-axis) while 4 load cells support the lid. In addition, the displacements of the x-and y-axis are measured using extensometers. (With the lid on, there is zero displacement in the z-direction). This instrumentation provides complete information on the loads and displacements in all three directions at all times during a test. With this information, and knowing the initial starting positions of the sidewalls and the dimensions of the box, the stresses and strains can be computed at all times during a test. Then, by normalized out the time, the stress-strain behavior of the rubble can be determined.

The simplest analysis of the data treats the situation with a 2-dimensional analysis. For tests conducted with the lid of the compression chamber in place, the rubble is considered to undergo plane-strain deformation. Compressive strains and stresses are taken to be positive (extensions are negative). The natural logarithmic strain components for this case are

$$\begin{aligned} \varepsilon_x(t) &= \ln\left[L_{x0} / L_x(t)\right] \\ \varepsilon_y(t) &= \ln\left[L_{y0} / L_y(t)\right] \\ \varepsilon_z &= 0 \end{aligned} \tag{2}$$

where L_{x0}, L_{y0}, L_x, and L_y are the original and deformed sample lengths in the x- and y-directions. When the lid is removed, the rubble is free to deform upwards in the vertical z direction. In this case,

$$\varepsilon_z = \ln\left[L_{z0} / L_z(t)\right] \tag{3}$$

where L_{z0} is the initial sample length and L_z is the deformed length.

The directional strains are used to define a two-dimensional *shear strain*

$$\gamma(t) = |\varepsilon_x(t) - \varepsilon_y(t)| \tag{4}$$

and a two-dimensional *volumetric strain*

$$\Delta(t) = \varepsilon_x(t) + \varepsilon_y(t)\ . \tag{5}$$

The shear strain provides a measure of the change in the sample's shape, while the volumetric strain indicates the change in volume. For tests where the sample is vertically unconfined, these two-dimensional strains do not represent the true total strain in the sample, since deformations in the vertical z-direction are not included.

The principle stresses in the x-, y- and z-directions are

$$\begin{aligned} \sigma_x(t) &= F_x(t)/\, A_x(t) \\ \sigma_y(t) &= F_y(t)/\, A_y(t) \\ \sigma_z(t) &= F_z(t)/\, A_z(t) \end{aligned} \tag{6}$$

where F_x, F_y, and F_z are the forces acting on the boundary of the sample over the areas A_x, A_y, and A_z.

With a 2-dimensional analysis, the *mean stress* is defined as

$$p_{2D}(t) = \tfrac{1}{2}\left[\sigma_{\text{major}}(t) + \sigma_{\text{minor}}(t)\right] \tag{7}$$

and the *shear stress* is

$$\tau_{2D}(t) = \tfrac{1}{2}\left[\sigma_{\text{major}}(t) - \sigma_{\text{minor}}(t)\right] \tag{8}$$

where σ_{major} and σ_{minor} are the major and minor principle stresses, defined as

$$\sigma_{\text{major}} = \max\left\langle \sigma_x, \sigma_y, \sigma_z \right\rangle \tag{9}$$

$$\sigma_{\text{minor}} = \min\left\langle \sigma_x, \sigma_y, \sigma_z \right\rangle\ . \tag{10}$$

By analogy to the linear Mohr-Coulomb failure criterion, these two-dimensional stress invariants were used to define a *mobilized angle of shearing resistance* as

$$\phi_{2D}(t) = \sin^{-1}\left[\tau_{2D}(t) / p_{2D}(t)\right] . \tag{11}$$

This time-varying shearing resistance assumes that the ice rubble behaves like a cohesionless material.

7. Results

Only a limited number of tests have been performed with freshwater ice in the test apparatus. Most tests to date have used saline ice. In general, however, it has been found that the mechanical properties of both saline and freshwater ice rubble depend significantly on the stress history of the material and on the loading scenario.

The dependence of friction angle on the strain ratio during the loading phase is illustrated in Figure 8 for the case of saline ice rubble. These tests were performed using the constant-velocity control for the apparatus with a range of velocity ratios $\{v_x/v_y\}$, which are proportional to the strain ratios $\{\varepsilon_x/\varepsilon_y\}$. At low pressures, the friction angle decreases with increasing mean pressure. Above 10 kPa, the friction angle approaches a constant limiting value that varies significantly with strain ratio. Samples that were loaded at higher strain ratios (greater confinement) support smaller friction angles. Limiting values for friction angle have been determined by calculating average values of ϕ for mean pressures above 10 kPa. Figure 9 shows the limit friction angle plotted as a function of the absolute value of the strain ratio, $|\varepsilon_x/\varepsilon_y|$, for both freshwater and saline ice rubbles loaded under constant strain ratio. The data clearly show a distinct decrease in limiting friction angle with increasing strain ratio. The physical explanation for this behaviour remains somewhat elusive – one possible explanation is that with greater confinement (and higher stress levels) the ice pieces can fracture, allowing the rubble to flow more easily.

It is also possible to perform tests in which the rubble is constrained laterally at a constant set-pressure. Figure 10 shows results on friction angle for saline ice rubble loaded at a rate of 4 mm/s with lateral confining pressures of 5, 10 and 15 kPa. As in Figure 8, there is a trend of decreasing friction angle with increasing confining pressure.

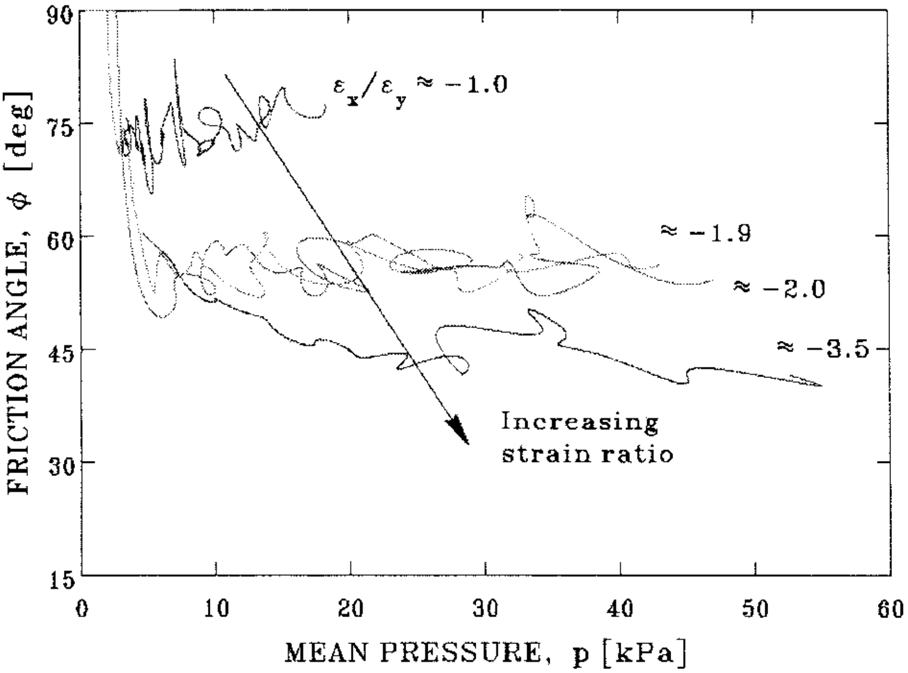

Figure 8. Influence of strain ratio on friction angle for saline rubble.

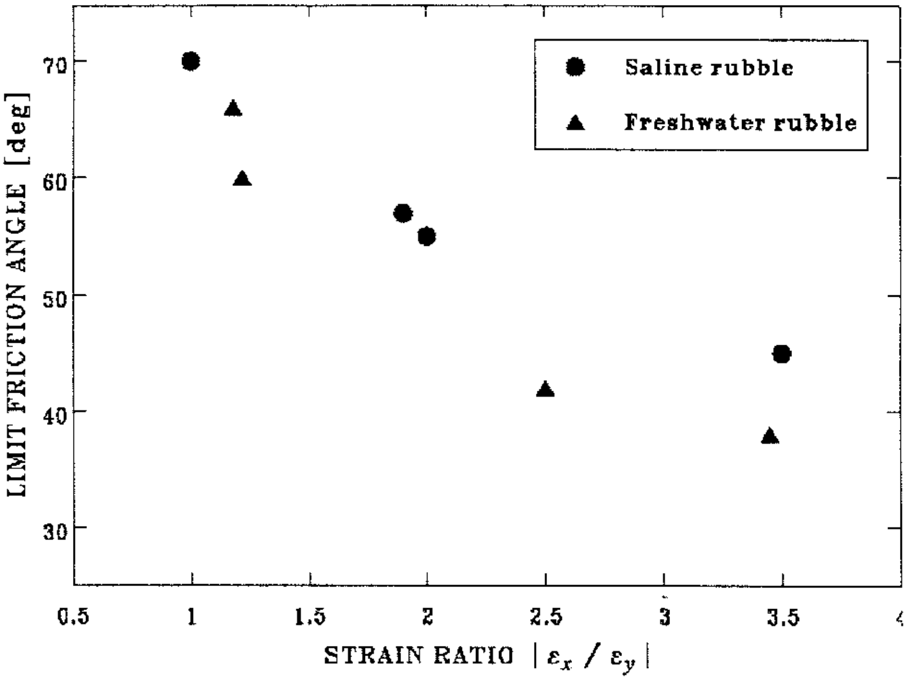

Figure 9. Influence of strain ratio on the limiting friction angle for saline and freshwater ice rubble.

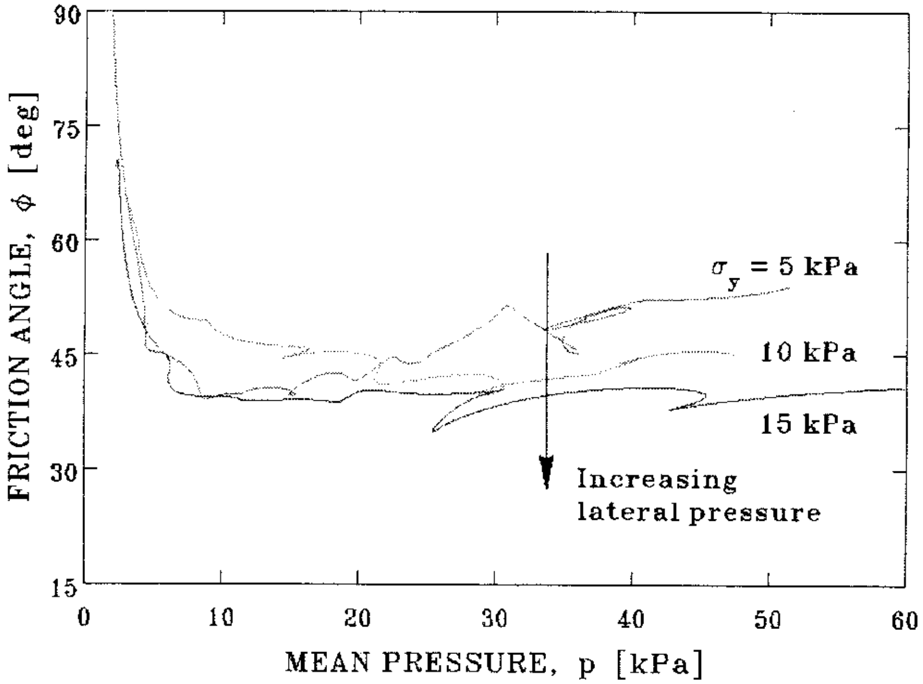

Figure 10 Friction angle versus mean pressure for saline rubble under constant lateral pressure.

8. Discussion and Conclusions

In many numerical models of river ice jams, the mechanical behaviour of ice rubble is assumed to follow the Mohr-Coulomb model, originally developed to describe soils. Previous measurements on ice rubble behaviour with shear boxes have provided conflicting information on the properties of ice rubble. These tests gave results that showed a wide range of scatter, and no systematic behaviour for the rubble properties. Based on these measurements, a constant value for friction angle ϕ was often quoted as an ice rubble property. However, the test results presented here, which were obtained using controlled boundary conditions in the CHC's bi-axial compression apparatus, illustrate that the friction angle ϕ is not constant – rather it behaves in a systematic manner depending on the stress history of the rubble, and on the loading scenario. It has been shown that for confined rubble loaded under a constant strain ratio, the friction angle decreases with increased confinement. For wet rubble that has been compacted with a pre-stress, the unconfined compressive strength depends strongly on the duration and magnitude of the pre-stress.

The CHC Bi-axial Compression Chamber is an effective apparatus for testing the bulk mechanical properties of ice rubble. It has numerous advantages over other devices that have previously been used to investigate the properties of ice rubble, including a digital controller that makes it possible to control the displacements, velocities and/or pressures on two perpendicular boundaries of the sample. Up to 0.5 m^3 of rubble can be tested at stresses up 100 kPa with linear strains up to 20% and shear strains up to 45 %.

Although these results are very encouraging, further testing is required to develop an understanding of the behaviour of broken river ice over the range of conditions encountered in nature. This is one of the main outstanding problems in river ice mechanics. Information is required on both the mechanical and thermal properties of ice rubble. The behaviour of broken ice plays such a key role in so many ice problems that a better understanding of its properties is essential to improve numerical models predicting ice jams, break-up, clearing, etc. These improved models will greatly assist in the prediction and management of river ice jams.

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