# ANCHOR ICE GROWTH IN CHANNELS

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## ABSTRACT

In turbulent river flows, anchor ice can form on the river bed. The formation of anchor ice has both physical as well as biological implications. Anchor ice can have significant effect on discharge and water level changes, since it raises the effective bed elevation and alters the bed roughness. Anchor ice can also have serious effects on invertebrates and fish, since it can block and freeze the intergravel flow in areas which are often preferred for spawning for cold water fish. This analysis shows the importance of vertical mixing, frazil size distribution as well as the geometry and distribution of roughness elements on the bed. It also provides a framework for further analytical and experimental studies.

# INTRODUCTION

Anchor ice is defined as submerged ice attached or anchored to the bed (Kivisild 1970). Formations of anchor ice have been observed in all types of rivers, ranging from shallow streams with steep slopes to deep rivers with mild slopes (Arden and Wigle 1972, Tesaker 1994, and Parkinson 1984). The existence of anchor ice has both physical and biological implications. Anchor ice can cause significant discharge and water level changes, since it raises the effective bed elevation and alters the bed roughness. It can be a major cause of hydropower production losses in the winter (Arden and Wigle 1972, and Marcott 1984). Anchor ice can also have serious effects on invertebrates and fish, since it can block the intergravel oxygen-bearing water flow into the gravel beds, and dislodge fish eggs from redds by the scouring action during its release (Prowse and Gridley eds. 1993).

Anchor ice forms on the river bed under turbulent supercooled river water. It has been observed that the growth of anchor ice is due to the accumulation of frazil ice on the bottom roughness and the in-situ growth contributed by the heat exchange betwen the supercooled river water and the bed material or deposited ice. However, a clear theoretical understanding on the anchor ice process is not available. Marcott and Robert (1986) developed an empirical simulation model for anchor ice. The growth of anchor ice was considered to be due to turbulent heat exchange betwen the supercooled river water and the bottom ice. The frazil attachment process, which has been suggested to be the most important process in anchor ice formation (Ashton ed. 1986), was neglected. Wang and Shen (1991) developed a one-dimensional simulation model for coupled frazil and anchor ice evolution in rivers. Both heat exchange and frazil attachment processes were considered. However, the vertical mixing in the flow was not explicitly formulated. and the detailed mechanism of frazil attachment was not examined. Hammar and Shen (1994) proposed a theoretical model for frazil ice accumulation on rough channel beds. They used the customary logarithmic law with a displacement height for the velocity profile outside the roughness layer (Jackson 1981). Inside the roughness layer, an empirical formulation for velocity profile derived for uniform rod-like roughnesses was used. The equivalent grain roughness parameters in these formulas cannot be directly determined from the bed roughness geometry. In this paper, the work of Hammar and Shen (1994) is refined by introducing a new formulation for velocity distribution in rough channel flow, which can easily be related to the size distribution of the bed material.

## PROBLEM FORMULATION

The attachment of frazil ice to channel bed is a process of particle deposition from a turbulent stream to a rough boundary. Besides the frazil particle characteristics, the deposition process is governed by the flow velocity profile and turbulent mixing. Deposition of fine particles from a turbulent gas stream to a solid boundary has been studied extensively (Papavergos and Hedley 1984). These studies are limited to boundaries with small or intermediate roughness heights. In this study, a formulation for frazil deposition in rough open channel flows will be developed. The formulation considers the frazil accumulation on the bed roughness as a result of the turbulent diffusion of frazil crystals from the turbulent core to the roughness layer and the attachment of frazil inside the roughness layer.

### Velocity Profile in a Rough Open Channel Flow

The velocity distribution and the associated turbulent mixing govern the transport and deposition of frazil ice. The commonly used logarithmic velocity profiles with an equivalent grain roughness and a displacement height (Jackson 1981, Song et al. 1994) do not directly relate to the roughness geometry, such as size, shape, and distribution. These formulations are also limited to conditions where the bed roughness height is relatively small comparing with the flow depth. In this study a generalized formulation for velocity distribution is developed based on a discrete element approach. This formulation can directly relate the velocity distribution to the roughness geometry and the roughness height is not limited. Such a formulation is more suitable for analyzing the anchor ice phenomenon, which often occurs in shallow streams with large poorly sorted roughness elements. It also has the advantage of being more adaptable to the changing bed roughness during the anchor ice evloution. A similar formulation was developed by Wiberg and Smith (1991), which appears to have errors.

By applying the momentum balance to a small control volume in the flow containing roughness elements with defined statistical distribution of size and geometry, the vertical velocity gradient can be obtained as:

$$\frac{\delta u}{\delta y}=\frac{u_*}{L}\left[1-\frac{\sum_{m=i}^{M}\left(\frac{3}{4}\rho C_{Dm}C_m/D_{xm}\right)\int_{y}^{D_{ym}}u\,dy}{\rho gHS(1-y/H)-\rho gS\sum_{m=i}^{M}C_m(D_{ym}-y)}\right]^{1/2} \tag{1}$$

where H is the total flow depth; u is the flow velocity; S is the channel slope; ρ is the water density; g is the gravity; m is an index for roughness size classes; $C_{Dm}$ is the drag coefficient for bed particles in the mth size class; $C_m$ is the area concentration of the mth size bed particles; $D_{xm}$ and $D_{ym}$ are the bed particle diameters in the downstream and vertical directions, respectively. The friction velocity $u_*$ may be expressed as

$$u_*=\left[gHS(1-y/H)-gS\sum_{m=i}^{M}C_m(D_{my}-y)\right]^{1/2} \tag{2}$$

and the length scale L can be expressed as

$$L=(1-C_s)\kappa y(1-y/H)^{1/2}+\kappa\sum_{m=i}^{M}C_mD_{zm} \tag{3}$$

in which, $C_s$ is the area concentration of bed particles at level y; κ is the von Karman constant; and $D_{zm}$ is the bed particle diameter in the transverse direction. This formulation gives the velocity profile from a selected bed level y = 0, where the flow is insignificant, to the free surface. In the free stream, this velocity profile reduces to the traditional logarithmic velocity profile.

## Vertical Mixing Outside the Roughness Layer

For a uniform open channel flow, assuming the longitudinal concentration gradient is negligible in relation to the vertical gradient, the vertical distribution of frazil concentration is governed by the following diffusion equation.

$$-\omega_b C(y) + \varepsilon_p \frac{dC(y)}{dy} = 0 \tag{4}$$

in which, C(y) = frazil concentration; $\varepsilon_p$= diffusion coefficient; $\omega_b$= buoyant velocity of frazil. The diffusion coefficient can be assumed to be the same as the eddy difussivity of the water, since the density difference betwen water and ice is small. Eq. 4 can be integrated with the boundary condition at y = h, the edge of the roughness layer.

$$C = C_h, \quad at \ \ y = h \tag{5}$$

Equations 4 and 5 can be nondimensionalized by introducing the following dimensionless variables:

$$[y^+; C^+; u^+; \omega_b^+; h^+; \varepsilon_p^+] = [y/H; C/C_h; u/\overline{U}; \frac{6\,\omega_b}{\kappa u_*}; h/H; \frac{6\,\varepsilon_p}{\kappa u_* H}] \tag{6}$$

in which, H = flow depth, U = depth-averaged velocity, and $u_*$ = shear velocity. The dimensionless form of Equations 4 and 5 are:

$$-\omega_b^+ C^+(y^+) + \varepsilon_p^+ \frac{dC^+(y^+)}{dy^+} = 0 \tag{7}$$

with

$$C^{+}=1 \quad at \quad y^{+}=h^{+} \tag{8}$$

The diffusion coefficient can be obtained from the logarithmic profile using the Reynolds' analogy as

$$\varepsilon^{+}(y^{+})=6y^{+}(1-y^{+}) \tag{9}$$

This expression leads to $\varepsilon^{+}(y^{+}) = 0$ at the free surface. However, due to surface renewal, one could reason that a non-zero downward mixing should exist at the free surface. Eq. 9 is therefor modified to

$$\varepsilon_{p}^{+}(y^{+})=6y^{+}(1-y^{+})+\varepsilon_{s}^{+}y^{+} \tag{10}$$

in which, $\varepsilon_s^{+}= 6\varepsilon_s/\kappa u_* H$; and $\varepsilon_s^{+}$ = nondimensionalized eddy diffusivity at the free surface.

The solution for Eqs. 8 and 9 becomes

$$C^{+}(y^{+})=\left(\frac{y^{+}}{h^{+}}\left(\frac{6(1-h^{+})+\varepsilon_{s}^{+}}{6(1-y^{+})+\varepsilon_{s}^{+}}\right)\right)^{\frac{\omega_{b}^{+}}{6+\varepsilon_{s}^{+}}} \tag{11}$$

Equation 11 shows that the frazil concentration distribution is a power function governed by the buoyant velocity and surface eddy diffusivity. An increase in buoyant velocity will shift the concentration toward the water surface, while an increase in the surface eddy diffusivity will enhance the downward mixing.

Since the surface eddy diffusivity is relatively small, the buoyant velocity plays a dominate role. Figure 1. shows dimensionless profiles of frazil concentration outside the roughness layer, assumming $h^{+} = 0.1$. This shows the significance of the buoyant velocity and hence the importance of frazil size distribution in anchor ice developement.

Based on Eq. 11, the concentration at the boundary of the roughness, y = h, can be related to the depth-averaged concentration, $\overline{C}$ , as

$$C(h) = \frac{\overline{C}}{\int_{h^{+}}^{1} C^{+}(y^{+})\, U^{+}(y^{+})\, dy^{+}} \tag{12}$$

This expression can be integrated numerically. The small amount of frazil inside the roughness layer is neglected in Eq. 12.

## Transport and Deposition Inside the Roughness Layer

Inside the roughness layer the situation is more complex. The rate of deposition of frazil on the roughness elements is dependent on the frazil concentration as well as the geometry and distribution of the roughness elements. The diffusion equation may be written as

$$\frac{d}{dy}\left(-\omega_{b} C(y) + \epsilon_{r}(y)\, \frac{dC(y)}{dy}\right) = \beta\, (y)\, u(y)\, C(y) \tag{13}$$

The removal of frazil by the roughness elements is represented by the sink term, on the R.H.S., in Eq. 13. The coefficient $\beta$ may be related to the size, density, and removal efficiency of the roughness elements (Lee and Liu 1982). Since the concentration decreases rapidly in the roughness layer due to accretion, the boundary conditions for Eq. 13 are: $C = C_{h}$, at $y = h$; and $C = 0$

at y = 0. The roughness layer eddy diffusivity may be obtained from a numerical solution of Eq. 1 as

$$\varepsilon_r=\frac{u_*^2}{\delta u/\delta y} \tag{14}$$

Equations 13 can be solved numerically to give the concentration profile inside the roughness layer. The rate of deposition of frazil per unit thickness of the roughness layer, which varies in the vertical direction, can be determined from

$$S=\beta\,(y)\,u(y)\,C(y) \tag{15}$$

This shows that the deposition rate, $\dot{m}$ , is proportional to the coefficient, $\beta$.

The total deposition rate over the depth of the roughness layer becomes

$$\dot{m}=\int_0^h \beta\,(y)\,u(y)\,C(y)\,dy \tag{16}$$

The above formulation indicates that the anchor ice growth due to frazil attachment on the bed is affected by the intensity of vertical mixing of the flow, the size of frazil particles, and the size and geometrical distribution of the bed roughness elements. Since the bed roughness changes with the growth of the anchor ice, the process of frazil accretion is a transient phenomenon. In the following section, simple numerical examples assuming constant bed roughness geometry will be given to illustrate the application of the above theoretical analysis.

## NUMERICAL EXAMPLES - CALCULATION PROCEDURE

The actual calculation procedure goes as follows:

1. The velocity profile is numerically calculated from Eq. 1.

2. With this knowledge the free stream frazil distribution, Eq. 11, can be

solved.

3. Given the value on the mean frazil concentration and size distributrion, the frazil concentration on the edge of the roughness layer, $C_h$, is determined.
4. The frazil distribution within the roughness layer, Eq. 13, may now be numerically solved as sufficient number of boundary conditions are defined.
5. With known frazil and velocity distributions within the roughness layer, the frazil deposition rate - Eq. 16, may be solved.

Numerical examples are made to illustrate the solution of Eq. 13, and the effect of surface eddy diffusivity and particle size. Calculation results are summarized in Table 1.

Table 1. Parameters and calculation results

| | ---------- Steep slope | | | ------------------ | Mild slope --- | |
|---|---|---|---|---|---|---|
| Slope, S | 0.005 | 0.005 | 0.005 | 0.005 | 0.0001 | 0.0001 |
| Flow depth, H (m) | 1 | 1 | 1 | 1 | 1 | 1 |
| Roughness, $D_y$ (m) | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| Mean Conc, $\bar{C}$ | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| Buoyant vel, $\omega_b^+$ | 0.001 | 0.001 | 0.001 | 0.005 | 0.001 | 0.005 |
| Friction vel, $u_*$ (m/s) | 0.22 | 0.22 | 0.22 | 0.22 | 0.07 | 0.07 |
| Surf eddy diff, $\varepsilon_s^+$ | 0.15 | 0.015 | 0.0015 | 0.15 | 0.15 | 0.15 |
| Mean velocity, U (m/s) | 1.3 | 1.3 | 1.3 | 1.3 | 0.69 | 0.69 |
| $C_h/\bar{C}$ | 0.97 | 0.97 | 0.97 | 0.87 | 0.86 | 0.46 |
| Ice growth rate (m/h) | 0.0195 | 0.0195 | 0.0195 | 0.017 | 0.0056 | 0.0030 |

All of the calculations are based on a constant $\beta$ value of $3*10^{-3}$. This value is only chosen as to give reasonable calculation results on the accretion. In reality, β is a function of roughness distribution and geometry. To make it even more complicated, β also varies with time as the anchor ice growth or decay changes the roughness pattern. The value of the coefficient β can only be quantified by well-planned laboratory experiments and field studies.

Another point to be noted is that the accretion is almost independent of the choice of surface eddy diffusivity. This can also be understood by analyzing Figure 2, where the frazil distribution in the free stream is plotted for different surface eddy diffusivities. As can be seen the effect of the surface eddy diffusivity on frazil conentration distribution is confined in a thin layer near the free surface and does not significantly change the concentration at the edge of the roughness layer, which actually governs the deposition rate. The buoyant velocity, and inherently the frazil size distribution, is another factor that strongly influences the deposition rate. This is because of the fact that the larger particles tend to be concentrated closer to the surface, and less frazil will be transported into the roughness layer for anchor ice growth.

## CONCLUSIONS

A theoretical analysis on the process of anchor ice growth due to frazil deposition is made based on a new velocity profile formulation. The analysis shows that the rate of frazil deposition on the bed is governed by the intensity of vertical mixing of the flow, the efficiency of the bed roughness to "catch" frazil, and the size distribution of the frazil suspension. The analysis improved the earlier analytical formulation by explicitly explaining the influences of the distributions of bed particle size and geomentry.

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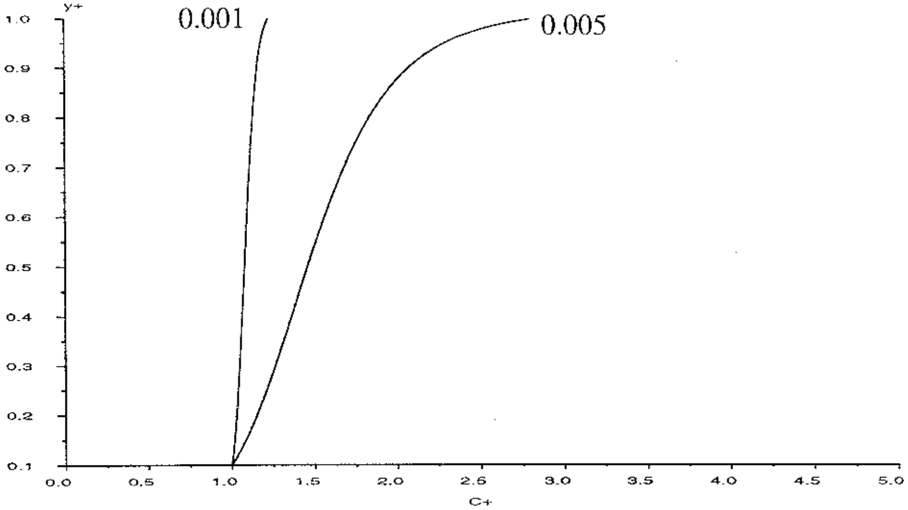

Fig 1. Non-dimensionalized concentration profiles for two different buoyant velocities.

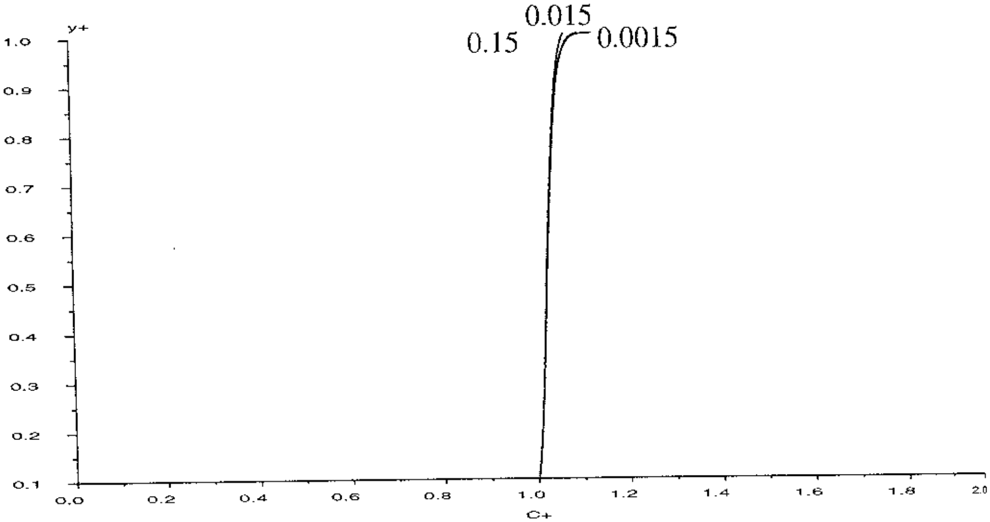

Fig 2. Non-dimensionalized concentration profiles for 3 different surface eddy diffusivity values.

## DISCUSSION

David Andres
Trillium Engineering and Hydrograph. Inc.

Given the natural buoyancy of the frazil particles, what is the minimum turbulence level required to transport them to the bottom of the stream? Since the turbulence is a function of the shear velocity, is there some limiting flow depth and/or slope for which anchor ice will not form?

Reply: The distribution of frazil in the flow is governed by Eq. 7 in the paper. As can be seen this equation constitutes a balance betwen uplifting (buoyancy) and turbulent mixing (eddy diffusivity). Solving this equation will lead to Eq. 11 which gives the non- dimensional frazil distribution.Theoretically, Eq. 11 says that given the existence of frazil in the flow there will always be frazil present at the edge of the roughness layer. In reality, there are of course limiting factors. One possibility is the heat flux from the bottoms (4-6 W/m$^2$) ability to melt the frazil ice to a certain extent, other sources migth be the heat generated by turbulent dissipation or short-wave radiation. In accordance to what is stated above, I can presently only say that anchor ice growth will decrease with increasing depth and decreasing slope but there are obvious possibilities to define certain "criterion" for growth or non growth.

S Beltaos
National Water Research Institute

The predicted bottom concentrations of frazil ice are comparable in magnitude to the surface concentration. Measurements indicate that the bottom concentration should be much smaller than the surface one (e.g. experiments by Dr. G. Tsang). Could you comment on this.

Reply: From Fig. 1, it can be seen that the concentration profile is strongly dependent on the frazil size distribution. In the presented calculations only single size, small particles are used (e.g. low buoyant velocity). If larger particles or conglomerats of particles should have been used the results should have looked a lot more like you have expected. In Dr Tsang's experiment the particles probably were fairly "old" and the concentration was fairly high (which promotes collissions and flocculation). This all together skews the concentration profile to the water surface. So in fact I can't see any contradictions between his observation and our.

K.S. Davar
Univ of New Brunswick

It seems the statistical distribution approach for assesing roughness of bed particles may be unduly involved and give an impression of rigour which is not justified for practical applications due to:

- bed particles are irregular
- roughness exposure is only for part of the element
- the lodgement is very random

Also the bed topography could have a greater effect on resistance than the particle size. Assesing river bed characteristics by field examinations and inspections of velocity profiles could lead to simpler and realistic assesment of the required bed resistance characteristics.

Reply: What has been shown in the paper is the dependancy of anchor ice growth on a local description of the roughness layer. In order to determine the deposition rate one must know the velocity distribution within the roughness layer. This velocity distribution was approximated in the customary lumped-parameter formulations, which cannot provide the information in the roughness layer that is needed for the anchor ice study. The presented approach takes into account the irregularities of the bed roughness as well as the random distribution. We agree that the presented approach appears to be somewhat cumbersome and tedious. However, statistical descriptions of gravel bed particles have been successfully used in analyzing flow resistance. It's application to anchor ice study should not be prohibitive. The bed topography is, as pointed out, an important factor affecting the flow resisrance, although I tend to believe that the topography exposes itself mostly into a large scale eddy diffusivity, and thus could be treated accordingly.

M Milko
DFO/Habitat Management

I'm no engineer, but a biologist. Should we be concerned with variable depth of a real stream/river? In small rivers the depth is highly variable, dunes, riffles can change depth/velocity profiles. Will the predictive formula still work? Or will the system break down? How would it react even with uniform particles.

Reply: In principle this is an averaged formulation both when it comes to the roughness description, depth and slope. All descriptions are based on average values and accordingly if your description is averaged in a good manner then the formulation should do.

T.D. Prowse
NHRI

What would be the major physical effects, and their significance to your modelled results for "steep turbulent" and "ordinary" rivers, of variable roughness height?

Reply: Introducing a complete size distribution of roughness heights would alter the velocity profile within the roughness layer as well as the connection point and level of velocity on the edge of the roughness layer. A change of the roughness description thus would change the deposition rate as well as the deposition distribution on the roughness with respect to level. In a roughness layer with uniform roughness elements the velocity profile tends to be fairly linear. In principle, adding a complete size distribution of roughness elements would make the roughness appear denser closer to the bottom and conseguently reduce the velocity in the bottom region of the roughness layer. This would then lead to a higher deposition rate in the upper part of the roughness layer. The change of roughness on the other hand would also change the complete concentration profile and thus the value on the frazil concentration on the upper roughness boundary, C(h). This would then change the amount of frazil available for deposition. To sum it up, the entire calculation is fully coupled and based on the roughness description. Any change in the roughness description would change the velocity profile and consequently the frazil concentration profile and the deposition rate.