THE EFFECTS OF HYDROPOWER PEAKING OPERATIONS ON THE THICKNESS OF ICE ACCUMULATIONS

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ABSTRACT

Hydropower operations alter the natural levels of discharge in a river. In general, the effect of a hydropower dam is to average the flow, cutting off the very high and very low periods of discharge which may result in flooding or drought conditions on a seasonal scale. Peaking operations, however, may reverse this trend, resulting in flows that are much higher or lower than the natural daily flow levels for that time of the year. During winter, natural discharge levels are low, and peaking operations may result in periods of abnormally high and low discharge in rivers under ice-covered conditions. These large variations in discharge may cause ice movement or grounding over the course of the cycling period. Therefore, the range of cycling is often limited during ice formation and breakup periods when the ice cover is most likely to move. Restrictions are often based on the peak discharge in the cycle and the water levels expected downstream. Due to the additional resistance offered by an ice cover, the attenuation of the peaking wave as it travels downstream can be much greater than for open-water conditions. This paper examines the effects of this attenuation on the peak discharge, water levels, and ice thickness experienced downstream of the hydropower facility.

INTRODUCTION

Hydropower peaking operations are used to provide energy when needed most, typically on a diurnal cycle of morning and evening, when electrical demands for cooking and heating are the highest. Peaking allows the most efficient use of the often limited flow volumes available during the winter months. Peaking generally refers to relatively large fluctuations of discharge at facilities with adequate storage. Hydropower cycling refers to smaller fluctuations of the average daily flow at run-of-river facilities and those with small storage capability. Peaking is used not only on small hydropower operations but also at many larger installations (e.g. Oahe Dam on the Missouri River near Pierre, South Dakota) to provide peak flow during the morning and evening hours (USACE, 1995). Due to favorable hydraulic conditions and the presence of another dam 30 miles downstream whose backwater reaches Oahe Dam, the flow is able to be reduced to zero during the storage portion of the peaking cycle. While the flow is not reduced to zero at most facilities, the peak flow is often two to three times the off-peak level. The peak flow is sometimes given as a percentage or multiplier of the base flow, the flow that would exist under non-peaking conditions. Figure 1 shows an example of a hypothetical peaking discharge hydrograph and the base flow level associated with it.

The formation of an ice cover may present problems to peaking operations. Ice cover formation is best accomplished at low, steady flows, and once stabilized (i.e.

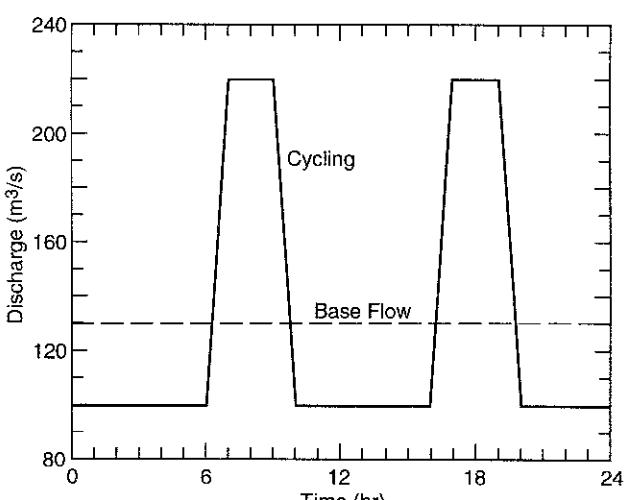

Figure 1. Peaking and base flow discharge hydrograph.

frozen in place), the cover can withstand higher flows without movement or failure. The extreme ranges of discharge resulting from peaking operations can inhibit the formation of an ice cover or cause an existing cover to fail and transport downstream. Continued failure and generation of ice covers downstream of a hydropower facility results in greater overall ice volumes contributing to the cover or accumulation. Larger ice volumes generally translate into higher water levels and increased risk of flooding during the spring breakup period. Higher water levels may reach far enough upstream to increase the tailwater level of the hydropower facility and thus reduce generating capacity and efficiency.

As a result, peak discharge or range limits are often instituted during the times when an ice cover is present or at least during the likely formation and breakup periods. These limits are sometimes developed through experience, resulting in occasional flooding when the appropriate limits were not chosen. Often, some form of numerical model is used to estimate the ice thickness and therefore the resulting water levels expected (or allowable) for different levels of peak discharge. Some facilities take great care in limiting the peak discharge during the initial cover formation period and then raise the peaking discharge gradually as the ice cover stabilizes. Typically, the expected ice thickness is determined from a static equilibrium ice thickness equation such as that presented by Beltaos (1983):

$$\eta_{eq} = \frac{BS}{2\mu(1-s_i)}\left(1+\left[1+\frac{(2f_o)^{\frac{1}{3}}\mu(1-s_i)}{s_i}\frac{f_i}{f_o}\frac{\left(\frac{q^2}{gS}\right)^{\frac{1}{3}}}{BS}\right]^{\frac{1}{2}}\right) \tag{1}$$

where η_{eq} is the equilibrium ice thickness, or the thickness that would be expected if all the forces acting on the cover were in balance, B is the river width, S is the slope of the water surface (which is equal to the friction slope and bed slope for a uniform flow condition), f_i and f_o are the ice and composite (combined ice cover and bed) Darcy friction factors, s_i is the specific gravity of ice, q is the unit discharge, g is the acceleration due to gravity, and μ is a coefficient describing the strength properties of the ice.

This equation is often applied at the peak discharge and would thus represent the worst conditions expected if the discharge was uniform for all points downstream of the dam. A hydrograph wave, however, is attenuated as it travels downstream under open water conditions due to frictional losses with the bed and banks. An ice cover, whether smooth or rough, will further attenuate the wave, resulting in water storage within the river reach and an increase in the ice-covered water levels. The peak discharge decreases with distance downstream, as do the resulting ice thickness and water levels expected. Because of this wave attenuation and its effects on the resulting ice thickness and water levels, areas of potential flooding downstream may be at less risk than has previously been thought. Limits instituted on the cycling operations that take into account the wave attenuation might be less restrictive than those based on equilibrium ice thickness. In order to demonstrate the effects of the wave attenuation on the discharge, ice thickness, and water levels downstream, a numerical model was utilized.

THE NUMERICAL MODEL

The model is a 1-D, unsteady, moving-ice model that simulates the water and ice movement within a river reach. The model is based on the de St. Venant equations of mass and momentum conservation for water and ice:

Conservation of water mass:

$$\frac{\partial d}{\partial t} + u\frac{\partial d}{\partial x} + d\frac{\partial u}{\partial x} = 0 \tag{2}$$

Conservation of water momentum:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\left(\frac{\partial d}{\partial x} + s_i\frac{\partial \eta}{\partial x} - S_o\right) + \frac{f_b u^2 (B+2d)}{8Bd}\left[1 + \frac{f_i}{f_b}\frac{B}{(B+2d)}\left(\frac{u-\upsilon}{u}\right)^2\right] = 0 \tag{3}$$

Conservation of ice mass:

$$\frac{\partial \eta}{\partial t} + \upsilon\frac{\partial \eta}{\partial x} + \eta\frac{\partial \upsilon}{\partial x} = 0 \tag{4}$$

Conservation of ice momentum:

$$\frac{\partial \upsilon}{\partial t} + \upsilon\frac{\partial \upsilon}{\partial x} + g(1-s_i)K_p(1-p)\frac{\partial \eta}{\partial x} + gs_i\frac{\partial \eta}{\partial x} + g\frac{\partial d}{\partial x} - gS_o - \frac{f_i(u-\upsilon)^2}{8s_i\eta} + \frac{gK_o\lambda K_p(1-p)(1-s_i)\eta}{B} = 0 \tag{5}$$

In the above equations, d is the depth beneath the ice cover, u is the water velocity, η is the ice thickness, υ is the ice velocity, g is the acceleration due to gravity, s_i is the specific gravity of ice, S_0 is the bed slope, B is the width, f_b and f_i are the Darcy friction factors for the bed and the ice, respectively, K_p is the passive pressure coefficient, p is the porosity, K_0 is the coefficient of lateral stress, λ is the coefficient of friction of ice on ice, and x and t are the space and time coordinates.

The solution variables are the water velocity (u), the depth beneath the ice cover (d), the cover thickness (η), and the ice velocity (υ). As a simplification, a uniform (constant-width) rectangular channel cross section is assumed. The ice cover is assumed to behave as a particulate mass following passive pressure failure criteria. The equations are solved in an uncoupled mode: first the ice equations and then the water equations. The equation solution is by a four-point, implicit finite difference scheme using a Newton-Raphson iteration procedure. A flow diagram of the model is shown in Figure 2.

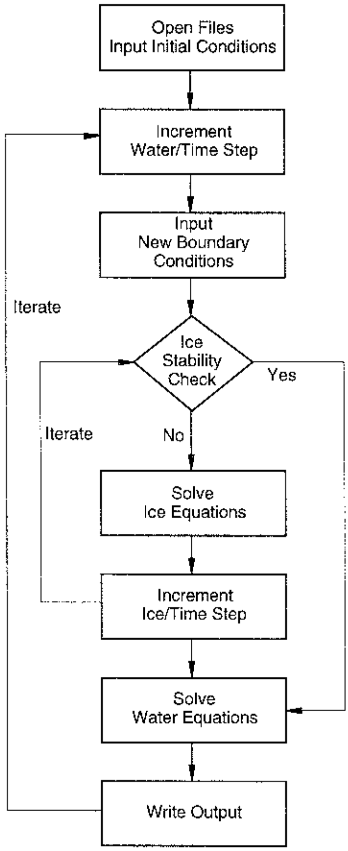

Figure 2. Numerical model flow diagram.

The program tests the stability of the ice cover at each time step by applying a complete force balance based on the current values of the variables. The ice cover stability check identifies whether the downstream-acting ice forces at each cross section are more or less than the resistive forces provided by the current ice thickness. The full ice equations (eq. 4 and 5) are only solved for the cross sections considered unstable. At cross sections where the cover is deemed stable, simplified equations are used which set the ice velocity to zero and the thickness to remain at its current value. If the ice cover is completely stable (no movement or unbalanced throughout the entire system), then there is no need to solve the ice equations and the program jumps to the solution of the water equations.

The values of ice velocity and thickness change faster than those of water velocity and depth due to the differences in celerity of the two media. The celerity of the water can be taken as the gravity wave speed, while that of the ice is closer to the speed of the force transmission (compression wave) through the cover, which depends on the ice properties. For this reason, the ice equation solution time step is set at an even fraction of the water equation solution time step (e.g. $\Delta t_{ice} = \Delta t_{water}/4$). In this way, several ice iterations can be computed prior to the computation of the next water iteration. As a result, smaller changes occur over each ice time step, leading to a smoother solution. Output from the program can be displayed in a variety of tables and graphs.

EXAMPLE APPLICATION

A hypothetical river reach downstream of a peaking hydropower plant was developed to demonstrate the effects of wave attenuation on the downstream discharge, ice thickness, and water levels. This generic reach and the peaking hydrograph are not meant to mirror any particular facility and were developed by combining the hydraulic and geometric characteristics of several hydropower installations.

The hypothetical peaking hydrograph is shown in Figure 1. It is a diurnal flow regulation scheme with flows rising from the off-peak to peak values at 6 AM and 4 PM. The flow remains at the peak value for two hours before returning to the off-peak value at 10 AM and 8 PM. The channel downstream of the hydropower facility is a uniform 100-m-wide rectangular channel with a bed slope of 0.0005. Cross sections are spaced every 100 m for a total channel length of 10 km. Additional channel and ice properties are given in Table 1.

Table 1. Additional Channel and Ice Properties

Property	Value
Darcy friction factor for the channel bed, f_b	0.08
Darcy friction factor for the ice cover, f_i	0.12
Water time step, Δt_{water}	60 s
Ice time step, Δt_{ice}	15 s
Passive pressure coefficient, K_p	3.85
Lateral stress coefficient, K_o	0.5
Ice on ice friction coefficient, λ	0.65
Ice cover porosity, p	0.4

For this example, the initial conditions are those that would be expected under a steady flow rate at the off-peak level of 100 m^3/s. The initial ice thickness is 1.47 m everywhere along the channel, which corresponds to the equilibrium thickness given by equation 1. The upstream boundary conditions are the water discharge, which changes with time, and equilibrium ice thickness (by equation 5). The downstream boundary conditions are normal water depth beneath an ice cover and zero ice velocity. The model output is compared for conditions at 0 km (the upstream end), 1 km, 5 km, and 10 km (the downstream end).

RESULTS

Figure 3 presents the discharge at the four locations mentioned above. While the discharge at 0 km is simply the upstream boundary condition (water discharge of Figure 1), the values at 1 km, 5 km, and 10 km are calculated by multiplying the channel width by the solution variables of water velocity and depth. It is evident from the figure that there is both an attenuation of the discharge peak and a lag in the time to reach the peak as the discharge wave travels downstream. The slopes of the rising and falling limbs of the hydrographs for 0 km and 1 km are very similar, which indicates that there is very little numerical dissipation (smoothing) in the solution. The 1-km, 5-km, and 10-km hydrographs show high-frequency fluctuations superimposed on the general rise of discharge. These fluctuations occur only on the rising limb and correspond to times when the ice velocity is fluctuating. In general, when the ice cover has a positive velocity, the frictional resistance of the cover on the water flow is reduced due to the reduction in relative velocity of the water compared to the cover. This results in an increase in the local discharge due to an increase in the water

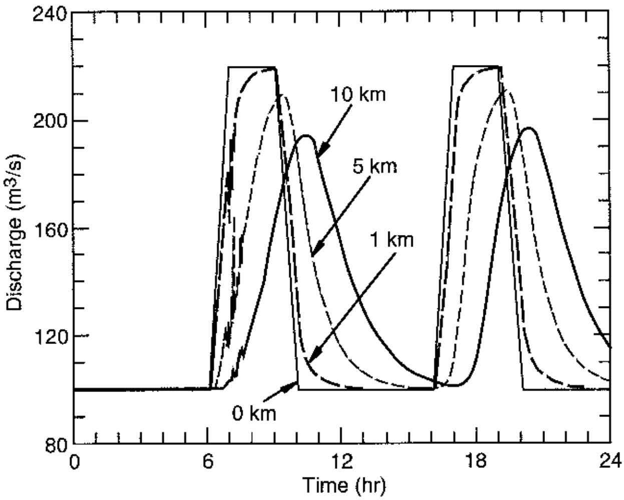

Figure 3. Water discharge vs time at 0, 1, 5, and 10 km downstream from the dam.

velocity. These fluctuations are not seen on the second discharge wave, since very little ice movement takes place during the second discharge wave. There is also a slight increase in the peak discharge on the second discharge wave for 5 km and 10 km. If the upstream discharge was increased and held steady, one would expect that the discharge at the downstream locations would also eventually increase to that steady level. With continued cycling, the levels of the peak discharges at 5 km and 10 km could reach the 220 m^3/s level, but only if the duration of the upstream peak inflow is of long enough duration. It appears from Figure 3, however, that the travel time of the peak discharge to reach 10 km is roughly 2 hours, the same as the duration of the upstream peak inflow. Therefore, it is doubtful that the discharge levels at 5 km and 10 km will rise much above the levels experienced with the second discharge wave.

Figure 4 shows the ice cover thickness and its changes with time. The areas of constant thickness correspond to the times of ice cover stability (zero ice velocity). Thickness begins to change at hour 6 from its initial value of 1.47 m (static equilibrium thickness for a flow of 100 m^3/s) due to the increase in water discharge at the upstream end. As mentioned above, when the ice cover at the upstream end is considered unstable, the new thickness is determined by the upstream boundary condition, which is a simplified form of equation 5. Since this equation is applied at a point, the

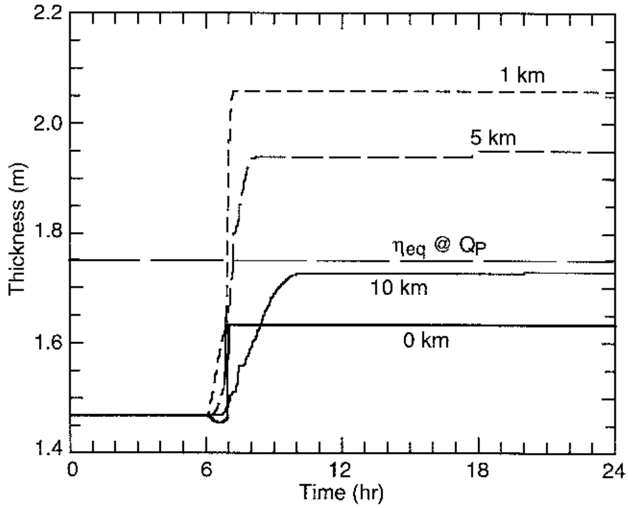

Figure 4. Ice cover thickness vs time at 0, 1, 5, and 10 km.

differential terms are neglected. The effect is to reduce equation 5 to a steady-state equilibrium ice thickness equation (although including ice velocity) which neglects the differences between water surface slope and bed slope in times of accelerating and decelerating water flow. The decrease in thickness at 0 km after hour 6 arises from the reduction of shear on the underside of the cover due to a positive ice velocity (and thus a smaller relative velocity between the water and ice). Hour 7 represents the time of maximum discharge and thus maximum water velocity at 0 km. Even though the discharge remains at this high level for two hours, the water depth increases with time, reducing the water velocity and shear on the underside of the cover. As a result, the greatest thickness at 0 km is expected at hour 7, when it reaches 1.63 m. Due to the increased water levels and reduced water velocity at 0 km (which is also partly due to the backwater effect of increased ice thicknesses downstream), the ice thickness at 0 km never reaches the value of static equilibrium thickness for a flow of 220 m^3/s (also plotted on Figure 4). The thickness at 1 km begins to rise immediately at hour 6 and quickly reaches its maximum of 2.06 m shortly after hour 7. The thicknesses at 5 km and 10 km rise slower, requiring additional time for the increased discharge and the effects of increased upstream thickness to reach those downstream sections. These two locations also show a slight increase in thickness with the second discharge wave.

The final point to be made from Figure 4 is that of ice momentum. The thickness at 10 km is not affected significantly by ice momentum, since the ice velocity is always zero. The ice velocity at 5 km approaches a maximum of 0.18 m/s, at 1 km a maximum of 0.24 m/s, and at 0 km a maximum of 0.28 m/s. Figure 4 depicts the thicknesses at 0 km and 10 km as below the static equilibrium level and the thickness at 1 km and 5 km as above the static equilibrium level. It was stated above that the point equation used as the boundary condition at 0 km assumes that the water surface slope and bed slope are equal. This assumption leads to an underprediction of ice thickness when the water surface slope is in fact greater than the bed slope (conditions of increasing discharge). It was also noted that the effects of attenuation of the discharge wave as it travels downstream results in smaller ice thicknesses with increasing distance downstream. Figure 5 shows the final, stable ice thickness vs x-location and presents a better illustration of the effects of ice momentum. Only the ice thicknesses at 0 km and 10 km are below the static equilibrium value. The thickness at 0.1 km (equation 5 takes into account the slope of the water surface by the differential terms for this reach) rises to 1.81 m and continues to its maximum value of 2.1 m at 2.0 km. From this point downstream, the maximum ice velocity as well as its effect on ice thickness decreases. The steep drop in thickness from 9 km to 10 km demonstrates that the 2-hour duration at the peak flow rate was not sufficiently long enough to develop a steady flow condition beyond 9 km, resulting in smaller discharge levels and ice thicknesses. Overall, however, the effects of ice momentum on ice thickness

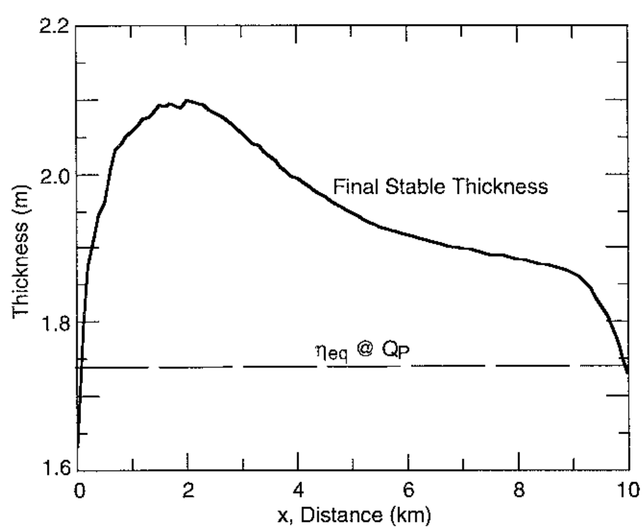

Figure 5. Final stable ice cover thickness profile.

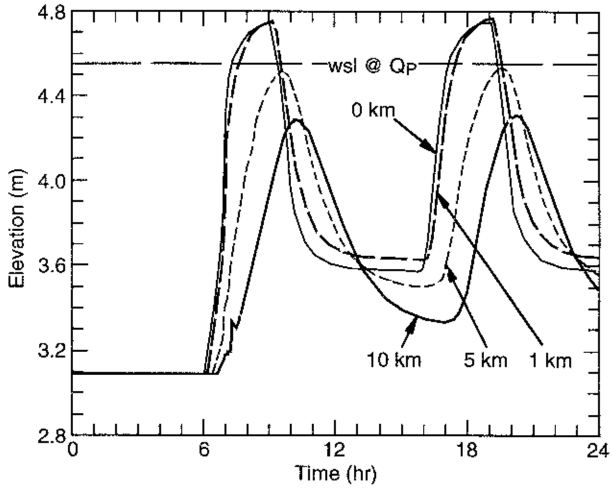

Figure 6. Water surface level vs time at 0, 1, 5, and 10 km.

are significant, as the thickness is greater than the static equilibrium value almost everywhere.

The final variable plotted in Figure 6 is the water surface level or total depth, equal to the underice water depth plus the submerged ice cover thickness. The high-frequency fluctuations which appeared on the rising limb of the first discharge wave in Figure 3 are absent except for very minor fluctuations for 10 km. This is because the total depth is a function of water depth and ice thickness, which are calculated over different time steps and tend to oscillate out of phase with each other, thereby damping these high frequency oscillations somewhat. Due to the attenuation of the discharge wave, the maximum water levels decrease with increasing distance downstream. The maximum water levels increase very slightly with the second discharge wave and would be expected to differ little with future discharge waves. This is because almost all of the ice motion expected occurred during the first wave, with only minor movement in the second wave. The water levels at 0 km, 1 km, and 5 km appear to reach their steady off-peak values (and not quite steady at 10 km) between the two discharge waves. Also plotted on Figure 6 is the water level which would be expected with an equilibrium thickness calculated by equation 1 coupled with the normal depth at a steady flow of 220 m^3/s. Since the ice thicknesses are greater than the static equilibrium level, one would also expect increased backwater effects, resulting in higher water levels. This is especially evident for 0 km and 1 km but also at 5 km, where the peak discharge was only 210 m^3/s yet the water level still reaches the static equilibrium thickness water level. The water level at 10 km is less, since both the thickness and the peak discharge are less than the static equilibrium levels.

CONCLUSIONS

This paper has attempted to show the benefits of using a fully unsteady, moving-ice numerical model to determine the effects on discharge, water level, and ice thickness downstream of a peaking hydropower facility. While static equilibrium ice thickness equations coupled with normal flow depths at the peak flow are often used in determining water levels, they may both under- and over-estimate true water levels. At the upstream end of the system, ice momentum effects translate into greater ice thicknesses and higher water levels than static equilibrium levels. The attenuation of the discharge wave as it travels downstream translates into less stress on the ice cover and results in generally thinner covers and lower water levels as the distance downstream increases. A model such as the one described here would be useful in determining the effects of hydropower peaking on downstream water levels or for determining the maximum range of cycling allowable. The model not only accounts for the attenuation of the discharge wave but also includes the important effects of ice cover momentum.

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DISCUSSION

Faye Hicks,
University of Alberta:

Can you elaborate on the degree to which the flow equations are coupled to the ice equations? That is, are they solved sequentially or are they in fact solved simultaneously, coupling at least some of the terms/variables?

Reply: In this version of the model, the ice and water equations are what might be termed "loosely coupled," that is, the ice equations are solved one or more times (at a smaller time step) and then the water equations are solved based on these new values of the ice variables. One would expect that as the size of the time step was reduced, the solution would approach that of a "fully coupled" system. The "loosely coupled" model requires the sequential solution of two banded coefficient matrices (one for the ice and one for the water), which each have 5 diagonals. I have, since the presentation of this paper, developed another version of the model that is truly "fully coupled," with the ice and water equations being solved simultaneously. That version requires the solution of only one banded coefficient matrix but which has 11 diagonals. In the fully coupled version, the effects of the changes in each solution variable on the others are part of the solution. In general, the fully coupled solution provides a "smoother" solution with lower-frequency fluctuations in the solution variables. The thicknesses calculated are slightly higher in the upstream reaches also, which results in slightly higher water surface levels.

Spyros Beltaos,
National Water Research Institute:

1. Have you checked the four-point implicit numerical scheme for possible numerical dispersion effects?
2. Does the continuity of water equation used in the model account for water within the pores of the accumulation cover, which may be moving along with the cover?

Reply: In response to the first question, there are both numerical dissipation and dispersion attendant in most numerical models, especially when linearizing such non-linear terms as those in equations 2–5. Dissipation, or smoothing, of the solution tends to attenuate some of the high frequency fluctuations. Dispersion, however, arises from the acceleration or deceleration of terms due to linearization. Theta-weighting factors will usually reduce the amount of dispersion at the expense of introducing more dissipation. The final word is the effects of natural damping, such as frictional resistance. If the natural damping effects dominate, then numerical dispersion and dissipation become less important. I made several runs with decreasing frictional resistance of the bed, banks, and ice cover. At very low frictional resistance, the dissipation was greatly reduced, showing that the natural damping was dominant over the numerical dissipation. Also, with stationary ice covers (thick enough to guarantee stability), there is very little numerical dispersion. The apparent "dispersion" effects shown on the rising limbs of the hydrographs in Figure 3, for example, are actual fluctuations arising from the movement of the ice cover.

In response to the second question, the water within the pores of the cover is included in both the continuity and momentum equations for the ice cover. It is treated simply as "ice" with a higher specific gravity; it moves and stops in the same way that the cover moves and stops, and there is no provision for mass transfer between the cover (and its pore water) and the water flow beneath.

Sylvester Petryk,
Petryk Consultants, Inc.:
The results show high-frequency fluctuations of conditions (in discharge, water levels, etc.) which are superimposed on the normal low-frequency transient conditions. It seems that the accumulation cover should dampen any high frequency fluctuations, rather than generate them. Can you give a physical and/or theoretical explanation of why they occur?

Reply: I welcome this question as it helps explain one of the benefits of using an unsteady, moving-ice model. One would expect that with a static ice cover, fluctuations in discharge, for example, introduced at the upstream end of a system would be damped. This holds not only for the low-frequency (cycling hydrograph) transient but also for high-frequency fluctuations. When the streamwise movement of the ice cover is in fact one of the solution variables, however, opportunity arises for the generation of fluctuations within the system. As an instability occurs and the ice cover moves, the frictional resistance to water flow (due to the ice cover) is reduced, since it is related to the square of the difference in the water and ice velocities. This reduction in resistance results in an increase in water velocity and the release of some water that had previously been stored within the channel. The many instabilities occurring within the cover can be thought of as "mini-dam breaks" which release or generate high-frequency fluctuations in the solution variables. Once ice cover motion is initiated, the order of the system appears to be lost with changes in ice motion affecting water velocity, which affects shear on the underside of the cover and thus ice thickness, ice velocity, etc. Unfortunately, I have not found an easy method to display the changes in four variables with time at many cross sections. Figure 7 below helps to explain the reliance of the local discharge on the ice velocity and the resulting high-frequency fluctuations. I plotted the local water discharge at 1 km from hour 6 to 8 along with the ice velocity. One can see that when the ice moves, the local discharge will increase and that discharge decreases when the ice cover slows or stops due to increased frictional resistance.

Dave Andres,
Trillium Engineering and Hydrographics, Inc.:

Is the natural frequency of the pack a meaningful parameter? How does this value compare to the frequency of discharge fluctuations? What is the minimum frequency of discharge fluctuations to which the cover can respond?

Reply: I would consider the natural frequency of the pack to be the speed of transmission of a compression wave through the ice cover. This value could vary greatly depending on the condition of the cover and its past history (e.g. whether it has undergone repeated shoving and thickening events). At one extreme would be a fully thermally consolidated or thermally grown sheet ice cover. For this case, the compression wave speed would approach the speed of sound in the medium. At the other extreme might be a loose (less than 100% surface coverage) single-layer accumulation of ice pieces. With an increase in the force exerted on this type of cover, some consolidation and jostling might occur within the cover as the porosity decreases, significantly decreasing the natural frequency or compression wave speed through the medium. Overall, the compression wave speed could range in value from the order of the water wave speed to very much higher. The current version of the model assumes that increases in forces at the upstream end are felt throughout the cover, simulating a very high compression wave speed. The loose-type cover could induce some force transmission lag, which would alter the solution somewhat.

In addressing the third question, theoretically, the model should respond to any discharge fluctuation, that is, providing that the cover was at the limit of stability. Physically, an instability that results in ice motion will itself progress through the system until it reaches a location where the resisting forces are great enough to restrict ice motion. However, the arrest of ice motion results in changes not only in thickness but also in water velocity and depth. It is the ice momentum that causes the final rejamming thickness to be slightly greater than that which would be expected at the limit of stability for the new hydraulic conditions. Therefore, a discharge fluctuation which would result in a new instability would have to be slightly greater. In other words, the threshold value of discharge fluctuation (resulting in an instability) would be dependent on the magnitude of the ice momentum attendant in the previously arrested ice motion.