A METHOD FOR CALCULATING FLOW RESISTANCE IN ICE-COVERED ALLUVIAL CHANNELS

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ABSTRACT

Presented herein is a semi-empirical method for calculating flow resistance in ice-covered alluvial channels. The method is based on data from experiments conducted with a laboratory flume and on data published from prior studies. Interpretation of the prior and present data, in accordance with dimensionless parameters describing flow resistance in an ice-covered alluvial channel, led to the development of a relationship for predicting the proportions of flow resistance attributable to the ice cover and to the bed. That relationship, combined with the Einstein-Barbarossa bar-resistance curve, comprises the key component for an iterative procedure for estimating flow depth in ice-covered alluvial channels. Other openwater resistance relationships, besides the Einstein-Barbarossa curve, could be used in the method.

INTRODUCTION

It is well understood (e.g., Ashton, 1986) that ice-cover presence imposes a floating solid boundary on the upper surface of flow, thereby increasing channel resistance to flow and redistributing flow over its depth. Less understood is the influence of cover presence on the dimensions and geometry of alluvial bedforms and, thereby, on bed resistance. The present paper seeks to clarify that influence and, in so doing, proposes a semi-empirical method for calculating flow resistance in ice-covered alluvial channels. The method is based on dimensional analysis backed by results from experiments with a laboratory flume, and a re-interpretation of findings from prior studies. The experiments are documented by Smith and Ettema (1995).

A review the prior laboratory studies summarized in Table 1 shows that they investigated different ranges of bed regimes; i.e., beds with dune, ripple, or mixed morphology. Viewed in the context of Fig. 1, which shows the influence of different bed regimes on openwater flow resistance, evidently flows producing different bedforms would respond differently to imposition of an ice cover. The additional solid boundary of a cover increases flow resistance and, thereby, reduces mean flow velocity and increases flow depth. In accordance with whether the bed regime is in ripple, dune, or mixed regime, Fig. 1 suggests that cover presence may either negligibly or substantially alter bedform geometry and bed resistance.

PARAMETERS AND FUNCTIONAL DEPENDENCIES

Dimensional analysis of the variables shown in Fig. 2 provides a useful framework for discussing flow resistance in ice-covered alluvial channels, and re-examining the findings from prior studies. An important feature of the dimensional analysis is that the total flow depth, Y, is taken as the characteristic length scale for ice-covered flow. Although it is possible to define flow depths for conceptual upper and lower layers of a composite flat-bed flow (e.g., as discussed in Ashton, 1986), these composite flow

Table 1. Prior studies of ice-covered alluvial channel flow

Parameter	Sayre and Song (1979)	Lau and Krishnappan (1985)	Wuebben (1988)	Series A and B (Present study)
Particle Reynolds Number	$7.5 < Re_* < 11$	$2.5 < Re_* < 4.5$	$14 < Re_* < 22.5$	$44 < Re_* < 60$
Bedform Regime	ripples and dunes	ripples	ripples and dunes	dunes
Channel Aspect Ratio	$5.6 < b / Y < 9.0$	$5.5 < b / Y < 11.6$	$4.3 < b / Y < 13.6$	$4.4 < b / Y < 5.7$
Transport Capacity	$5.5 < \eta < 12$	$2 < \eta < 7$	$5 < \eta < 12$	$2 < \eta < 3$

Table 2. Dimensionless parameters for an ice-covered alluvial channel

Dimensional Parameter	Dimensionless Parameter
Viscosity	$Re \equiv UY / \nu$ or $Re_* \equiv u_{\tau b} D / \nu$
Particle Weight	$D_* \equiv D [\gamma_s / \rho \nu^2]^{1/3}$
Particle Roughness	$\varepsilon_b \equiv D / Y$
Cover Roughness	$\varepsilon_i \equiv k_i / Y$
Gravity	$Fr \equiv U / (gY)^{1/2}$
Bed Resistance (Tractive Force)	$f_b \equiv 8\tau_b / \rho U^2$ or $\theta \equiv \tau_b / \gamma_s D$
Cover Resistance	$\alpha \equiv \tau_i / \tau_b$
Bedform Length	$L_* \equiv L / D$
Bedform Height	$\delta \equiv H / L$
Pressure Gradient (Slope)	S

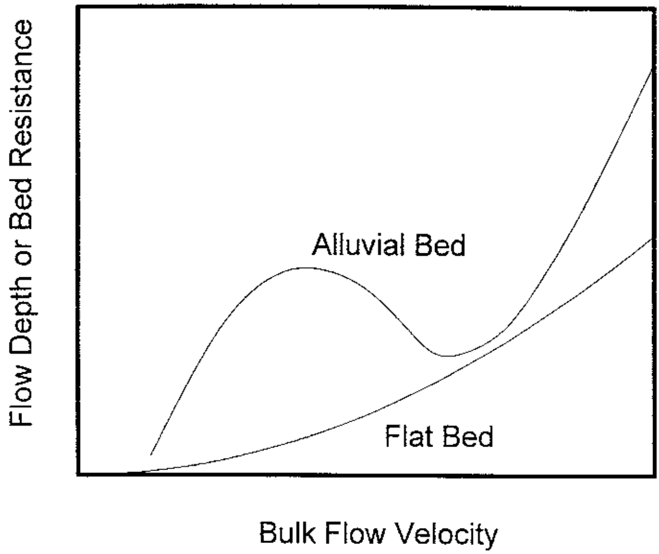

Fig. 1. Depth-Velocity relationships for fixed (flat-bed) and alluvial-bed channels

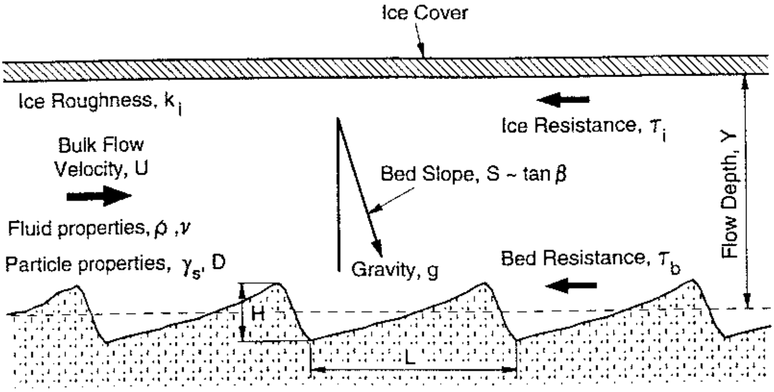

Fig. 2. Variables influencing flow in ice-covered alluvial channels

depths are only surrogate parameters for shear stress at the upper and lower flow boundaries--a definition which becomes obscure when the bed or ice cover is not flat. Furthermore, the spectra of turbulent eddies that shape dunes and ice cover surfaces evolve over the entire flow depth. They are not limited to a conceptual layer. Thus, the following dimensional analysis defines the flow Reynolds number, Froude number, and relative roughnesses in terms of the total flow depth, Y.

The thirteen variables shown in Fig. 2 normalize as the ten dimensionless parameters listed in Table 2. The influence of viscosity on the interaction of fluid and particle is expressed equally well by either the flow Reynolds number, Re, or the particle Reynolds number, Re_*, since $Re_* \equiv Re\,(f_b/8)^{1/2}\varepsilon_b$. Similarly, the bed resistance may be described equally well by the friction factor, f_b, or the Shields parameter, $\theta = (f_b/8)Re^2\varepsilon_b^{\,2}/D_*^{\,3}$. The particle Reynolds number and the Shields parameter are appropriate for describing the state of sediment movement over an alluvial bed because they are independent of both flow depth and flow velocity. They depend only on the bed resistance, τ_b, and the fluid and particle properties.

Bed Resistance

The functional dependence for bed resistance may be written as

$$f_b = \varphi_b\left(Re_*, D_*, \varepsilon_b\right) \tag{1}$$

The Froude number, $Fr = U/(gY)^{1/2}$, is omitted from Eq. (1) on the premise that ice-cover formation on high-velocity streams, in which Froude number effects are important, is rare. Also implicit in Eq. (1) is the assumption that ice cover affects alluvial bed resistance and sediment transport only by modifying flow depth, bulk flow velocity, and the tractive force exerted on the bed by the flow. This key assumption enables the use of openwater relationships, such as those given by Einstein and Barbarossa (1952), Engelund and Hansen (1967), or Alam and Kennedy (1969), to predict bedform geometry and bed resistance in ice-covered flows. The validity of this assumption for ripple-bed channels is supported by Lau and Krishnappan's (1985) data, which show that ripple dimensions depend only on the tractive force and sediment and fluid properties (θ and Re_*, for example). Its validity for dune-bed flows is confirmed by the results of the writers' experiments, which are discussed subsequently.

Components of Bed Resistance

The openwater bedform resistance formulations of Einstein and Barbarossa (1952), Engelund and Hansen (1967), or Alam and Kennedy (1969) divide bed resistance into surface friction and bedform drag, employing a flat-bed friction formula for the particle friction factor, f_b', and an empirical relationship for the bedform friction-factor, f_b'', determined from laboratory and field alluvial resistance data. This approach is used herein, with the modification that flat-bed friction is computed from the covered-channel formulas of Parthasarathy and Muste (1994),

$$\left(\frac{8}{f_b'}\right)^{1/2} = \frac{1}{\kappa}\log\left[\left(\frac{f_b'}{8}\right)^{1/2}\frac{Re}{1+0.3\left(k_b/Y\right)\left(f_b'/8\right)^{1/2}Re}\right] + 5.5 - \frac{1}{\kappa}\left[\log\left(1+\alpha'\right) + \frac{1+\alpha'^{3/2}}{1+\alpha'}\right] \quad \text{(2a)}$$

$$\left(\frac{8}{f_i'}\right)^{1/2} = \frac{1}{\kappa}\log\left[\left(\frac{f_i'}{8}\right)^{1/2}\frac{Re}{1+0.3\left(k_i/Y\right)\left(f_i'/8\right)^{1/2}Re}\right] + 5.5 - \frac{1}{\kappa}\left[\log\left(\frac{1+\alpha'}{\alpha'}\right) + \frac{1+\alpha'^{3/2}}{\alpha'^{1/2}\left(1+\alpha'\right)}\right] \quad \text{(2b)}$$

in which κ is the von Karman constant, and $\alpha' = f_i'/f_b'$ is the resistance ratio for the flat-bed channel. Note that the flat-bed ice-cover friction factor, f_i', represents the ice-cover resistance that would prevail in the absence of bedforms. It is distinct from the ice-cover friction factor, f_i, for a dune-bed channel. Given values of Re, ε_b, and ε_i, Eq. (2) can be solved iteratively for f_b' and f_i' using a Newton-Raphson algorithm. The value of f_b' may then be used to obtain a corresponding value for f_b'' from one of the aforementioned openwater alluvial resistance relationships. This sequence of calculations is an approximation, albeit a simplified one, of Eq. (1). Its strength is that it employs many openwater empirical data on bedform resistance. Its weakness is that it does not reveal explicitly the role that bedform geometry plays in determining ice-covered alluvial channel resistance. That role is discussed in the following sections.

Bedform Geometry Influence on Bed Friction Factor

The dependence of the bed friction factor on the size and spacing of bedforms has been enveloped by the following considerations. Rather than attempting to express friction factors as explicit functions of bedform geometry, meaning

$$f_b = \tilde{\varphi}_b\left(\delta, L_*, \varepsilon_b\right) \tag{3}$$

researchers more often express bed friction factors as functions of bed shear stress and bed particle properties, meaning Re_* and D_*. One method of so doing is to consider the equivalent bed roughness, $K = K_{form} + k_b$, as in

$$f_b = \varphi_b\left(Re, \frac{K}{Y}\right) \tag{4}$$

Bed particle roughness, k_b, is most often expressed a multiple of some representative particle size. For example, Yalin (1992) indicates that $k_b/D_{50} = 2$ for $\theta < 1$. Bedforms may be viewed as macroscopic roughness elements, presenting to the flow an equivalent bedform roughness, K_{form}, dependent only on bedform geometry:

$$\frac{K_{form}}{H} = \varphi_{Kf}(\delta) \tag{5}$$

The concept of equivalent bedform roughness is not unique to this study. For example, Van Rijn (1984) and Raudkivi (1990) provide specific expressions for φ_{Kf}. The functional dependencies of bedform size and spacing are

$$\delta = \varphi_{\delta}\left(\eta, D_*, \varepsilon_b\right) \tag{6}$$

$$L_* = \varphi_L\left(\eta, D_*, \varepsilon_b\right) \tag{7}$$

The transport capacity, $\eta = \theta/\theta_c$, is specified in Eqs. (6) and (7) rather than the particle Reynolds number, since it reveals more clearly the ability of the flow to transport sediment particles. Specification of η and D_* is equivalent to specification of Re_* and D_*, by way of the modified Shields curve, $\theta_c = \varphi(D_*)$, and the identity $\theta = Re_*^2 / D_*^3$. Yalin (1992) presents a thorough analysis of bedform mechanics and data leading to Eqs. (6) and (7). Combination of Eqs. (5) through (7) yields the functional dependence of K_{form}:

$$\frac{K_{form}}{D_{50}} = \frac{K_{form}}{H}\delta\, L_* = \varphi_{Kf}\varphi_{\delta}\varphi_L = \tilde{\varphi}_{Kf}\left(\eta, D_*, \varepsilon_b\right) \tag{8}$$

whence the functional dependence of the equivalent bed roughness becomes

$$\frac{K}{Y} = \frac{K_{form} + k_b}{\varepsilon_b D_{50}} = \varphi_K\left(\eta, D_*, \varepsilon_b\right) \tag{9}$$

Combination of Eqs (4) and (9) yields the functional dependence expressed in Eq. (1).

Ice-Cover Resistance

The resistance of the ice cover, τ_i, is expressed by the resistance ratio, $\alpha \equiv \tau_i/\tau_b$. The writers propose the following functional dependence for the resistance ratio:

$$\alpha = \varphi_{\alpha}\left(Re, \frac{k_i}{K}\right) = \varphi_{\alpha}\left(Re, \frac{\varepsilon_i}{\varphi_K\left(\eta, D_*, \varepsilon_b\right)}\right) \tag{10}$$

in which use is made of the identity $k_i/K = \varepsilon_i/(K/Y)$ and Eq. (9). Since the transport capacity is the principal determinant of equivalent bed roughness, Eq. (10) suggests combining η, ε_b, and ε_i into a single parameter, $\eta\varepsilon_b/\varepsilon_i = \eta D_{50}/k_i$, which reduces the functional dependence for α to

$$\alpha = \varphi_{\alpha}\left(Re, D_*, \eta \frac{D_{50}}{k_i}\right) \tag{11}$$

When used in conjunction with a bed friction factor predictor, such as that of Einstein and Barbarossa (1952) or Engelund and Hansen (1967), Eq. (11) enables calculation of the depth-discharge relationship for ice-covered flows. At present however, there are few data with which to determine the exact form of Eq. (11). A semi-empirical expression is proposed for Eq. (11) later in this paper.

EXPERIMENTS

Two series of experiments aimed at obtaining data on bedform geometry and bed resistance in a covered dune-bed channel were conducted using a 27-m long, 0.91-m wide by 0.45-m deep laboratory flume. The flume was not refrigerated, and ice-covers were replicated with sheets of roughened plywood. Both series of experiments comprised four runs simulating a different ice-cover, but the same water discharge, energy slope, and bed material. Measured were flow depth, bulk flow velocity, bedload transport rate and bedform geometry. The second series repeated the first series under the same conditions to confirm findings. Table 3 lists the parameters for the two series of runs, herein termed Series A and B.

Cover resistance, τ_i, was measured directly during the Series B runs. Series A cover resistances were estimated by assuming the resistance ratio, α, to be identical for corresponding runs in the two series. Fig. 3 depicts the apparatus used to measure the shear force on the cover. A load cell mounted at the upstream end of the flume first measured the shear force on all 22 panels in the flume. The last panel at the downstream end of the flume was removed, and the shear force on 21 panels measured. Six or seven of these iterations occurred for each of the Series B runs. A linear regression of shear force versus number of panels yielded the shear force per panel, from which the cover shear stress, τ_i, was calculated.

RESULTS

The flume data were analyzed on the assumption that the flume flows can be treated as being essentially two-dimensional. The hydraulic radii for the flows are computed as $r_o = Y$ and $r_c = Y/2$ for openwater and covered flows, respectively. This approach is used in lieu of a sidewall correction procedure, as it similarly takes into account the fact that the flume walls are hydraulically smooth and offer much less resistance to the flow than do the bed and cover. Use of the sidewall correction procedure of Vanoni (1975), instead of the above procedure, does not affect the results or conclusions significantly.

Flow Resistance

The resistance ratio, $\alpha \equiv \tau_i/\tau_b$, increased with increasing cover roughness, not only because τ_i increased, but also because τ_b decreased. These results are evident in Fig. 4, which illustrates ice-cover influence on the bed friction factor, f_b, and its particle and form drag components, f_b' and f_b'', respectively. The decrease in the bedform friction factor is consistent with decreased dune steepness, a result of increased flow depth and decreased bulk flow velocity. The effect of ice cover on dune geometry is discussed later.

The particle friction factor increased with increasing cover roughness because it is affected by ice cover through two mechanisms. First, cover presence and increasing cover roughness reduce the bulk flow velocity, which in turn reduces the surface

Table 3. Experimental Parameters

Run	ε_b	ε_i	*Re*	*Fr*	*S*	Temp. (°C)	f_b'	f_b	α	L_*	δ
F2	0.0160	-	83323	0.41	0.138	20.2	0.0289	0.0642	0	926	0.023
S2	0.0141	0.0001	97241	0.34	0.137	25.2	0.0346	0.0660	0.42	1030	0.024
M2	0.0131	0.0102	83818	0.31	0.129	20.6	0.0373	0.0585	0.83	973	0.019
R2	0.0123	0.0950	84847	0.28	0.133	21.2	0.0433	0.0609	1.25	991	0.012
F4	0.0160	-	85777	0.41	0.129	21.9	0.0289	0.0612	0	838	0.026
S4	0.0140	0.0001	87216	0.34	0.134	21.9	0.0346	0.0638	0.42	879	0.027
M4	0.0134	0.0104	86051	0.32	0.130	21.9	0.0375	0.0559	0.83	910	0.012
R4	0.0122	0.0945	83703	0.28	0.130	20.8	0.0432	0.0610	1.25	1080	0.008

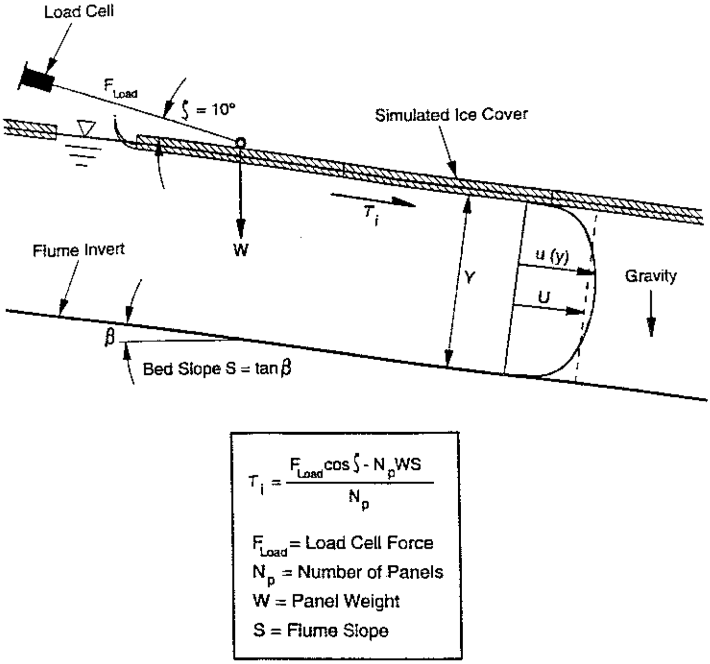

Fig. 3. Arrangement used to measure cover shear stress

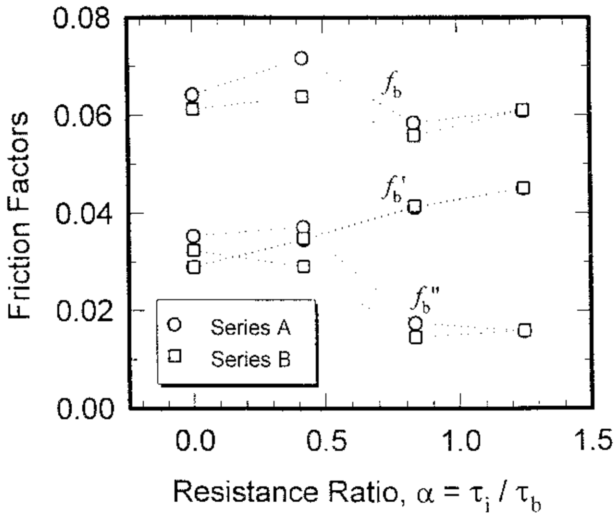

Fig. 4. Effect of ice cover on bed resistance

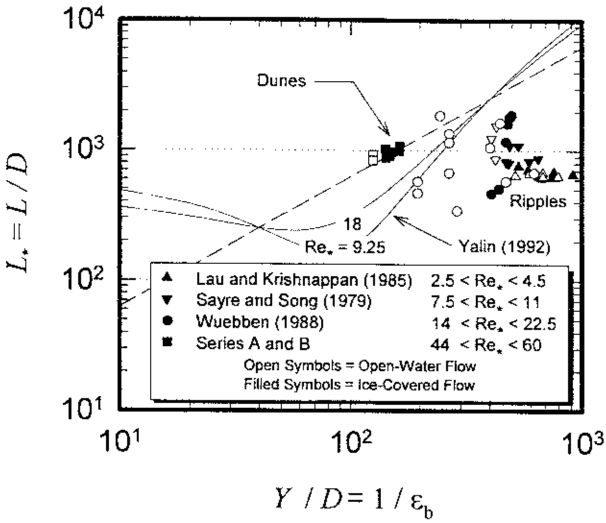

Fig. 5. Comparison of Ice-Covered Bedform Length Data from Prior and Present Studies

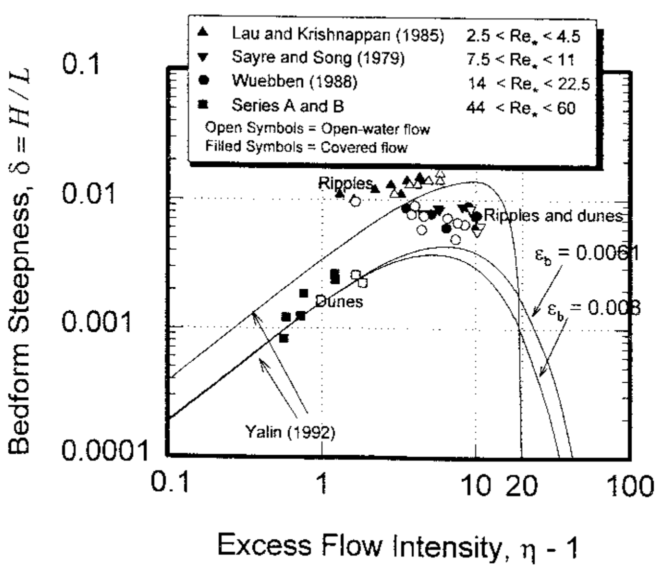

Fig. 6. Comparison of ice-covered bedform steepness data from prior and present studies

Lau and Krishnappan's data are insensitive to flow depth and cluster around the line $L_* = 650$. Values of Re_* for their experiments place them at the lower end of the transitional roughness regime, very close to the smooth regime ($Re_* < 2.5$). Lau and Krishnappan assumed their experiments were smooth flows; they used a smooth boundary relation for estimating flat-bed friction. The bedforms developed in their experiments are uniquely ripples. Ice cover influences ripple beds differently than dune beds. Whereas Lau and Krishnappan's (1985) experiments prove the two-layer hypothesis to be a valid simplifying assumption for ripple-bed flows, the present experiments show it to be inappropriate for dune-bed flows.

The bedform length data of Sayre and Song and Wuebben lie in between these extremes, in the transitional roughness regime. Hence, the data do not follow any regular trend because ripples and dunes are present in both sets. Sayre and Song's bedform data lie far to the right of the dune relations. Bedforms in their runs are predominantly ripples.

It is not surprising that neither Sayre and Song (1979) nor Wuebben (1986 and 1988) reached firm conclusions about the appropriate depth scale or the influence of ice cover for their bedforms. With the placement and subsequent roughening of the ice covers in their experimental series, the decreasing particle Reynolds number caused the prominence of ripples to increase and that of dunes to decrease. It is interesting to note that Sayre and Song employed a zero-crossing analysis of their bed profiles, which gives only average bedform dimensions; it is insensitive to the multi-modal power spectra of mixed regime bed profiles.

Fig. 6 shows the variation of bedform steepness with excess transport capacity, $\eta - 1$, for prior and present data. The lines in the figure are Yalin's (1992) empirical relations for ripple and dune and steepness. Two dune steepness curves are shown, for $\varepsilon_b = 0.0061$ and 0.008, encompassing the range of flow depths used for Series A and B.

Although the ripple data of Lau and Krishnappan do not coincide exactly with the ripple steepness relation, they are within the uncertainty band of the data used to determine the curve. Lau and Krishnappan (1985) demonstrated this result. The effect of the ice cover in their experiments was to reduce the transport capacity, thereby decreasing ripple steepness. The ultimate effect of the ice cover is the same for the dune data of Series A and B, although the mechanism for steepness reduction is different. For dune-bed flows, steepness is reduced through decreases in transport capacity as well as lengthening of dunes in response to the increasing flow depth. Since the experiments of Wuebben (1988) and Sayre and Song (1979) include ripples and dunes, the average bedform steepness for each of their experiments should lie between the ripple and dune steepness curves, which they do. The range of flow intensities studied in these experiments places them near the apexes of the ripple and dune steepness curves, which explains why the measured steepness values were relatively insensitive to changes in transport capacity induced by ice cover presence and increasing ice cover roughness.

FLOW RESISTANCE IN ICE-COVERED ALLUVIAL CHANNELS

The Series A and B data, together with data from Lau and Krishnappan (1985), Sayre and Song (1979), and Wuebben (1988), are arranged in Fig. 7 in accordance with Eq. (11). The resistance ratio is plotted versus $\eta D_{50}/k_i$. Values of k_i for the present experiments were derived from shear force measurements on the simulated ice panels (Smith and Ettema, 1995). Values of k_i for the data of Sayre and Song (1979) were computed using the formula of Webb et al. (1967). Values of k_i for the data of Wuebben (1986 and 1988) and Lau and Krishnappan (1985) are based on their descriptions of their simulated ice covers. These roughness values are gross estimates. Precise values of k_i are not required in Fig. 7 because $\eta D_{50}/k_i$ typically varies over several orders of magnitude.

Values of the resistance ratio for the Series A and B experiments were computed from direct measurements of cover shear stress. Lau and Krishnappan (1985) computed values of τ_b and τ_i using a k-e turbulence model. The values of α from these two sources may be considered with confidence. Because direct measurements of boundary shear stresses were unavailable, values of α from Sayre and Song (1979) and Wuebben (1988) were computed using the two-layer hypothesis; i.e., $\alpha = (1 - Y_v)/Y_v$, in which Y_v is the elevation above the average bed level of the maximum streamwise velocity.

The regression line and the 95 percent prediction intervals in Fig. 7 are proposed as an interim predictor for a until a more complete determination of the functional form of Eq. (10) becomes available. The equation of the regression line is

$$\alpha = 0.84\left(\eta \frac{D_{50}}{k_i}\right)^{-0.20} \tag{12}$$

Its range of applicability is $1 < \eta < 11$, corresponding to the range covered by the data.

Although the relationship shown in Fig. 7 embodies the essential physics of ice-covered alluvial resistance, it certainly requires further study to be of use in natural channels. The roles of viscosity and particle weight, played by Re and D_*, are not considered in Eq. (12). Furthermore, flow depth, Y, does not appear in Eq. (12). That parameter influences the scale of dunes, which in turn sets the equivalent bedform roughness of the bed. Ignorance of these parameters contributes to the uncertainty of values of a predicted by Eq. (12). A more complete treatment of their effects requires more data, particularly data from large Reynolds number flows typical of natural channels.

CALCULATION OF RESISTANCE IN ICE-COVERED ALLUVIAL CHANNELS

Eq. (12) may be used to estimate the depth-discharge relationship for ice-covered channels. The procedure requires as input cover roughness, k_i, median particle diameter, D_{50}, unit water discharge, q, bed slope, S, and an initial estimate of flow depth, Y. It predicts values of flow depth, Y, bulk flow velocity, U and boundary shear stresses, τ_b and τ_i. As illustrated here, the procedure uses the Einstein-

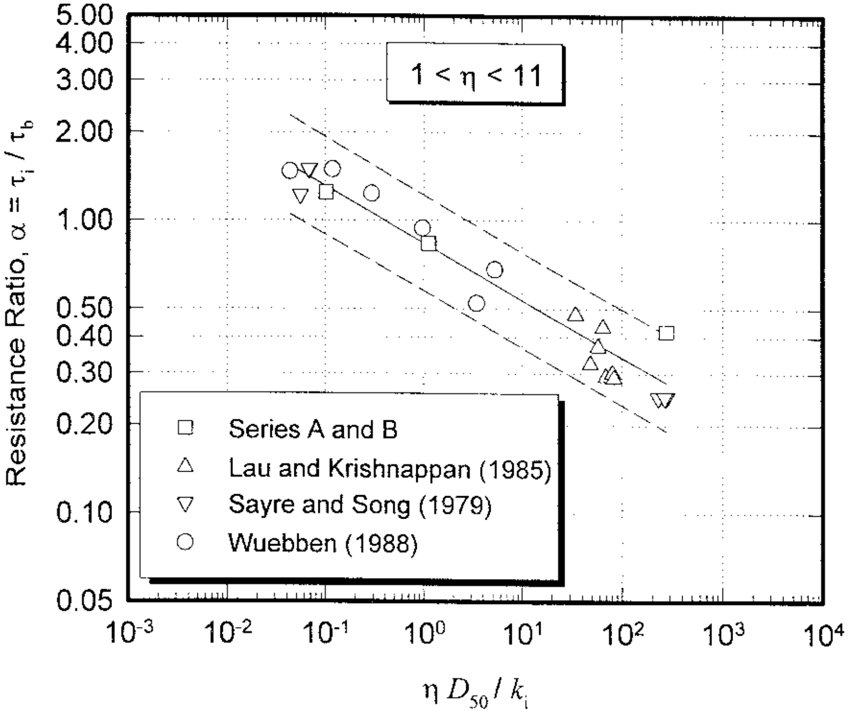

Fig. 7. Resistance Ratio, α, as a function of $\eta D_{50}/k_i$

Barbarossa method for predicting bedform resistance, but most other bedform resistance predictors can be adapted for use as well. The steps for the procedure are listed in Table 4. As the iterations proceed, the bedform resistance decreases, flat-bed resistance increases, the resistance ratio increases, and the flow depth increases. This adjustment process concurs with that observed in the loose-bed experiments.

Presently, a lack of field data prevents validation of the method for flow in an actual ice-covered river. This lack of data is surprising. The writers were unable to find a set of data that included flow stage and discharge, ice-cover characteristics (thickness, roughness) and bed characteristics (sediment size and dune topography). The development of the flow-resistance method described herein suggests the need for field investigations to obtain such data in order to develop and validate methods for calculating flow resistance in ice-covered alluvial channels.

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Table 4. Depth-discharge calculation procedure for ice-covered alluvial channels

Begin with specified values of S, q, D_{50}, ν, k_i		
Initial calculations:	$Re = q / \nu$; $k_b = 2D_{50}$	$D_* = D_{50} (\gamma_s / \rho\nu^2)^{1/3}$; $\theta_c = \varphi(D_*)$ (modified Shields curve)
Make an initial guess for flow depth, Y.		
Step 1. Compute flat-bed friction.		
1(a)	Relative roughnesses	$\varepsilon_b = D_{50} / Y$; $\varepsilon_i = k_i / Y$
1(b)	Flat-bed friction factor (Eqs. 18 and 19 of Parthasarathy and Muste, 1994)	$f_b' = \varphi(Re, \varepsilon_b, \varepsilon_i)$
Step 2. Compute bedform resistance.		
2(a)	Flat-bed Shields parameter	$\theta' = (\varepsilon_b Re)^2 f_b' / 8D_*^3$; $\psi = 1 / \theta'$
2(b)	Bedform friction factor (Einstein and Barbarossa, 1952)	$(8 / f_b'') = \varphi(\psi)$
Step 3. Compute total bed resistance	Bed friction factor	$f_b = f_b' + f_b''$
Step 4. Compute ice-cover resistance.		
4(a)	Shields parameter	$\theta = (\varepsilon_b Re)^2 f_b / 8D_*^3$
4(b)	Transport capacity	$\eta = \theta / \theta_c$
4(c)	Resistance ratio (Eq. 11)	$\alpha = 0.84 (\eta D_{50} / k_i)^{-0.20}$
Step 5. Compute channel resistance.	Total friction factor	$f = (1+\alpha) f_b$
Step 6. Update flow depth		$Y = (q^2 f / 8gS)^{1/3}$
Return to Step 1, repeating until acceptable convergence is obtained for Y.		

DISCUSSION

David Andres,

Trillium Engineering and Hydrographics Inc.:

It is gratifying to see someone grappling with quantifying changes in bed roughness under an ice cover. I think the approach is very ingenious. For many of the mobile streams that I have looked at, the grain roughness is a very small part of the overall roughness. How bad would the approximation be if one were to neglect the grain roughness in the overall formulation? Would this approximation make the technique easier to apply?

Reply: The grain roughness component of bed resistance (in other words, the surface resistance factor, f_b') should not be neglected. The overall resistance is a composite of surface and bedform resistance components; $f_b' + f_b''$. For beds with well-developed dunes and ripples, the bedform resistance, f_b'', is the dominant component, as indicated in Table 3, though the surface resistance component still contributes substantially. As the bedforms flatten and, thereby, bedform resistance diminishes, the extent of surface resistance increases. What happens is that the sheltering wakes behind bedforms get smaller and the area of bed surface exposed to streamwise flow increases. In effect, a the reduction in bedform resistance is partially compensated by an increase in surface resistance. This trend is reflected in the values of f_b' given in Table 3.

An additional consideration is that most loose-bed resistance methods divide resistance into surface and bedform components. Calculation of the bedform component usually requires determining first the surface resistance component, as is shown in Table 4 for the Einstein-Barbarossa method.