The Effects of Freezing on the Stability of a Juxtaposed Ice Cover

David D. Andres, M.Sc.CE, P. Eng.
Trillium Engineering and Hydrographics Inc.
210-4812 87 Street Edmonton, Alberta, T6E 5W3
dandres@trilliumeng.ab.ca

On all but very slow-moving streams, the initial ice cover is formed by frazil pans and slush accumulating against a solid ice cover formed at locations hydraulically predisposed towards producing lodgement. Once lodgment has occurred, two main stability criteria must be met for a stable accumulation to form and progress upstream: entrainment and internal strength. If floe entrainment does not occur, the cover initially accumulates by the juxtaposition of floes one layer thick and the head of the ice cover advances upstream at a rate determined by the ice supply. After the juxtaposed cover forms, its strength continues to increase due to downward freezing through the frazil-filled voids between the pans.

The forces on the ice cover at any location are from the downstream component of the weight of the cover and the drag on its under surface. Both increase proportionately as the upstream length of the ice cover increases. When these forces exceed the internal strength of the cover, produced by mostly freezing at the surface and to a lesser extent from internal friction, the cover shoves and consolidates. This produces a thicker ice cover, a higher stage, and a slower rate of ice cover advance than would be evident with a juxtaposed cover. A stability number has been derived that differentiates between juxtaposed and consolidated ice covers on the basis of the river characteristics, discharge, and air temperatures. Prototype-scale data from four locations on the Peace River in Alberta were used to verify the index.

1. Introduction

Numerical modelling of ice processes in rivers is becoming common practice. A number of one and two dimensional models are available to simulate the generation of ice, the formation of stable ice covers, the timing of freeze-up, and the duration over which ice covers can exist (Andres and Spitzer, 1989; Shen, Wang, and Lal, 1993; Carson and Groeneveld,1997; for example). These models use both hydraulic and thermal inputs as part of the modelling processes. Determining the stable ice thickness is one of the important modelling issues because freeze-up levels and the rate at which an ice cover develops in a particular reach of interest depend upon the ultimate thickness of the cover. These issues become particularly important in regulated rivers where increased flows for hydropower production (for example) may produce severe ice conditions that lead to high ice-related flood levels; and some means are required to adjust flow releases so that both hydropower and environmental concerns can be optimized.

Current simple approaches to calculating stable ice covers make use of the "narrow" and "wide" channel ice jam concepts (Pariset, Hausser, and Gagnon, 1966). The characteristics of "narrow' channel jams are determined by conditions at the head, whereby the thickness of the cover is determined by whether or not an ice floe can be entrained under the ice cover and transported or deposited on the ice underside. Typically, an ice floe Froude number approach is used to determine if a floe can juxtapose at the head of an ice cover without being entrained. If the criterion is satisfied, a cover with a thickness of about one floe will form.

The "wide" channel ice jam approach requires that the thickness of the ice cover be sufficient to provide enough internal friction to transfer the down-channel gravity and shear forces to the banks. Once an ice cover forms by the juxtaposition of ice floes, it will collapse and thicken if the "wide" channel jam criterion is not met by the thin juxtaposed cover. This produces an ice cover whose thickness reflects the flow at the time of the collapse without the effects of freezing because the formation of a stable cover of this type is a very rapid process.

It is evident that both types of ice covers can form while a stable ice cover is in the process of developing, albeit the juxtaposed ice cover ice often a transient condition. Neither the "narrow"or "wide' channel models provides a mechanistic description of how ice covers form in rivers. Furthermore, neither of these approaches provides for methods to include the effects of freezing on the stability of the juxtaposed ice cover.

This paper examines the role played by freezing on the stability of an ice cover. Freeze-up processes are described briefly for a typical large river that is regulated for hydropower purposes, and as such is subjected to fluctuating flows . Equations are presented for rates of upstream advancement of an ice cover, for the attendant increases in stresses within the ice cover, and for increases in the strength of the cover by freezing in the upper layers. Observations of ice conditions on the Peace River in northern Alberta are used to determine a dimensionless stability number that allows for the differentiation between the formation of juxtaposed and consolidated ice covers on the basis of the channel characteristics, the discharge and the ambient air temperature.

2. Freeze-up Processes

The production of ice in a flowing stream occurs in two modes, border ice growth and frazil production. In most cases border ice is first to appear. Frazil production begins once the entire column of water cools to a supercooled condition, and it is the dominant ice producing mechanism on most streams. The frazil coalesces into buoyant, loose accumulations (slush), rises to the surface, and forms a crust on contact with the cold air. These masses of frazil take on a round pancake-like appearance, with a solid flat surface and a mass of porous frazil slush suspended underneath (Figure 1).

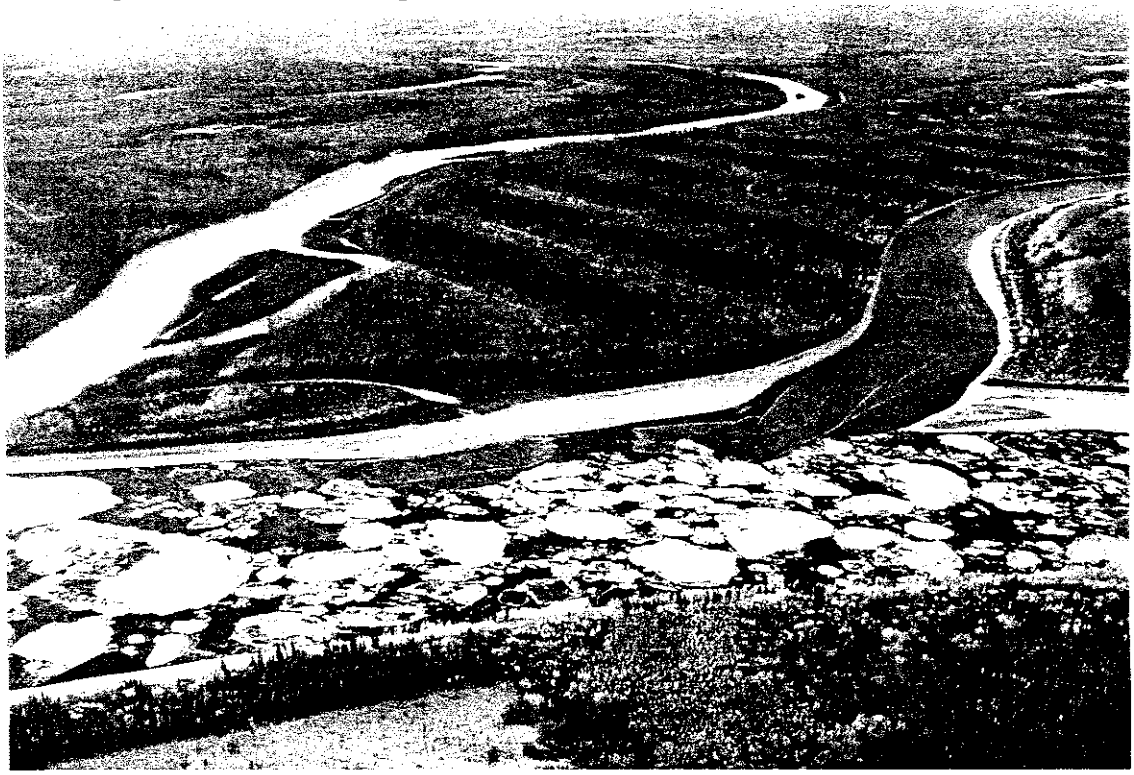

Figure 1 Ice floes and large rafts arriving at the head of an advancing ice cover

On all but very slow-moving streams, the initial ice cover is formed by frazil pans and slush lodging against a solid ice cover formed by surface growth at special low-velocity locations such as at deep pools, the entrance to a lake, or the head of a reservoir. Lodgement or bridging may also occur when frazil pans lodge at a surface contraction in a long narrow reach, or where shorefast ice has grown outward from the bank, usually in a sharp bend. The ice cover then progresses upstream and an "accumulation" type of cover forms, solidifying and thickening by a variety of processes, depending upon the hydraulic and meteorologic conditions.

Once lodgment has occurred, two main stability criteria must be met for a stable accumulation to form and progress upstream: entrainment and internal strength. If a stable lodgement forms and the velocity is low enough that entrainment of floes does not occur, then the cover initially accumulates by the juxtaposition of floes one layer thick. On many large streams the frazil pans are sufficiently mature to form large rafts (Figure 1) of sufficient size and integrity to resist entrainment at the head of the cover (Andres, 1995). The solid crusts of the floes and rafts juxtapose against each other, and advance upstream at a rate determined by the surface concentration of ice arriving at the ice front. The loose frazil attached to the floe may be removed from the crust by the shear of the flow and redistributed downstream under the previously formed cover. As soon as the juxtaposed cover begins to form (Figure 2), its strength begins to increase due to downward freezing through the frazil-filled voids between the pans. The thickness of the frozen layer increases with the length of time the juxtaposed cover remains in place. When added to the internal strength of the thin frazil accumulation, this frozen frazil can significantly increase the strength of the accumulation.

Figure 2 Typical juxtaposed ice cover formed form the accumulation of ice floes

As the ice cover advances upstream, the downstream component of the weight of the cover and the drag on its under surface increase proportionately to its length. These forces are resisted by the frozen cover downstream and to a lesser extend by shear along the bank. When, at any location, these forces exceed the combined internal strength of the cover produced by freezing at the surface, and by internal friction, the cover shoves and consolidates (Figure 3). The thickness increases to develop greater internal friction forces, partially to compensate for the loss of the strength of the frozen layer. The result is a very rough and thick accumulation of frazil and solid floes (Figure 4) that can produce extremely high water levels. Neill and Andres, 1983 documented one example of a particularly severe consolidation event.

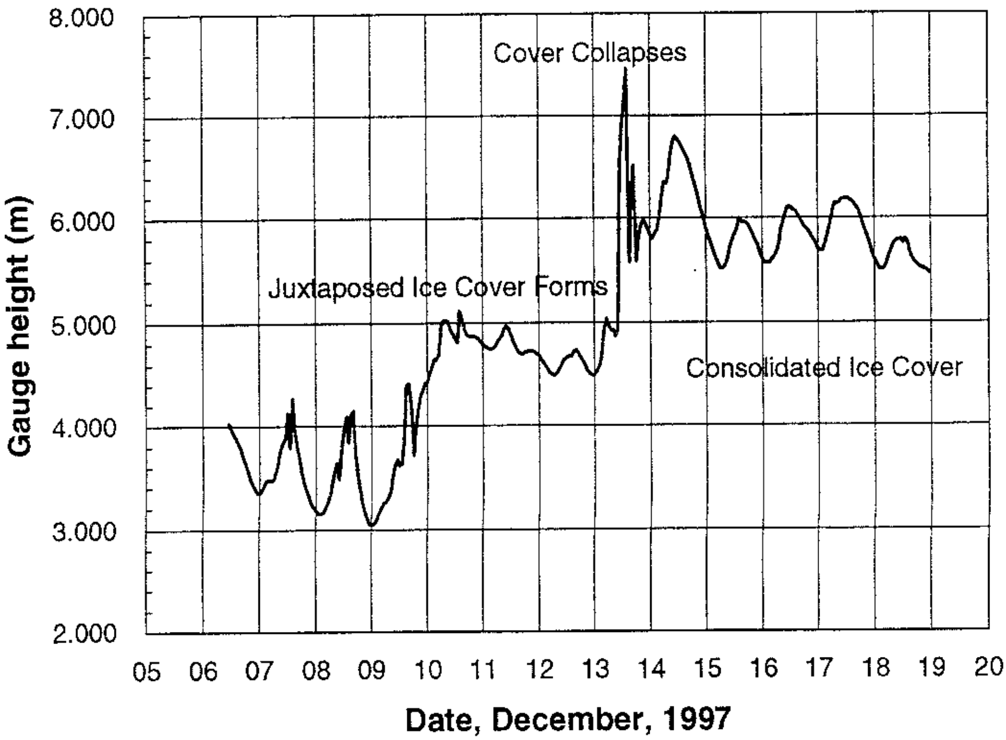

Figure 3 Water level response due to the collapse of a juxtaposed ice cover

Figure 4 Surface characteristics of a consolidated ice cover

3. Stability Considerations and Ice Cover Characteristics

The characteristics of an ice cover in any particular reach of the river depend on (1) the rate at which ice arrives at the ice front (this determines the rate at which the ice cover is progressing upstream and the rate at which the force on the cover is increasing) and (2) the rate at which the strength of the ice cover is increasing due to the growth of the interstitial ice between the juxtaposed floes. The former depends on the air temperature because it affects the generation of the frazil and the subsequent supply of ice arriving at the ice front; and the channel width, slope, and discharge because they determine the forces on the juxtaposed ice cover. The latter depends on the thickness of the juxtaposed ice, because it provides the initial strength of the cover prior to the growth of the thermal ice; and the air temperature, because it determines the rate of growth of the thermal ice.

The rate at which the force on the juxtaposed ice cover, F increases is given by

$$\frac{dF}{dt} = c\,(\rho g W R_i S + \rho_i g W h_j S) \qquad [1]$$

where c is the celerity of advancing ice front, the first term in the brackets is the shear on the ice underside, and the last term in the brackets is the weight of the ice cover. Note that W is the stream width, R_i is the hydraulic radius of the flow associated with the ice cover, h_j is the thickness of the juxtaposed ice cover which is typically one floe thickness or less thick, and S is the slope of the river. Furthermore, the celerity of the advancing ice cover is given by

$$c = \frac{C W h_f (1-p_f) V}{W h_j (1-p_j)} \qquad [2]$$

where C is the concentration of surface ice floes upstream of the head of the cover, V is the mean velocity in the uniform flow upstream of the head of the cover, and h and p are the thickness and porosity of the ice. The subscripts f and j denote the individual floes and the juxtaposed ice cover, respectively. Adopting the friction factor form of the flow equation,

$$V = \frac{Q\,S^{1/3}}{\left(\frac{f_{bo}}{8g}\right)^{1/3}\left(\frac{Q}{W}\right)^{2/3} W} \qquad [3]$$

where Q is the discharge upstream of the ice cover and f_{bo} is the open water bed friction factor.

Prescribing a value for R_i is somewhat more complicated because allowance must be made to account for the reduced flow under the ice cover, Q_o due to flow abstraction into channel storage as the ice cover advances upstream. Never the less

$$Q_o = Q - \Delta Q \qquad [4]$$

and

$$\Delta Q = c \, \Delta H \, W \qquad [5]$$

where

$$\Delta H = H_o + \frac{\rho_i}{\rho} h_j - H_{bo} \qquad [6]$$

in which H_o is the total flow depth under the ice cover and H_{bo} is the open water depth upstream of the ice cover. Again, using the friction factor form of the flow equation, and combining equations [2], [4], [5], and [6] results in

$$Q_o = \mathrm{A} \, Q \qquad [7]$$

where

$$\mathrm{A} = [1 - C[\frac{h_f(1-p_f)}{h_j(1-p_j)}][(\frac{2f_o}{f_{bo}})^{1/3} + (\frac{\rho_i}{\rho})(\frac{h_j}{H_{bo}}) - 1]] \qquad [8]$$

and where f_o is the composite roughness under the ice cover. Writing R_i in terms of the total discharge upstream of the ice cover, inserting its expression into equation [1], and assuming that $h_j < R_i$ produces an equation for the rate at which forces on an advancing ice cover increase. This is given as

$$\frac{dF}{dt} = \rho g \, W S (\frac{f_i}{f_o})(\frac{f_o}{f_{bo}})^{1/3}(\frac{\mathrm{A}}{2})^{2/3}(\frac{Q}{W}) C [\frac{h_f(1-p_f)}{h_j(1-p_j)}] \qquad [9]$$

Formulation of the resisting force that develops due to downward freezing depends upon the failure mode of the juxtaposed ice cover. Both crushing failure and buckling failure are a possibility. The crushing strength varies linearly with the thickness of the solid ice and depends upon the its compressive strength. Buckling strengths depends upon the total thickness of the ice cover to the three halves power and the composite elastic modulus of the ice cover.

It is difficult to prescribe a rigorous formulation of the resisting force because of numerous unknowns such as the local variability of the shear stresses on the cover, the local variability in cover thickness, the micro scale interaction between individual floes within the the juxtaposed cover, and the role played by open between the floes on the overall strength of the juxtaposed cover. Therefore, it is convenient to describe the resisting force as an average compressive strength related to the thickness of the solid ice cover, however it may be manifested. Therefore, the rate at which the resisting force R increases is given by

$$\frac{dR}{dt} = \frac{dh}{dt} W \sigma \qquad \textbf{[10]}$$

where dh/dt is the growth rate of frozen layer at the surface of the ice cover, σ is the strength of the frozen ice layer, and the internal strength due to friction of the thin juxtaposed ice cover is assumed to be much less than the strength of the frozen layer at its surface. The growth rate of the frozen surface layer can be described by the well known ice growth equation (Ashton, 1986) that includes both the thermal resistance of the surface air layer and the frozen ice, but including both terms makes for a non-linear equation. Since the surface air layer provides the most thermal resistance when the frozen layer is thin (Figure 5), it is convenient to linearize the growth rate according to the following equation

$$\frac{dh}{dt} = \frac{H_{ia} T_a}{(1-p_j)\rho_i L} \qquad \textbf{[11]}$$

where H_{ia} is the convective heat transfer coefficient between the air and ice surface, T_a is the air temperature, and L is the latent heat of fusion of water. The equation also provides an allowance for the porosity of the frazil within the juxtaposed ice cover in increasing the growth rate.

Insipient stability of a juxtaposed ice cover occurs when $dR/dt = dF/dt$. Combining equations [9], [10], and [11], rearranging to isolate the discharge, channel characteristics, and temperature on one side, and dividing both sides by k_i to make the equation dimensionless results in a dimensionless stability number that differentiates between the tendency to form juxtaposed or consolidated ice covers in rivers during freeze-up.

$$\frac{T_a W k_i}{Q S \rho_i L} = \frac{\rho(1-p_f) g k_i}{\sigma H_{ia}} \left(\frac{f_i}{f_o}\right) \left(\frac{f_o}{f_{bo}}\right)^{1/3} \left(\frac{A}{2}\right)^{2/3} C \left[\frac{h_f(1-p_f)}{h_j(1-p_j)}\right] \qquad [12]$$

The left side of the equation represents the effects of the discharge in terms of its contribution to the rate at which ice arrives at the ice front and the shear force under the juxtaposed ice cover, the air temperature as it relates to the growth of the solid ice within the juxtaposed cover, and the slope and width of the channel which also affect the shear stress. The right side length of the equation represents the strength of the newly formed ice between the ice floes and a number of constants that reflect the roughness of the stream and the underside of the ice, the ice floe characteristics, and the growth mechanism of the solid ice. For typical values of friction factors, heat transfer coefficients, ice strengths, channel storage losses, etc., *it could be possible for the stability number to range between 0.00002 and 0.001. Its actual* value would need to be confirmed from actual observations of stable juxtaposed ice covers and the attendant hydraulic characteristics.

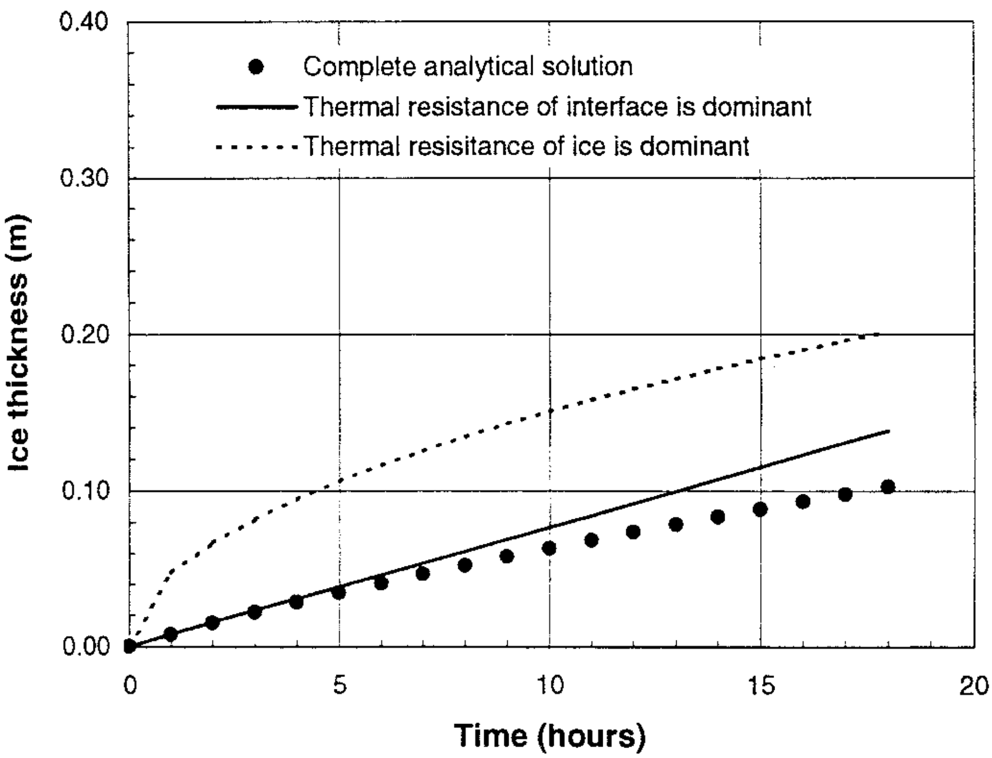

Figure 5 Effect of various solution techniques on the calculated growth rates of a thin ice sheet, T_a = -20 ^{0}C, H_{ia} = 15 W/m^2-^{0}C

4. Determination of Critical Stability Number

The critical stability number was determined from measured and observed freeze-up levels on the 1200 km long Peace River in northern Alberta (Figure 6). The river drains the eastern slopes of the Rocky Mountains, crosses the Alberta-British Columbia border at Clayhurst, and at Peace River town the course of the river changes to a northerly direction before the river enters the Slave River downstream of Peace Point. The river has been regulated since 1972 by the Bennett Dam. This has substantially increased the discharge during the winter period. For example at Peace River, the natural winter flows were in the order of 200 to 500 m^3/s. Since regulation, the winter flows have been in the range of 1500 to 2000 m^3/s.

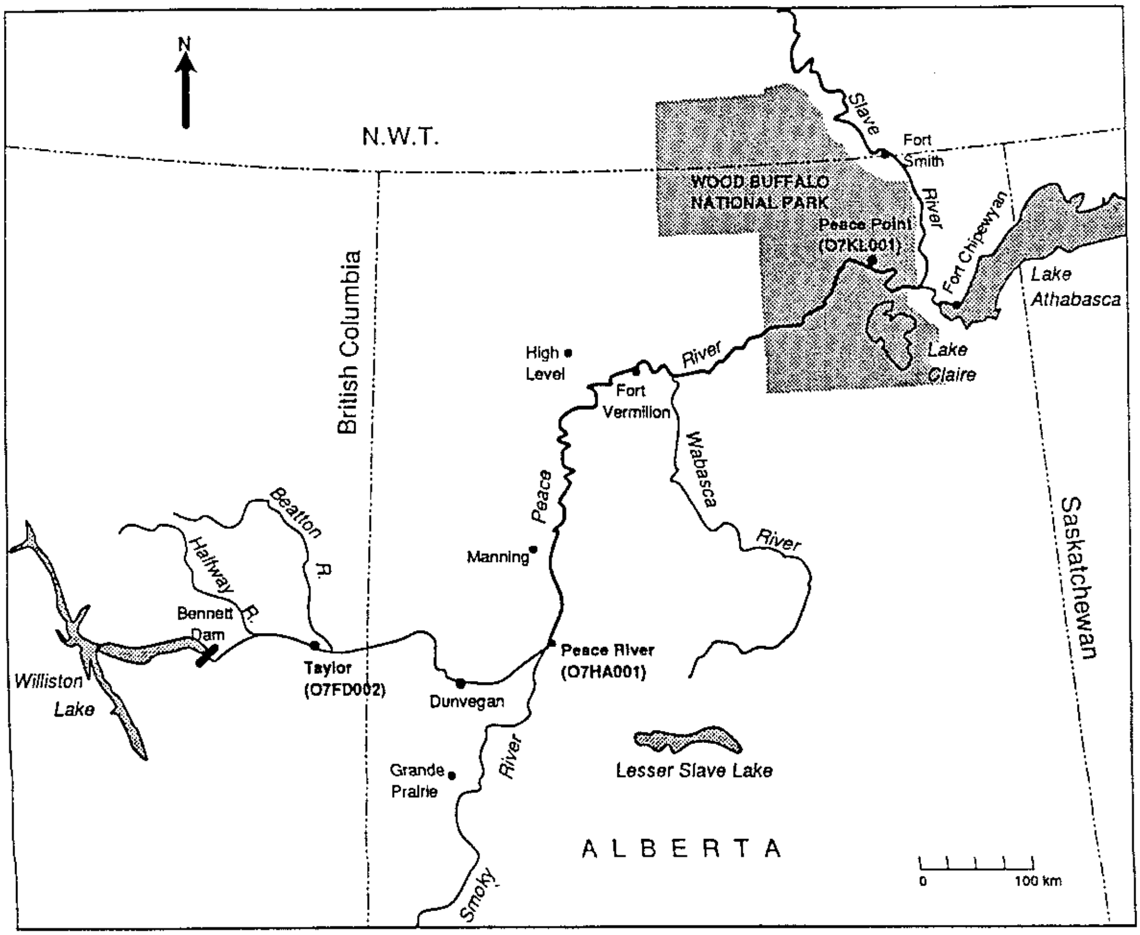

Figure 6 Study area, Peace River in northern Alberta, Canada

Water Survey of Canada (WSC) has operated a number gauges through the study area since at least 1958. Three gauges, currently located at Taylor, Peace River, and Peace Point, provide site-specific information on water temperatures, discharges, freeze-up dates, and freeze-up stages. Hydraulic and geomorphic data from Kellerhals, Neill, and Bray (1972) describe the hydraulic characteristics of the salient reaches between Taylor and the Slave River. Since the late 1970's, Alberta Environmental Protection (AEP) has observed the progression of the ice cover over the entire winter to assist in the mitigation of high water levels caused during freeze-up and breakup at Peace River town (Fonstad and Garner, 1984).

This river is a convenient source of data to verify the stability number because the river undergoes numerous changes in character along its course. Upstream near Taylor, its typical slope is about 0.0004 to 0.0005, with a width of about 400 m. In its middle reaches, near Peace River town, the slope is about 0.0003 with a width of 420 m. In the lower reaches the width approaches 600 m and the slope is as low as 0.00006. Thus, as an ice cover forms on the river, there is an opportunity for a variety of ice cover types to form under more or less similar discharges but under vastly different hydraulic conditions.

Post-regulation data from the three gauging sites and from other ice surveys for which the relevant parameters could be determined were used in the calibration. Figure 7 illustrates a plot of the stability parameter as a function of the stage increase at freeze-up. Each data point was characterized as *either being a consolidated cover, a juxtaposed cover, or a* transitional cover on the basis of either direct observation of the ice cover, the measured ice thickness, or the stage increase at freeze-up. The very thick covers and/or large stage increases at freeze-up were classified as consolidated ice covers, the thin covers and/or low stage increases at freeze-up were classified as juxtaposed ice covers, and those for which the cover characteristics were not obvious were termed transitional.

From the plot it is evident that critical value of the stability number is about 0.0003. Below this number, the ice cover is predominantly thick, suggesting consolidation, and above this number the ice cover is predominantly thin, suggesting juxtaposition. There is some scatter in the data which can be attributed to local conditions at the measurement site not being representative of the ice characteristics of the reach, and perhaps due to inaccurate representations of the floe characteristics. For typical values of floe porosity, storage losses, and channel and ice roughness, the corresponding effective compressive strength of the solid ice that contributes to the stability of a juxtaposed cover on the Peace River is about 1200 kPa.

5. Conclusions

It is evident that both hydraulic conditions and ambient temperatures have a dramatic effect on the characteristics of a stable ice cover. In typical large scale rivers that are not too steep, low temperatures will tend to produce juxtaposed ice covers, while higher temperatures tend to facilitate the formation of consolidated ice covers. In very mild sloped rivers, juxtaposed ice covers will form regardless of the temperature, while on steep rivers, it can be guaranteed that only consolidated ice covers will form. The stability number defined herein provides a mechanism to estimate the likelihood of what type of ice cover will form for any given hydrometeorological condition.

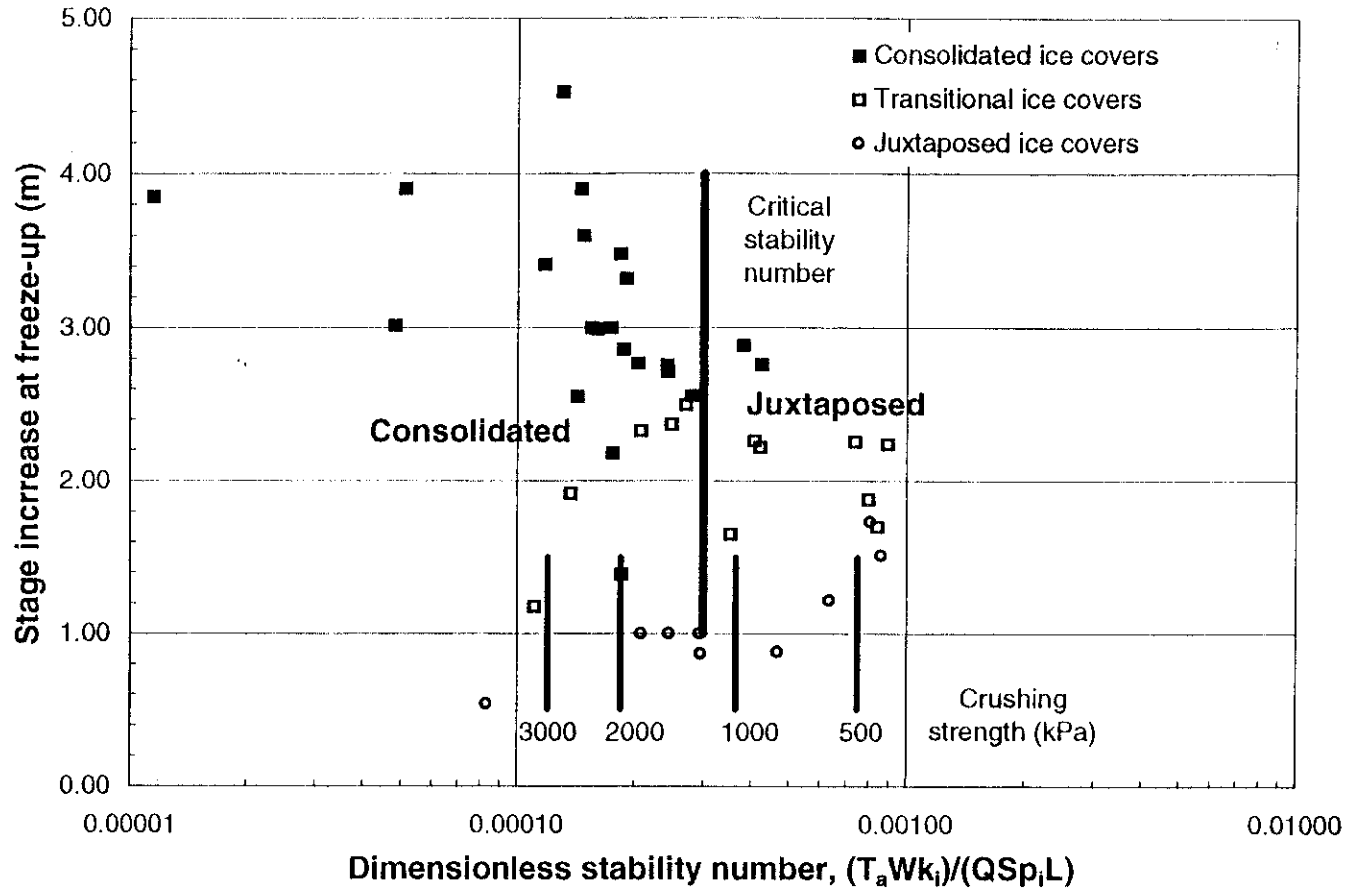

Figure 7 Differentiation between juxtaposed and consolidated ice cover on basis of dimensionless stability number

More work is required, however, to investigate further the failure mechanisms of ice covers, particularly how to define the buckling strength of juxtaposed ice covers. Also, there is a need to examine the role played by channel planform characteristics on the transfer of longitudinal forces to the channel banks. A sinuous channel probably can form juxtaposed ice covers more readily than a straight channel because the length of the cover that produces stresses at any location may be limited by the meander length of the channel.

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