Design Considerations for the use of Ice as a Construction Platform

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The purpose of this paper is to provide straightforward, practical design advice to contractors and others who wish to use ice as a construction platform. The paper begins with a comparison of criteria for short term loading and then goes on to consider the potential for accelerated creep during long term loading. Equations have been fitted to several of the graphical relationships presented in the literature to facilitate spreadsheet calculations.

Based on a comparison of four methods, Gold's "safe" criterion is recommended for the determination of design ice thickness for short term loading situations (in the order of 3 minutes, or less). It is noted, however, that none of the available methods are based on actual loads above 1400 kN (~75tons); therefore, an additional factor of safety should be applied when designing for higher loads. Particular care should also be taken when moving loads on the ice cover, ensuring that vehicle speeds are kept below 75% of the critical velocity.

As present, the strain-energy criterion is the only potentially reliable failure criterion for long term loading. Application of this method requires continuous monitoring of both the load history and the resulting ice deflection, and the load should be removed before the deflection of the ice cover reaches the maximum allowable value. Furthermore, serious consideration should be given to limiting the allowable deflection to the freeboard, if the freeboard is less than the allowable value obtained using the strain energy criterion.

1. Introduction

The load bearing capacity of floating ice has received considerable attention in the literature (e.g. Nevel, 1968, 1970b; Gold, 1971; Frederking and Gold, 1976; Michel, 1978; Beltaos, 1978, to cite only a few), and valuable reviews of both theory and practice are available (e.g. Kerr, 1975; Lipsett and Harrington, 1981, respectively). However, it was felt that a review of design methods for construction applications was pertinent at this point for several reasons. First, much of the available literature on the topic concentrates on the application of this theory to the design of ice bridges, which involve short term loading situations and smaller loads than some construction applications. Secondly, current practice depends on numerous reference sources and this material is still somewhat scattered. Ashton's (1986) review is helpful, but for some applications one must still reference the original source material in order to develop design equations. Finally, many of the design guidelines used in current practice are based on complex analytical solutions which have been presented in graphical form for practical application; however, that form is inconvenient for modern computer application. Here, key solutions are presented in equation form, for easy application in a spreadsheet or for implementation in a simple computer program.

This paper begins by considering the basic theory necessary to distinguish between short and long term loading conditions, and the methodology of design for each case. The determination of design ice thickness for short term loading situations is then covered, revisiting Gold's (1971) criterion and comparing it with other criterion proposed in the literature. The paper then focuses on the more difficult aspects of the design associated with the potential for accelerated creep during long term loading. Moving loads on an ice cover are also considered as construction equipment must be deployed to the position on the ice cover where it will be used.

2. Behaviour of Ice Under Short Term Loading

Short term loads on a floating ice cover are defined as those which occur for a short enough duration that the ice behaves in an elastic manner. Specifically, if the ice is capable of sustaining the load it will deflect (strain) under the load, but it will recover completely upon removal of the load (i.e. there is no residual strain within the ice cover). If the load is too large, brittle failure (breakthrough) occurs.

For a detailed review of the literature on the theoretical analyses of the elastic behaviour of ice, the reader is referred to Kerr (1975). To summarize, it has been determined that if we place a vertical load on a homogeneous and isotropic floating ice cover for a short period, the resulting instantaneous deflection which occurs can be described by the theory of an elastic plate on an elastic foundation, for which (Kerr, 1975):

$$\frac{D}{\gamma} \nabla^4 w + w = \frac{q}{\gamma} \tag{1}$$

where D is the flexural rigidity of the plate; γ is the elasticity of the foundation (which in the case of a floating ice cover is simply the specific weight of water: 9.805 kN/m^3 at 0°C); w is the plate deflection at spatial co-ordinates (x,y) on the plate; and, q is a uniformly distributed load. D depends upon the properties of the ice and is defined as:

$$D = \frac{E\,h^3}{12\left(1-\nu^2\right)} \tag{2}$$

where *E is Young's modulus, h is the ice plate thickness, and* ν *is Poisson's ratio* (generally taken as $^1/_3$ for this situation).

From equation (1) we see that the deflection, w, is dependent only upon the loading condition and a parameter, D/γ, which has the units of length raised to the power of four. It is therefore convenient to define a single parameter, ℓ, which has the units of length, by taking the fourth root of D/γ:

$$\ell = \sqrt[4]{\frac{D}{\gamma}} = \sqrt[4]{\frac{E\,h^3}{12\gamma\left(1-\nu^2\right)}} \tag{3}$$

This parameter, ℓ, is commonly known as the "characteristic length" of the ice cover, though some have also termed it the "action radius" (e.g. Nevel, 1970b).

It has been found that the characteristic length is a much stronger function of ice thickness, h, than of either E or ν. Therefore, the primary objective in the design of the an ice cover for load bearing purposes is generally to determine the required ice thickness. Based on field measurements and laboratory tests involving short term loads, Gold (1988) reported values of ℓ for columnar grained, freshwater ice in the range of $15.9\,h^{3/4}$ to $17.5\,h^{3/4}$ (where h is in metres), and extending to $12.6\,h^{3/4}$ for granular, freshwater ice covers. Michel (1978), using typical values for E and ν, found $\ell = 17\,h^{3/4}$. In the ensuing discussions, we adopt $\ell = 16\,h^{3/4}$ as a practical value to use for short term loading situations.

Based on the solution of equation (1) for the case of a concentrated load, P, on an infinite plate (Kerr, 1975) the maximum deflection, w_o, which occurs right under the concentrated load, can be calculated as:

$$w_o = \frac{P}{8\sqrt{\gamma D}} = \frac{P}{8\gamma\ell^2} \tag{4}$$

Equation (1) is linear and, consequently, it appears that the method of superposition can be used to determine the deflections, $w(x,y)$, of multiple loads spaced apart from each other (Kerr, 1975). That is, their deflections can be computed separately and simply added at each (x,y) co-ordinate. Nevel (1968) presented a solution for the entire deflection bowl as a function of the radial distance, r, from a concentrated load (Figure 1). It is seen that the influence of a concentrated load on an ice cover is negligible beyond about $5r/\ell$. To facilitate the use of the relationship presented in Figure 1 when calculating combined deflections for loads placed at a spacing less than $7r/\ell$, we have fit a polynomial equation of the form:

$$\frac{w}{w_o} = a_0 + a_1\left(\frac{r}{\ell}\right) + a_2\left(\frac{r}{\ell}\right)^2 + a_3\left(\frac{r}{\ell}\right)^3 + a_4\left(\frac{r}{\ell}\right)^4 + a_5\left(\frac{r}{\ell}\right)^5 + a_6\left(\frac{r}{\ell}\right)^6 \quad (5)$$

where a_o = 1.00, a_1 = -0.195, a_2 = -0.315, a_3 = 0.171, a_4 = -0.0356, a_5 = 0.00342, and a_6 = -0.000126 (coefficient of determination = R^2 = 0.99992).

These solutions for equation (1) are for an infinite ice sheet so, practically speaking, this means that the load should be a minimum distance of $7r/\ell$ from any wet cracks (cracks which penetrate the full depth of a floating ice cover) or open water, and a greater distance is strongly recommended. Near wet cracks or open water, the ice cover should be considered as semi-infinite. Analytical solutions indicate that the load bearing capacity of a floating ice sheet should be reduced by half for this case (Kerr, 1975). Also, the maximum deflection is about 4 times that for the infinite ice sheet (i.e. about 4 times that predicted by equation (4)).

3. Behaviour of Ice Under Long Term Loading

To understand the behaviour of ice under long term loading conditions, it is necessary to understand the molecular behaviour of ice under loading. Michel (1978) provides an excellent and detailed discussion of this, from which the following brief explanation is drawn.

Ice is a crystalline material which derives it strength from the intermolecular bonds within a hexagonal crystal lattice structure. Imperfections in the lattice structure (e.g. extra molecules) create internal stresses within the ice cover and, when the ice is loaded for a sufficient duration, these imperfections can propagate through the crystal lattice creating permanent damage (deformation) within the ice cover. Typically what is observed in such situations is an initial elastic deflection as a load is placed on the ice cover (as described above in section 2), followed by a gradual increase in the magnitude of the deflection over time as imperfections propagate through the crystal lattice. This additional deflection with time is known as "creep" and, once it occurs, the ice remains deformed after removal of the load. The propagation of imperfections is initially impeded at the grain boundaries of the ice crystals (a process known as strain hardening) but at some point, if the load duration is sufficient, imperfections will be effective in propagating around the grain boundaries after which "accelerated creep" will be observed. This is a particularly dangerous situation since the rate of deformation of the ice cover increases dramatically at this point and complete failure can be expected soon after.

All crystalline materials are subject to such behaviour, but certain forms of defect propagation are temperature dependent, and since ice is used operationally at temperatures very near its melting point, it is particularly subject to creep deflection. Thus accelerated creep can occur after only a brief time of loading. Gold (1975) reported that for imposed stresses up to 480 kn/m^2, elastic behaviour was observed for load durations of up to 200 s (3.3 minutes) and minimal non-recoverable strain was observed for load durations of up to 400 s (6.7 minutes). Beltaos (1978) has suggested that a load duration in excess of 10 minutes should be considered a long term loading situation for floating ice covers.

A number of criterion have been suggested for determining the onset of failure as a result of creep deflection and Beltaos (1978) provides an excellent review of these. As he points out, any such criterion should be independent of time and should apply under any loading history. He proposed the strain-energy criterion which is based on the premise that *"failure occurs when the maximum work done by internal stresses on a unit volume of material equals or exceeds a critical value"*. As present, the strain-energy criterion is the only long term loading failure criterion which meets the two requirements set out above.

Using data obtained from field tests, Beltaos (1978) observed remarkable consistency in the cumulative strain energy at the "onset of failure", W_{OF}, which he defined at the point in time at which the deflection rates accelerate noticeably (i.e. at the onset of accelerated creep):

$$W_{OF} = 300\, h^{5/2} \tag{6}$$

when W_{OF} is in kN-m and the ice thickness, h, is in metres. Since he found that the time period between the onset of accelerated creep and ultimate failure (breakthrough) was quite random (due to the complexity of factors affecting the failure process), he suggested that the onset of accelerated creep (i.e. equation (6)) be taken as the long term loading failure criterion.

4. Design Ice Thickness for Short Term Loads

Some complications arise in the application of equation (1) because natural ice is neither isotropic nor homogeneous. To begin with, the mechanical properties of ice are dependent upon temperature, and the upper portion of an ice cover (if not insulated by snow) will be near the prevailing air temperature while the bottom of the ice cover will be at 0°C. The mechanical properties of ice are also dependent upon the crystal structure, and natural ice is non-isotropic and usually stratified (particularly in the case of river ice). Other factors affecting the mechanical properties of river ice include: the rate of freezing; the hydraulic conditions at the time of freezing (particularly flow velocity); and, the content of impurities, all of which are highly variable. Consequently, several of the practical methods currently employed to determine the design ice thickness are empirical in nature. Four alternative methods are considered here.

"Safe" Load Criterion (Gold, 1971)

Figure 3 presents failure loads reported by Gold (1971). The upper line defines a "safe" design ice cover thickness, above which few failures were reported:

$$P = 350\, h^2 \tag{7}$$

where P is a concentrated load (in kN) and h is the ice thickness in metres. The lower line defines a limit beyond which failures were certain (P = 7000 h^2), a relationship which is useful to the design of icebreakers. Based on the data in Figure 1, Gold (1971) also suggested that "sound" ice covers should not fail under a slow moving load for:

$$P = 1400\, h^2 \tag{8}$$

First Crack Criterion (Michel, 1978)

Michel suggested taking the load which produces the first radial crack as the design load capacity under short term loading. He favoured this criterion because it is easy to verify in the field. Also, since such a crack effectively changes the plate from infinite to semi-infinite, for which the load capacity is half, such a criterion provides for a factor of safety of about two for competent ice and factor of safety of barely one if wet cracks form. (The load capacity should therefore be halved if a crack occurs.)

Application of this criterion first requires determination of the tensile strength of ice. Under short term loading (elastic behaviour of ice) Michel (1978) found the tensile strength of ice, σ_T, to be relatively independent of temperature, but quite strongly dependent upon ice type. He proposed the following relationship:

$$\sigma_T = \frac{80}{\sqrt{d}} \tag{9}$$

where d is the grain size of the ice in metres. Typical grain sizes for freshwater ice types are: 1 to 4 mm for snow and frazil ice; and 5 to 25 mm for columnar ice (range from top to bottom). Note, these must be converted to metres for use in equation (9).

One must also know the stresses induced under loading to assess the design ice thickness. Nevel (1968, 1970b) presented a family of curves defining the stress under distributed loads based on thick plate theory from an analytical solution assuming a homogeneous, isotropic, elastic plate on an elastic foundation. Figure 2 presents an adaptation of the original figure, in which complete results for the thick plate theory are provided whereas, in presenting this same figure, Nevel (1970b) took thin plate theory (the $h/\ell = 0$ curve) as the limiting case.

Nevel's equations are quite onerous, but he reported that Westergaard's (1926) simpler formula could be applied with negligible error. This involves using thin plate theory, and replacing the radius of the load, *b*, with *a*, where:

$$a = \sqrt{1.6b^2 + h^2} - 0.675\,h \qquad (10)$$

when $b/h < 1.724$. Using this approach, an equation describing Figure 2 was obtained by fitting a polynomial to the thin plate theory ($R^2 = 0.9999$):

$$stress\ factor = \frac{\sigma h^2}{P(1+\nu)}$$
$$= 5.08 - 399\left(\frac{b}{\ell}\right) - 181\left(\frac{b}{\ell}\right)\ln\left(\frac{b}{\ell}\right) - 774\left(\frac{b}{\ell}\right)^2\ln\left(\frac{b}{\ell}\right) - 72.8\sqrt{\frac{b}{\ell}} \qquad (11)$$

and then substituting *a* for *b* in equation (11). Substituting Michel's (1978) equation (9) for the stress (i.e. setting $\sigma_T = \sigma$) and assuming $\nu = {}^1/_3$ and $\ell = 16\,h^{3/4}$, one can determine the ice thickness requirement for a given load iteratively. The solution converges quickly if a reasonable initial guess is taken for the ice thickness (e.g. 1 m).

Nevel (1970b) noted that for many situations, such as in the case of wheeled vehicles, the size of the load is small enough that it should be considered as concentrated. If that is the case, then the analysis is actually considerably simplified, since we use only the intercepts (the values at $b/\ell = 0$) in Figure 2, and the stress factor depends only upon h/ℓ. Fitting an equation to these intercepts we obtained ($R^2 = 0.9999$):

$$stress\ factor = \frac{\sigma h^2}{P(1+\nu)} = 0.82 - 0.48\ln\left(\frac{h}{\ell}\right) \qquad (12)$$

Westergaard (1948) presented a similar equation based on base ten logarithms.

Freeboard Criterion (Panfilov, 1961; Frederking and Gold, 1976)

It is important to remember that the density of freshwater ice is not significantly different than that of water (about 90 to 92% for natural ice), and so it floats with less than 10% of its total thickness above the water level. The distance between the top of the ice and the water level is commonly known as the "freeboard". Panfilov (1961, in Russian: see Kerr, 1975) noted that if the deflection of a loaded ice cover exceeds this amount, *and* if cracks are present in that ice cover, then water will flood the ice surface and the load bearing capacity of the ice sheet will be compromised. In addition, if water does flood onto the ice surface, it will bring the temperature there to 0°C. Since the top of the ice is in compression, and the since the compressive strength of ice is strongly dependent upon temperature (Michel, 1978), then the ice strength may be noticeably reduced in an area of high stress (Kerr, 1975). Based on this, Panfilov (1961) proposed that the appropriate high stress (Kerr, 1975). Based on this, Panfilov (1961) proposed that the appropriate design criterion for short term loads would be to limit the deflection to the freeboard. Frederking and Gold (1976) have also suggested this as a design criterion.

To apply this method, the maximum deflection (under the load) as defined in equation (4) is set to equal the freeboard:

$$w_o = \left(1 - \frac{\rho_i}{\rho}\right) h = \frac{P}{8\gamma \ell^2} \tag{13}$$

where ρ_i is the density of ice and ρ is the density of water. Assuming ice to have a density of 92% of that of water and adopting $\ell = 16\ h^{3/4}$, we get the following relationship for the freeboard criterion:

$$P = 1600\ h^{10/4} \tag{14}$$

Strain Energy Criterion (Beltaos, 1978)

In presenting his strain energy criterion for long term loading, Beltaos (1978) pointed out that if it was truly independent of time, that is representative of a genuine material property, then it should be applicable for both short and long term loading situations. Taking equation (6) as the failure criterion then, the strain-energy for instantaneous loading would be equal to the product of the load, P, and the deflection , w_o, with the latter defined by equation (4), such that

$$W_{OF} = 300\ h^{5/2} = w_o P = \frac{P^2}{8\gamma \ell^2} \tag{15}$$

Adopting $\ell = 16\ h^{3/4}$, we get the following relationship for the strain energy criterion:

$$P = 2455\ h^2 \tag{16}$$

Comparison of Short Term Loading Criteria for Construction Applications

Figure 4 presents a comparison of these short term loading criteria for design ice thickness over the range of loads applicable to construction applications, assuming concentrated loads. (It is important to note that none of these methods apply to punch through failures, however.) Note that a person weighs approximately 1 kN and that the crane weights indicated include the crane's rated load capacity. The documented failure loads reported by Gold are shown to provide a frame of reference. Of significance to construction applications is the range of applicability of each criterion, since much of the work in this area to date applies to the design of ice bridges. For example, Gold's reported failures are for loads up to only about 300 kN. Michel *et al.* (1974) reported that the maximum load safely carried on the James Bay Project ice bridges, which were reinforced with logs, was 75 tons (1470 kN). Beltaos' (1978) strain-energy criterion is based on field tests for ice thicknesses less than 0.76 m. At capacity, the load imposed by some cranes used in construction are an order of magnitude larger than the loads corresponding to these upper limits, as seen in Figure 4. Therefore extreme care should be exercised in applying any of these criterion in such applications. We suggest applying an additional factor of safety when specifying the ice thickness in this case.

It is also of interest to note the difference between the freeboard criterion (which is proportional to $h^{10/4}$) and the others (which are proportional to $h^{3/4}$). It would appear that the design ice thickness obtained from Gold's (1971) "safe" criterion (equation (7)) is always greater than that which would be prescribed by the freeboard criterion. However, this is not always the case for Michel's (1978) criterion (e.g. for the smaller grain sizes), nor for Gold's "minor risk" criterion (equation (8)), which would both dictate lesser ice thicknesses over at least part of the practical range of loads. It is also interesting to note that Beltaos' (1978) strain energy criterion plots below all of the others, illustrating its applicability in defining the onset of failure.

Based on the comparison presented in Figure 4 and the above discussion, and keeping in mind the non-homogeneity of river ice in particular, Gold's "safe" criterion (equation (7)) is recommended for the determination of design ice thickness for short term loading situations (in the order of 3 minutes or less).

5. Design Ice Thickness for Long Term Loads

At present, the strain energy criterion is the only potentially reliable failure criterion for long term loading, since it is the only one which is independent of time and applies under any loading history. To apply this method, we use equation (6) to compute the maximum allowable deflection of the ice cover under the long-term load:

$$w_{allowable} = \frac{W_{OF}}{P} = \frac{300\, h^{5/2}}{P} \qquad (17)$$

Of course, it is first necessary to ensure that the ice thickness is greater than the safe short term value, so that $w_{allowable}$ is greater that w_o (as defined by equation 4)) since elastic deflection of the ice will initially occur and creep deflections will go beyond that amount. The greater the ice thickness, therefore, the longer the load will be able to be left on the ice cover before reaching $w_{allowable}$.

Application of this method requires continuous monitoring of both the load history and the resulting ice deflection. As the deflection of the ice cover approaches $w_{allowable}$, the onset of failure should be expected and the load should be removed. In addition, consideration should be given to limiting the allowable deflection to the freeboard amount (if it is less than the allowable deflection prescribed by equation (17)).

The main disadvantage of the strain-energy approach is that it only allows us to *anticipate* the onset of failure, by continuously monitoring the deflection of the ice cover and removing the load before the amount of deflection reaches the allowable maximum value. It would be much more desirable to be able to *predict* the onset of failure so that we could place a load on the ice cover and know how long it could safely be left there. This problem has been addressed by Nevel (1968), Frederking and Gold (1976), and Beltaos and Lipsett (1978). Figure 5 (adapted from the latter study) presents a non-dimensional time versus deflection curve of the form:

$$\frac{w}{w_o} = 1 + \sqrt{\frac{t}{\tau}} \qquad (18)$$

Here, w is the deflection at any time t, and τ is a "characteristic time", arbitrarily defined as the time it takes for the deflection to reach twice the initial value, w_o. Documented values of τ are limited to those presented by Beltaos and Lipsett (1978) for only a few cases (for which they found it to vary between about 100 to 250 minutes). Therefore, the the value of τ must be determined on a site specific basis from actual deflection measurements taken for a short period after the load is applied (in the order of 60 minutes or more). This can be done by plotting the deflection (y-axis) against the square root of the time (x-axis). The slope of this line will be $w_o/\sqrt{\tau}$ With this approach, one does not have to wait for the measured deflections to actually reach $2xw_o$ in order to determine τ.

Knowing τ one can use equation (18) to determine an approximate deflection versus time curve and obtain a reasonably good estimate of the expected deflection for a given time. However, it is very important to note that the curve is very flat and there is considerable spread in the data for the inverse relationship. For example, depending upon the value of τ, the spread in the data of Figure 5 would suggest a duration of 5.5 to 24 h for the creep deflection to reach 3 times the initial value. *Therefore, it is not possible to accurately determine the time that it will take for a particular deflection to occur.*

Equation (18) does allow us to look at the importance of freeboard to the long term loading problem, however. In Figure 6 we present a comparison of the design ice thickness required based on these two criteria using equation (18) to compute the deflection for load durations of 1 hour (a typical crane lift duration), and 1 and 5 days (which is a typical range of time for crane setup). The freeboard requirement for short term loading is also provided, for reference purposes. As the figure illustrates, design ice thicknesses based on the strain energy criterion may be considerably less than those specified by the freeboard requirement, particularly for smaller loads. It should also be noted that Gold's "safe" (short term loading) criterion, favoured by some for this application, is not conservative when compared to the freeboard criterion.

6. Moving Loads

When moving equipment and material onto the ice cover; the deflection bowl under the moving load will push water aside, generating waves in the water. If the vehicle speed approaches the velocity of these waves (defined as the critical velocity), something very much like resonance can occur. Beltaos (1981) has shown that this can lead to severe stresses within the ice cover and excessive deflections (approaching twice the elastic deflection defined by equation (4)). Numerous breakthroughs have been documented as a consequence of high speed moving loads. A particularly dangerous aspect of this problem is that the ice cover might not necessarily fail in response to a load moving at or near to the critical velocity, but damage to the ice (which is not readily evident) may lead to breakthrough for the next vehicle passing over the affected area. Therefore it is vital that loads be moved at speeds less that the critical velocity. Beltaos (1981) has recommended that moving load velocities be limited to 75% of the critical velocity.

The critical velocity is a function of both the ice characteristics (represented by the characteristic length, l) and the water depth under the ice cover, H. The latter dominates when $l/H > 1$ such that the critical velocity, u_c, can simply be defined by the critical velocity for open channel flow:

$$u_c = \sqrt{gH} \tag{19}$$

where g is the acceleration due to gravity. However, for small ice thicknesses (particularly in deep flows) the characteristics of the ice cover cannot be neglected. Nevel (1970a) presented an analytical solution for the problem in graphical form. Beltaos (1981) has provided an updated version in metric units; a polynomial equation valid for $0 < l/H < 1.4$ has been fitted and is available from the first author.

7. Summary and Recommendations

Based on the comparison presented here, Gold's "safe" criterion is recommended for the determination of design ice thickness for short term loading situations (in the order of 3 minutes, or less). An additional factor of safety should be applied when designing for loads in excess of 1400 kN (75 tons). Also, particular care should be taken when moving loads onto the ice cover; speeds should be kept below 75% of the critical velocity.

As present, the strain-energy criterion is the only potentially reliable failure criterion for long term loading. As there is currently no reliable method for determining the time that it will take for a particular deflection to occur application of this method requires continuous monitoring of both the load history and the resulting ice deflection. The load should be removed before the deflection of the ice cover reaches the maximum allowable value. Furthermore, serious consideration should be given to limiting the allowable deflection to the freeboard if the freeboard is less than the allowable value obtained using the strain energy criterion.

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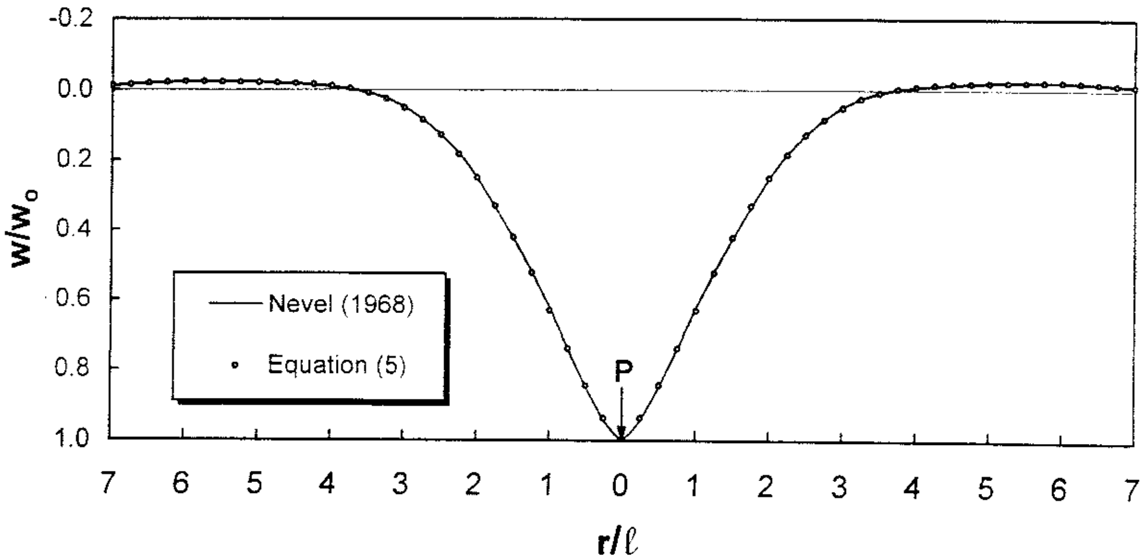

Figure 1. Deflection of a floating ice sheet under a short term concentrated load.

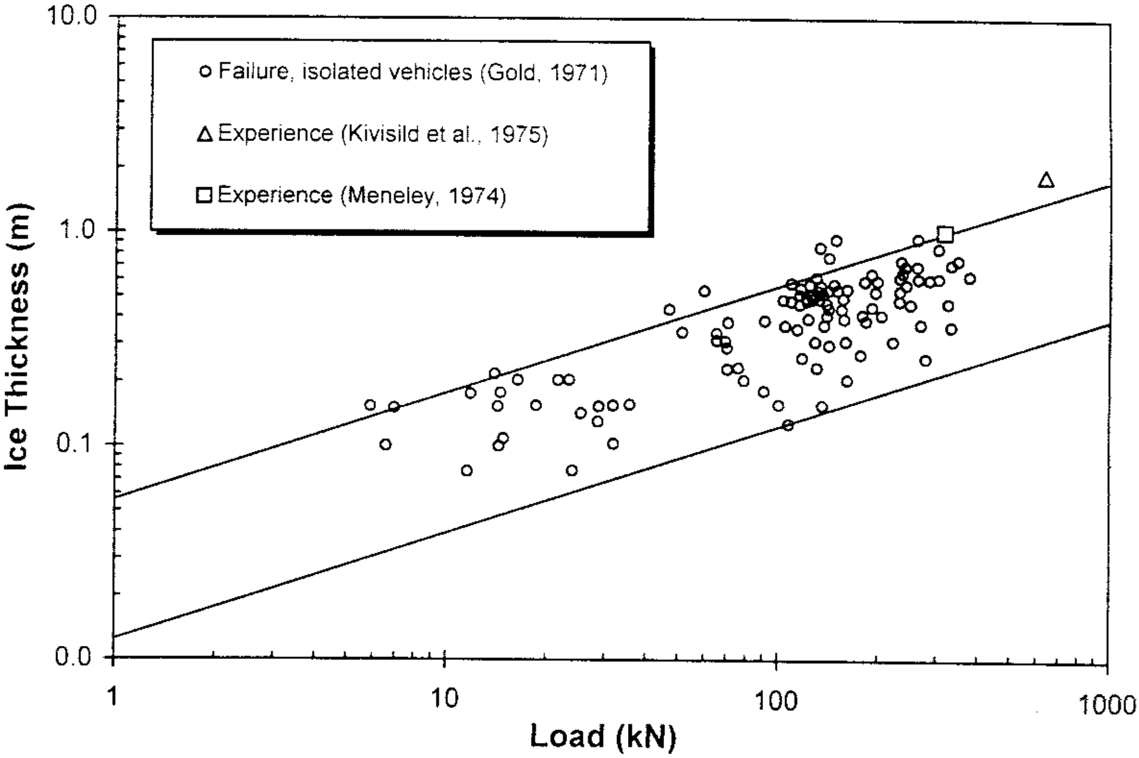

Figure 2. Failure loads reported by Gold (adapted from Gold, 1971).

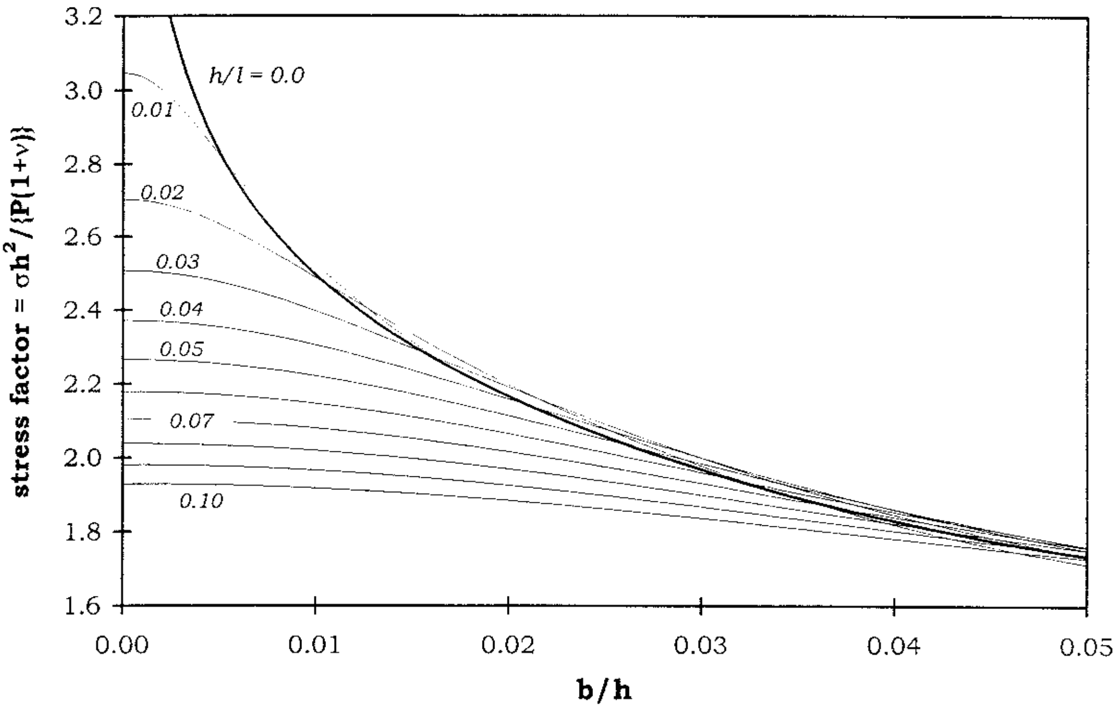

Figure 3. Stress factor calcualted based on Westergaard's equation (adapted from Nevel, 1970).

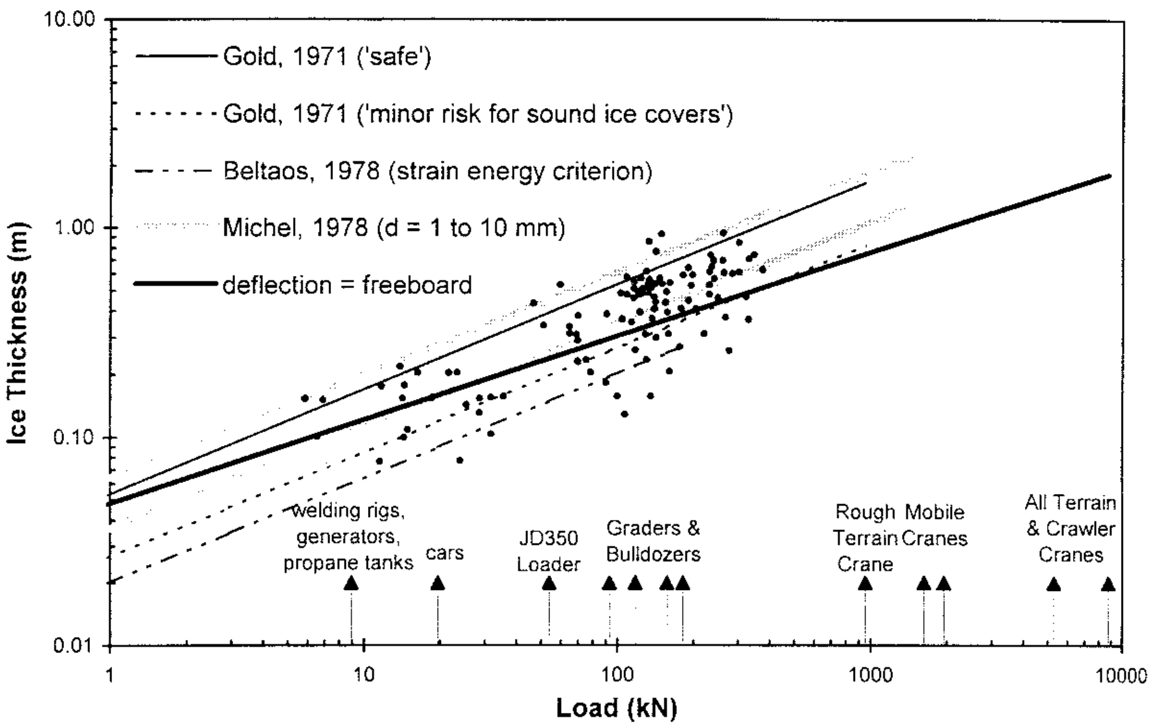

Figure 4. Comparison of various design criterion for determining the design ice thickness for short term loads (elastic behavior).

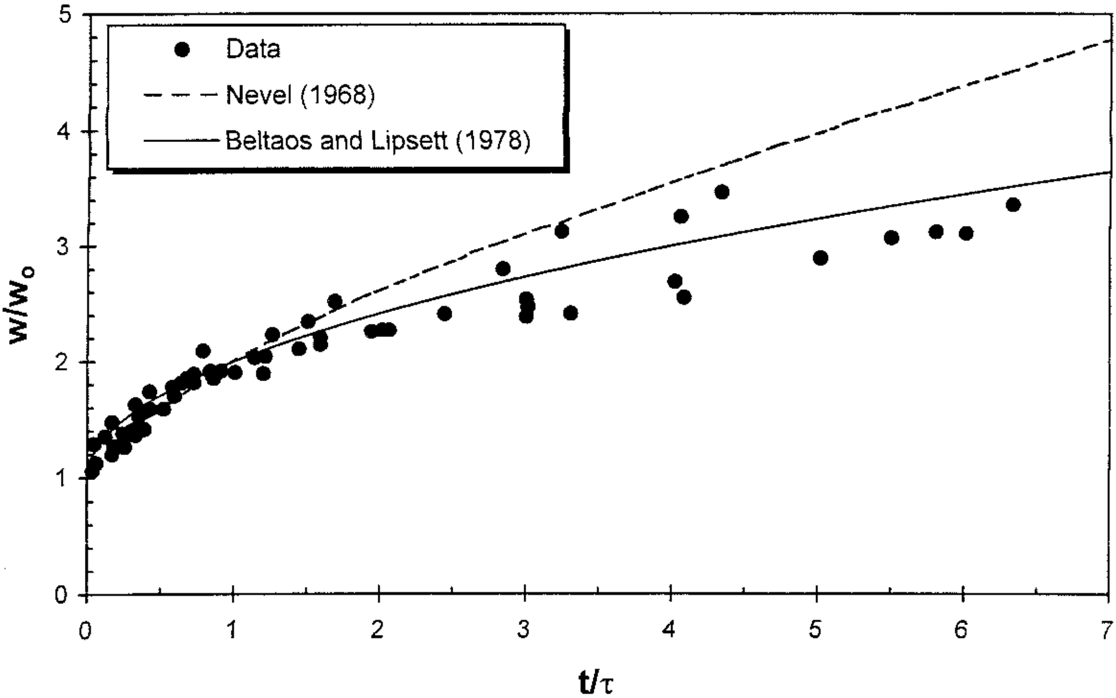

Figure 5. Plastic deflection as a function of time (Beltaos and Lipsett, 1978).

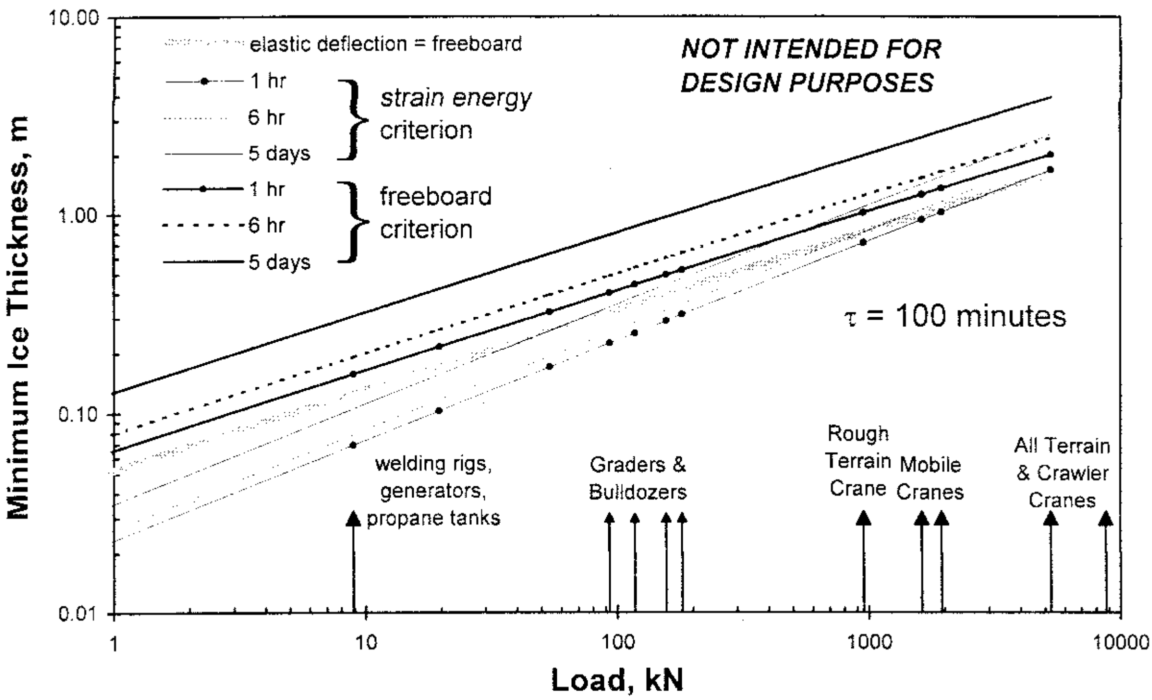

Figure 6. Comparison of strain energy and freeboard criteria for long term loading.