THE EFFECTS OF UNCERTAINTY IN ICE ROUGHNESS ON EQUILIBRIUM ICE THICKNESS AND STAGE

Kathleen D. White[1] and Steven F. Daly[2]

Abstract

The U.S. Army Corps of Engineers requires the use of risk and uncertainty methods in the analysis and design of hydraulic and flood control structures. When the uncertainties are quantified, the risk associated with different levels of protection can be quantified as well. At the present time, these methods address the uncertainties encountered in developing discharge-probability functions and stage-discharge functions for gaged and ungaged watersheds in open-water conditions. There are no established methods to perform similar analyses in ice-affected rivers. The additional sources of error include ice roughness, ice thickness, and ice properties such as porosity and cohesion. In addition, discharge measurement errors present in open-water cases are compounded by the measurement errors caused by the presence of ice, such as frozen recorders and ice-affected stages leading to overly high discharge estimates. This paper addresses the additional complexities introduced when risk and uncertainty analyses are attempted for ice-covered conditions. In particular, the effects of uncertainty in ice roughness on the calculated equilibrium ice jam thickness and stage are explored.

Résumé

Le Corps du Génie de l'armée americaine exige l'emploi des méthodes de risque et d'incertitude au cours de l'analyse et du projet de structures hydrauliques de contrôle d'inondation. Une fois que les incertitudes sont quantifiées, les risques associés à différents niveaux de protection peuvent aussi être quantifiés. Jusqu'á présent ces méthodes n'addressent que les incertitudes dans les fonctions de probabilité du débit et dans les relations entre débit et niveau en eau libre. Aucune méthode n'existe pour des analyses semblables dans le cas d'un couvert de glace. Les sources supplementaires d'erreur sont la rugosité de la glace, son épaisseur, et autres characteristiques telles que porosité et cohésion. De plus, les erreurs sur la mesure du débit en eau libre sont augmentées par la présence du couvert de glace, par example jauge gelées, ou effet du couvert sur le niveau d'eau qui conduit à une surestimation du dèbit. Dans cette communication, nous presentons les complexités supplémentaires dans l'application des analyses de risque et d' incertitudes au cas de cours d'eau avec couvert de glace. Nous examinons en particulier les effets de l'incertitude dans la rugosité de la glace sur l'épaisseur et dèbit d'un embâcle de glace en èquilibre.

---

[1] Research Hydraulic Engineer, U.S. Army Cold Regions Research and Engineering Laboratory, 72 Lyme Road, Hanover, NH 03755-1290

[2] Research Hydraulic Engineer, U.S. Army Cold Regions Research and Engineering Laboratory, 72 Lyme Road, Hanover, NH 03755-1290

INTRODUCTION

Ice jams in northern rivers can result in stages that are both higher and more rapidly attained than open-water flood events. Predicting the stages associated with ice jams under different ice and hydraulic conditions is important in developing effective and efficient ice jam mitigation measures. But because of limitations in our knowledge of the complex physical processes involved in ice jam formation and progression, a number of uncertainties are incorporated into estimates of ice jam stage. Uncertainty in hydrology is also a function of the complexity and variability of the natural systems (Kundzewicz 1995). The uncertainties may take the form of data uncertainties (e.g., measurement error), sampling uncertainties (e.g. missing data or small sample size), and model uncertainties (Plate and Duckstein 1987).

Traditional methods of analyzing flood damage measures have implicitly recognized the existence of these uncertainties through the use of freeboard or other factors of safety. With the advent of powerful computers capable of rapid, complex computations, it is now possible to quantify the uncertainties inherent in estimating stages. When the uncertainties in determining stages are quantified, the risk associated with different levels of protection can be quantified as well.

The U.S. Army Corps of Engineers (USACE) now requires that risk and uncertainty principles be incorporated into the analysis and design of hydraulic and flood control structures, using methods developed by the Hydrologic Engineering Center (USACE 1996). These methods address the uncertainties encountered in developing discharge-probability functions and stage-discharge functions for gaged and ungaged watersheds in open-water conditions. At the present time, there are no established methods to perform similar analyses in ice-affected rivers.

The additional sources of error introduced by the presence of ice may be broadly categorized as data uncertainties, sampling uncertainties, and model uncertainties, which includes parameter uncertainties. Data uncertainties affecting discharge measurement in open-water cases (Pelletier 1988) are exacerbated by measurement errors caused by the presence of ice, such as frozen recorders and ice-affected stages (Pelletier 1989). Sampling uncertainties are common in ice jam analyses because of the small sample sizes and questionable reliability of ice data and (see, e.g., Gerard and Karpuk 1979, USACE 1991).

Model uncertainties, defined by USACE (1996) as the lack of complete knowledge regarding the form of a hydrologic or hydraulic function due to incomplete scientific or technical understanding of the physical process, add further uncertainty depending on the particular method used to estimate ice-affected discharges from reported stages and on parameter uncertainties. Perhaps the most important source of model uncertainty is the determination of ice-affected discharge, upon which most ice-induced stage estimates rely. Model uncertainty can also be introduced through the method selected to estimating stages, which can range from empirical relationships to complex numerical models that include ice jam effects under unsteady flow conditions (Daly et al. 1995). Beltaos (1995) provides an overview of ice jam theory and numerical models used to estimate jam stage.

Parameter uncertainty arises from our limited knowledge of the physical relationships between the parameters and between each parameter and the outcome, and from errors in measurement or estimation of parameters. In addition, there is often limited information regarding the range and distribution of the functional parameters under different conditions. The selection of a model determines which parameters are considered. Among the parameters important in determining ice jam thickness and stage are discharge, ice roughness, ice piece size, and ice properties such as porosity and cohesion.

This paper addresses the effects of parameter uncertainty in estimating equilibrium ice thickness for breakup ice jams. In particular, the effects of uncertainties in the estimate of ice roughness on the calculated equilibrium ice jam thickness are explored. Values of Manning's "n" (a parameter often used to describe ice roughness) adopted in calibrated numerical models of several breakup ice jams are analyzed to determine the range and distribution of this parameter. This information is utilized in a numerical simulation to determine the range and distribution of equilibrium ice thickness and resulting stages in a rectangular channel with fixed slope and discharge.

UNCERTAINTY IN EQUILIBRIUM ICE JAMS

A number of expressions are available for calculating the equilibrium ice jam thickness in breakup ice jams, that is, the uniform section of an ice accumulation that has formed through shoving of the ice (Pariset et al 1966, Uzuner and Kennedy 1976, Beltaos 1978, 1995). The formulation selected for use in this study is that utilized by Wuebben et al (in press) in the ICETHK option of the USACE step-backwater computer program HEC-2 (USACE 1982):

$$\frac{d\eta}{dx} = \mu\left(1 - \frac{\rho_i}{\rho}\right)\rho_i g \eta^2 - \left(g\rho_i S_f B - 2C_i\right)\eta - \tau \qquad (1)$$

where $d\eta/dx$ is the change in the thickness of the ice accumulation with distance (x); η is ice thickness; μ is the coefficient of ice properties, $\mu = k_0 k_1 k_x (1-e)$; k_0 is the coefficient of friction between ice and bank = tan ϕ; ϕ is angle of internal friction; k_1 is the lateral stress coefficient, approximately 0.33; k_x is the passive pressure coefficient $= \tan^2\left[45° + \frac{\phi}{2}\right]$; e is the porosity of the equilibrium section; ρ_i is the density of ice; ρ is the density of water; g is gravity; S_f is the water slope; B is the river width; C_i is the cohesion, generally assumed to be 0 for breakup ice jams formed from unconsolidated rubble; τ is the friction on the underside of the ice cover $=\rho g R_i S_f$; R_i is the hydraulic radius $\approx \left(\frac{n_i}{n_c}\right)^{3/2} \frac{y_i}{2}$ for a wide channel jam; n_i is the Manning's "n" for underside of ice; n_c is the composite Manning's "n" $= \left[\frac{n_i^{3/2} + n_b^{3/2}}{2}\right]^{2/3}$; n_b is the Manning's "n" for the bed; yi is the under-ice depth $= \left[\frac{q n_c}{0.6299 S_f^{1/2}}\right]^{3/5}$; and q is the discharge per unit width.

Setting $d\eta/dx=0$ and solving for η, the following equation is obtained:

$$\eta = \frac{BS_f}{2\mu(1-s_i)}\left[1+\sqrt{1+\frac{4\mu(1-s_i)R_i}{s_i BS_f}}\right] \tag{2}$$

where s_i = specific gravity of ice = ρ_i/ρ. The water surface elevation in the equilibrium section of a breakup ice jam, h, is calculated by:

$$h = y_i + s_i\eta \tag{3}$$

From equations 1, 2, and 3, it is apparent that uncertainty can be introduced through the following variables: ϕ, e, ρ_i, ρ, S_f, B, n_i, n_b, , and q. Assuming that the density of water can be considered constant during the formation of the equilibrium portion of a breakup jam (i.e., temperature effects are negligible), and that the density and internal friction angle of the ice are also constant during this period, uncertainty can then be attributed to variability in B, n_b, q, S_f, e, and n_i. For the purpose of this study, we will assume a rectangular channel of width 100 m with n_b of 0.030, discharge 100 m^3s^{-1}, water slope 0.001 m/m, and equilibrium jam section porosity of 50%. With these assumptions in place, uncertainties in computed equilibrium ice jam thickness and stage will be directly related to uncertainties in ice roughness. Equation 2 suggests that there will be less uncertainty in ice thickness than the uncertainty introduced by ice roughness, while equation 3 suggests that stage will have larger uncertainty than ice thickness.

Uncertainty in ice roughness for breakup ice jams

There are various definitions of uncertainty, ranging from general (e.g. "not certain") to quite specific. Pelletier (1988), for example, defines uncertainty as "the range within which the true value of a measured quantity can be expected to lie." This definition is particularly useful in ice analyses because of the inherently unsteady nature of the hydrologic, meteorological, hydraulic processes involved. Each observation is subject to spatial and temporal variation such that the observed value can be considered to be representative of the true value, but not necessarily an exact representation of the true value. A probability distribution function can be derived to describe the distribution of the observed values. The true value (or the expected value in statistical terms), is the mean value of the observed values, and the uncertainty about the mean can be quantified using the distribution of the observed values.

The definition given by Pelletier (1988) does not involve distributional assumptions and therefore one may proceed in the determination of uncertainty via parametric or nonparametric methods as appropriate. The approach used in the present study is to first determine the distribution of the available data and then simulate a large sample of ice roughness based on the distribution of the existing data. Confidence limits, which define the uncertainty in each population, can be developed from the results of the simulation.

Determining the distribution of the ice roughness of the underside of breakup ice jams is the first step in the process of examining its effects on the uncertainty of equilibrium ice thickness. While there have been some reports of the ice roughness associated with ice covers (Carey 1967) and freezeup jams (Nezhikhovskiy 1964), ice roughness estimates for breakup ice jams are rare. A search of calibrated breakup jam models of the Winooski River at Montpelier, VT (Tuthill et al. 1996), Missouri River at Pierre, SD (Daly et al. 1995), and Aroostook River at Fort Fairfield, ME (White and Acone 1996) revealed 46 ice roughness values (Manning's "n"). The "n" values (Figure 1) ranged from very smooth (0.020) to the very rough (0.150), with a mean of 0.066 and standard deviation of 0.023. The smooth under-ice roughness value could arise from a situation in which thin ice plates form the equilibrium jam in a relatively deep channel. The rough under-ice roughness could arise when an ice jam composed of large or thick pieces of ice forms in a relatively shallow channel. This occurrence might violate some assumptions in equations 1 through 3 above, but is quite likely to be encountered in hydraulic modeling of ice jams.

The breakup jam "n" value sample shown in Figure 1 appears somewhat skewed and there is a high (but not extreme) outlier, but the Shapiro-Wilk test for normality indicates that the sample distribution is close to normal (p=0.03). If the sample were decidedly non-normal, a nonparametric method should be used to simulate a large number of samples. However, in this case, the distribution is close enough to statistically normal that a normal distribution can be used to simulate a large sample. Although the high outlier shown in Figure 1 ("n" = 0.150) appears to be extreme, it is not classified statistically as an extreme outlier. In addition, the fact that it has been used in a calibrated numerical model implies that such "n" values are likely to be encountered in other analyses. Therefore, its removal from further analysis is not justified. In fact, the authors believe that extreme outliers can be valuable in the analysis of ice jam events, which themselves can often be considered extreme events in terms of stage.

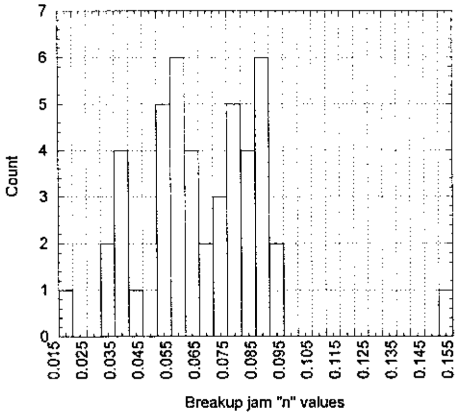

Figure 1. Distribution of Manning's "n" Values For The Underside of The Ice From Calibrated Numerical Models of Breakup Ice Jams at Three Sites.

The simulated sample of size 10,000, based on a mean of 0.066 and standard deviation of 0.023, is shown in Figure 2. The 95% confidence interval is (0.029, 0.105). This means that, using Pelletier's (1988) definition of uncertainty as "the range within which the true value of a measured quantity can be expected to lie", one can be 95 % confident that the Manning's "n" values for breakup jams will lie between 0.029 and 0.105, based on the data shown in Figure 1. This range varies from very smooth (0.029) to quite rough (0.105), and is not particularly helpful in advancing current knowledge of the subject beyond confirming the highly variable nature of breakup jam ice roughness.

Uncertainty in equilibrium ice thickness and stage based on uncertainties in ice roughness for breakup ice jams

The simulated Manning's "n" values shown below were used to compute equilibrium ice thickness using eq 2 for a rectangular channel of width 100 m, with water slope 0.001 m/m, ϕ = 45°, $s_i = 0.916$, $n_b = 0.030$, discharge = 100 m^3s^{-1}, and equilibrium jam section porosity of 50%. The resulting ice thickness values, shown in Figure 3, range from 1.26 m to 2.56 m, with a mean of 2.09 m and standard deviation of 0.19 m. The nonlinearity of eq 2 predicts departure from normal distribution, and indeed, the distribution is non-normal, exhibiting some skew and several extreme low outliers. The 95% confidence interval is (1.74, 2.36). As suggested by eq. 2, the uncertainty in the calculated ice thickness is less than that for ice roughness.

Equilibrium ice jam stage was calculated from the simulated ice thickness values using eq 3. The resulting stages (Figure 4), range from 2.12 m to 5.00 m, with a mean of 3.65 m and standard deviation of 0.44 m. The non-normal stage distribution is slightly skewed but has no extreme outliers. The 95% confidence interval is (2.86, 4.33), indicating more uncertainty than for ice thickness, but less than for ice roughness.

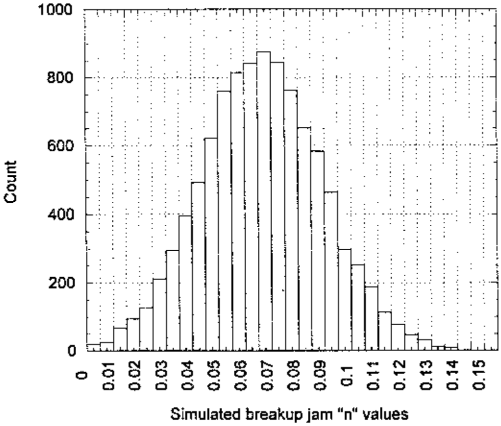

Figure 2. Distribution of Manning's "n" Values Simulated From Data Depicted in Figure 1.

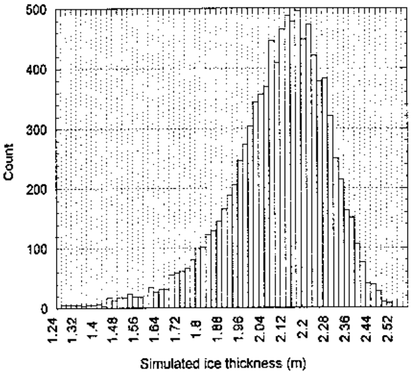

Figure 3. Equilibrium Ice Thickness Based on Simulated Manning's "n" Values.

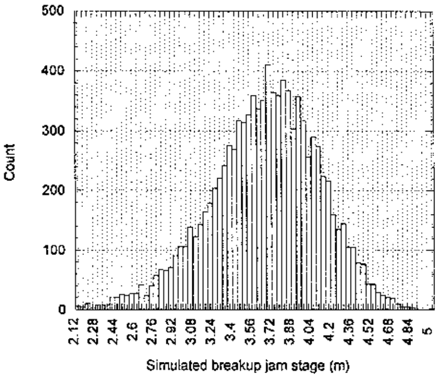

Figure 4. Equilibrium Ice Jam Stage Values Based on Simulated Ice Thickness.

Using Pelletier's (1988) definition of uncertainty, if an "n" value of 0.066 were selected under the given conditions, the equilibrium ice thickness is 95% likely to fall between 1.74 and 2.36 m with an expected value of 2.09 m. It appears that, under the given conditions, ice thickness is relatively insensitive to differences in ice roughness, with a variation of only 0.62 m between the 95% confidence limits corresponding very smooth and very rough ice covers. In practical terms, the simulations indicate that errors in estimating under-ice roughness would be expected to have a relatively small effect on computed equilibrium ice thickness, but a larger effect on equilibrium stage.

In order to evaluate the effects of differences in the central tendency and variation of ice roughness on equilibrium ice jam thickness and stage, the simulations were repeated for two cases: holding the standard deviation constant and varying the mean ice roughness (Table 1), and holding the mean ice roughness steady while varying the standard deviation (Table 2). In both cases, the simulated ice roughness distributions are normal when the coefficient of variation (ratio of the standard deviation to mean) is less than about 0.40; otherwise, the roughness distributions are no longer normal. The ice thickness and stage distributions are non-normal in all cases.

The mean equilibrium ice thickness increased 0.39 m (about 22%) as the mean roughness varied from 0.030 to 0.075 (150%) (Figure 5). The mean equilibrium ice jam stage showed a somewhat stronger correlation with roughness, increasing from 2.97 to3.80 m (about 31%) (Figure 6). Changing the standard deviation of the underlying distribution of simulated ice roughness from 0.018 to 0.028 (55%) had little effect.

The 95% confidence intervals for equilibrium ice thickness and stage do tend to shift upwards as the mean ice roughness increases, but there is substantial overlap considering the magnitude of the change in ice roughness (Figures 5 and 6). In fact, the mean ice thickness and stage calculated with an under-ice Manning's "n" of 0.030 fall within the confidence interval of those calculated for a Manning's "n" 0.066. The implication is that if one were to try to estimate ice roughness from a given stage or jam thickness observation, the estimate could range from quite smooth to moderately rough. For example, an ice thickness of 2 m results when using Manning's "n" values within the confidence limits for mean ice roughness ranging from 0.030 to 0.075. These results also suggest that small over- or under-estimations of mean ice roughness will have small effects on calculated mean equilibrium ice jam stage. For example, if one underestimated mean ice roughness to be 0.040 rather than 0.045, the corresponding mean equilibrium ice thickness and stage would be underestimated by about 0.05 m and 0.08 m, respectively.

TABLE 1. Effect of Variations in Mean of n_i on Equilibrium Ice Thickness and Stage.

n_i			**ice thickness (m)**			**stage (m)**		
mean	**std. dev.**	**distribution**	**mean**	**std. dev.**	**95% CI**	**mean**	**std. dev.**	**95% CI**
0.030	0.023	non-normal	1.77	0.25	1.33, 2.14	2.97	0.46	2.21, 3.74
0.045	0.023	non-normal	1.91	0.23	1.45, 2.24	3.23	0.47	2.38, 3.98
0.066	0.023	normal	2.09	0.19	1.74, 2.36	3.65	0.44	2.86, 4.33
0.075	0.023	normal	2.16	0.17	1.84, 2.40	3.80	0.43	3.04, 4.45

TABLE 2. Effect of Variations in Standard Deviation of n_i on Ice Thickness and Stage.

n_i			**ice thickness (m)**			**stage (m)**		
mean	**std. dev.**	**distribution**	**mean**	**std. dev.**	**95% CI**	**mean**	**std. dev.**	**95% CI**
0.066	0.018	normal	2.10	0.14	1.84, 2.31	3.65	0.35	3.05, 4.18
0.066	0.021	normal	2.09	0.17	1.78, 2.33	3.64	0.40	2.93, 4.26
0.066	0.023	normal	2.09	0.19	1.74, 2.36	3.65	0.44	2.86, 4.33
0.066	0.025	normal	2.08	0.21	1.68, 2.37	3.63	0.48	2.76, 4.37
0.066	0.028	non-normal	2.08	0.23	1.63, 2.39	3.63	0.53	2.67, 4.45

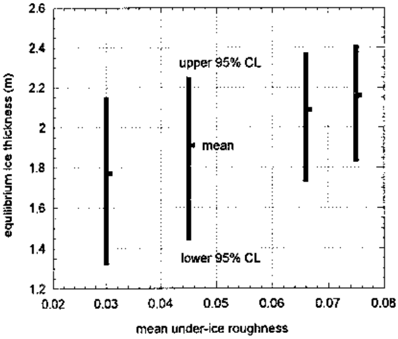

Figure 5. Effect of Change in Mean Ice Roughness on Equilibrium Ice Thickness.

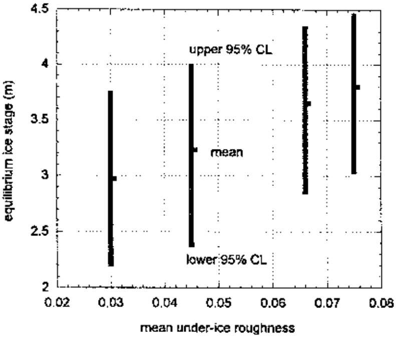

Figure 6. Effect of Change in Mean Ice Roughness on Equilibrium Ice Stage.

CONCLUSIONS

There are various sources of parameter uncertainty in the estimation of equilibrium ice thickness. This paper addresses the effects uncertainties in the estimate of ice roughness on the calculated equilibrium ice jam thickness. Values of Manning's "n" adopted in calibrated numerical models of several breakup ice jams ranged from very smooth (0.020) to the very rough (0.150), with a mean of 0.066 and standard deviation of 0.023 (n=46). The distribution was found to be nearly normal. The normal distribution was used to simulate a large sample (10,000) of ice roughness values, from which equilibrium ice jam thickness and stage were calculated for a rectangular channel. The 95% confidence interval for the simulated ice roughness ranges from very smooth to very rough, (i.e., there is a great deal of uncertainty in the estimation of under-ice roughness).

The equilibrium ice jam thickness and stage calculated from the simulated ice roughness values were no normally distributed. As suggested by the equilibrium ice thickness equation, the large uncertainty in ice roughness was reduced in the calculated equilibrium ice thickness. Equilibrium ice stage has a larger uncertainty than ice thickness, but less than ice roughness. Increases in the mean value of ice roughness resulted in larger increases in stage than ice thickness. Increases in the variation of ice roughness were shown to have little effect on thickness or stage.

The results suggest that small errors in estimation of mean ice roughness will have small effects on mean calculated equilibrium ice thickness and stage. Future work will examine the parameter uncertainties introduced by other variables (ϕ, e, ρ_i, ρ, S_f, B, n_i, n_b, , and q), both singly and as a group.

REFERENCES

Beltaos, S., ed. (1995) River Ice Jams. Water Resources Publications, LLC, Littleton, CO. 372 pages.

Carey K.L. (1966) "Observed Configuration and Computed Roughness of the Underside of River Ice, St. Croix River, Wisconsin." U.S. Geological Survey Professional Paper 550-B, Pp. B192-B198.

Daly, S.F, A.M. Tuthill, and W.R. McGilvary (1995) "Ice-Affected Flooding Oahe Dam to Lake Sharpe Reconnaissance Study." Contract Report to U.S. Army Corps of Engineers, Omaha District. USA Cold Regions Research and Engineering Laboratory, Hanover, NH. 63 p.

Gerard, R.L. and E.W. Karpuk (1979) "Probability Analysis of Historical Flood Data." J. Hydraulics Division, Vol. 105, No. HY9, pp. 1153-1165.

Kundzewicz, Z.W. (1995) "Hydrological Uncertainty in Perspective." Proceedings of the International Workshop on New Uncertainty Concepts in Hydrology and Water Resources. Madralin, Poland. pp. 3-10.

Nezhikhovskiy, R.A. (1964) "Coefficient of Roughness of Bottom Surface of Slush Ice Cover." Soviet Hydrology, Selected Papers, No. 2, pp.127-150.

Pelletier, P.M. (1988) "Uncertainties in the Single Determination of River Discharge: A Literature Review." Canadian J. Civil Engineering, Vol. 15, pp. 834-850.

Pelletier, P.M. (1989) "Uncertainties in Streamflow Measurement Under Winter Ice Conditions A Case Study: The Red River at Emerson, Manitoba, Canada." Water Resources Research, Vol. 25, No. 8, pp.1857-1867.

Plate, E.J. and Duckstein, L. (1987) "Reliability in Hydraulic Design." IN Engineering Reliability and Risk in Water Resources. (E.J. Plate and L. Duckstein, eds.), NATO ASI Seies, Series E: Applied Sciences, No. 124. Nijhoff, Dordrecht.

Tuthill, A.M., J.L. Wuebben, S.F. Daly, and K.D. White (1996) "Probability Distributions For Peak Stages on Rivers Affected by Ice Jams." J. Cold Regions Engineering, Vol. 10, No. 1, pp. 36-57.

U.S. Army Corps of Engineers (1991) "Ice-Influenced Flood Stage Frequency Analysis." Engineer Technical Letter No. 1110-2-325, Department of the Army, Washington, DC. 16 pages.

U.S. Army Corps of Engineers (1996) "Risk-Based Analysis for Flood Damage Reduction Studies." Engineer Manual 1110-2-1619, Hydrologic Engineering Center, Davis CA. 62 pages

White, K.D. and S.E. Acone (in press) "Assessing the effects of alternative project operation on upstream ice conditions: Aroostook River at Fort Fairfield, Maine." IN Winter Environments of Regulated Rivers, proceedings of the 8th Workshop on the Hydraulics of Ice Covered Rivers: [Aug 16-18, 1995, Kamloops, BC].

Wuebben, J.L., A.M. Tuthill, and J.J. Gagnon (in press) "ICETHK Users Manual Version 1." USA Cold Regions Research and Engineering Laboratory Special Report. Hanover, NH.