FRAZIL TRANSPORT AND EVOLUTION IN CHANNELS

L. Hammar[1] and H.T. Shen[2]

[1]Lulea University of Technology
Swedish Electric Utility Research and Development Co.
Stockholm, Sweden
[2]Department of Civil and Environmental Engineering
Clarkson University, Potsdam, NY, USA

ABSTRACT

A mathematical model for formation and evolution of frazil ice in channels is developed. In the simulation, the primary nucleation is assumed to be due to the mass exchange of seeding crystals at the free surface. The growth of frazil crystals is simulated based on the rate of heat transfer between individual crystals and the ambient turbulent flow. Secondary nucleation and flocculation are simulated based on binary collisions of frazil particles.

The model is validated with experimental data. It is capable of simulating the variation of water temperature during the frazil formation period and the evolution of size and concentration distributions of the frazil in the flow. Effects of the surface heat exchange rate, the rate of seeding, and the flow condition are examined.

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INTRODUCTION

Frazil ice is the most important form of river ice during the freeze-up period (Ashton, 1986). It is the origin of almost all other forms of river ice. The formation of frazil ice can lead to the surface ice run, which eventually develops into the ice cover. Frazil granules that are entrained to the underside of the ice cover may accumulate into hanging dams. Frazil crystals suspended in the turbulent supercooled water can attach to the bed material to contribute to the growth of anchor ice. All of these processes can have significant impacts on physical, chemical, and biological conditions of the river. Although the process of frazil evolution is not completely understood, it is generally accepted that frazil ice forms in supercooled turbulent river water. The presence of seeding crystals is also required. Once seeded, ice crystals will grow both in size and number. The growth of a frazil crystal is governed by the heat transfer between the ice crystal and the supercooled turbulent water. The multiplication of crystals is due to secondary nucleation. The water temperature variation is the result of the dynamic balance between the latent heat released by ice crystals and the heat exchanges at the water surface and the channel bed.

Mathematical models have been used to simulate frazil and grease ice formation in surface waters, Omstedt and Svensson (1984), and Omstedt (1985a,b) developed a series of numerical models for the formation of frazil ice and grease ice in the upper layers of the ocean. The vertical distributions of ice concentration and temperature are simulated. In these models, the ice crystals are considered to have a constant size, with a constant heat exchange rate between the water and ice crystals. The secondary nucleation is assumed to be directly related to the thermal growth of ice crystals. The flocculation of frazil crystals are modelled, but the effect of differential rising was neglected, Nyberg (1986) followed the model of Omstedt (1985b) developed a two-dimensional model for river channels. Mercier (1984), extended the work of Daly (1984), formulated a kinetic model of frazil growth. Mercier (1984) simulated frazil formation in channels using a Monte Carlo technique. Hammar and Shen (1991) developed a two-dimensional model for frazil formation in channels by generalizing Omstedt's formulation to variable crystal sizes. Distributions of water temperature, ice concentration, and crystal size are examined. In this paper, the model of Hammar and Shen (1991) is modified and extended to include kinetics of secondary nucleation and flocculation (Daly 1984, Mercier 1984). The present model simulates the variation of water temperature and the evolution of frazil size and concentration. The frazil formation is initiated by seeding small ice nuclei on the water surface. Secondary nucleation and flocculation are simulated based on collision of frazil particles. The model is validated with existing flume experiment data. Effects of surface heat exchange, seeding rate, rate of secondary nucleation, and the flow condition are examined.

MODEL FORMULATION

The present mathematical model uses the standard k-ε turbulence formulation to simulate the flow condition. A general equation solver PHONECS for heat and mass transfer problems (Spalding 1981) is used to solve the governing flow and transport equations. The frazil particles are assumed to be thin circular disks with a constant ratio of 1/10 between the thickness and diameter, based on the flume data obtained by Daly and Colbeck (1986). The size distribution of frazil particles is described by logarithmically spaced size groups. Seeding crystals as well as those produced by secondary nucleation are assumed to be in the lowest size group.

Mean Flow Equations

The governing equations for the mean flow, frazil concentration, and water temperature are:

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = - \frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} (\nu_t \frac{\partial U_i}{\partial x_j}) + g_i \tag{1}$$

$$\frac{\partial T}{\partial t} + U_i \frac{\partial T}{\partial x_i} = \frac{\partial}{\partial x_i}(\frac{\nu_t}{\sigma_T} \frac{\partial T}{\partial x_i}) - S_b + S_f \tag{2}$$

$$\frac{\partial C_k}{\partial t} + U_i \frac{\partial C_k}{\partial x_i} = \frac{\partial}{\partial x_i} (\frac{\nu_t}{\sigma_c} \frac{\partial C_k}{\partial x_i}) - \omega_k \frac{\partial C_k}{\partial x_3} + S_{Ck} + S_{floc,k} \tag{3}$$

in which, ρ is density of the mixture, $\rho_w + (\rho_i - \rho_w) \Sigma_k C_k$; U_i is the ith component of the mean velooity; P is the mean pressure, ν_t is the kinematic eddy viscosity; g_i is the ith gravity component; σ_T and ν_c are Prandtl/Schmidt numbers; T is water temperature; C_k is the volumetric concentration of frazil in the kth size fraction; ω_k is the frazil buoyant velocity of kth size fraction; S_b and S_f are source terms due to heat loss at the channel boundaries and the latent heat released from frazil growth; S_C is the source term due to the thermal growth of frazil; and S_{floc} is the source/sink term due to secondary nucleation and flocculation.

FRAZIL EVOLUTION

The frazil ice size distribution is divided into eight logarithmically spaced size groups. The sizes of these groups range from 4μm to 1.435 mm. Seed crystals and secondary nuclei are assumed to be in the lowest size group. In the following sections the formulation for frazil evolution including thermal growth, secondary nucleation, flocculation, and the treatment of source/sink terms will be discussed. Turbulent shear can cause flocculated particles to break up. However, since particle breakup is limited to large, weakly bounded aggregates under high shear (Mercier 1984), particle breakup will not be considered.

Thermal Growth of Frazil Crystals

The rate of growth of an ice crystal depends on the rate of transfer of latent heat from the crystal to the ambient turbulent flow. The rate of heat transfer per unit ice surface area, q, can be expressed in terms of the heat transfer coefficient, h, as:

$$q = h(T_i - T_w) \tag{4}$$

where T_i is the ice surface temperature and T_w is the ambient water temperature. The heat transfer coefficient, h, can be expressed in its dimensionless form as a Nusselt number defined by:

$$N_u = \frac{hl}{k_w} \tag{5}$$

where l is a characteristic length of the ice crystal, and k_w is the thermal conductivity of water. In this study, the face radius calculated based on the surface area of the ice crystal, A_s, i.e. $(A_s/4\pi)^{1/2}$, is used as the characteristic length. Combining Eqs. 4 and 5 gives the following equation for the rate of heat transfer:

$$q = \frac{N_u k_w}{l} (T_i - T_w) \tag{6}$$

The Nusselt number depends on the flow condition and the particle size. In this study, the following formulation developed by Batchelor (1980) and Wadia (1974), as summarized by Daly (1984) is used:

$$N_u = (\frac{1}{m^*}) + 0.17P_r^{1/2} \quad ; \textit{if } m^* < \frac{1}{(P_r)^{1/2}} \tag{7a}$$

and,

$$N_u = [(\frac{1}{m^*}) + 0.55(\frac{P_r}{m^*})^{1/3}] \quad ; \textit{if } \frac{1}{(P_r)^{1/2}} < m^* < 10 \tag{7b}$$

in which, $m^* = r/\eta$, is the ratio between the face radius of the ice crystal and the Kolmogorov length scale.

For large particles, i.e. $m^* > 1$;

$$N_u = 1.1[(\frac{1}{m^*}) + 0.80\alpha_T^{0.035} (\frac{P_r}{m^*})^{1/3}]; \textit{ if } \alpha_T m^{*4/3} < 1000 \tag{8a}$$

and

$$N_u = 1.1[(\frac{1}{m^*}) + 0.80\alpha_T^{0.24} P_r^{\frac{1}{3}}]; \textit{ if } \alpha_T m^{*\frac{4}{3}} > 1000 \tag{8b}$$

in which, $\alpha_T = \sqrt{2k}/U$, is the turbulence intensity. In the transition region, i.e. $1 < m^* < 10$ the variation of N_u is not well defined. In this study, Eq. 8 is used when $m^* > 1$. It is of interest to note that when m^* increases, the N_u decreases. Therefore, the thermal growth rate of frazil particles decreases rapidly with the increase in particle size.

Secondary Nucleation

The model for the kinetics of secondary nucleation developed by Evans et al. (1974) and Mercier (1984) is adopted. This formulation considers that the collision breeding is the primary mechanism for secondary nucleation. The number of nuclei produced due to collisions between particles in size classes v_i and v_j is:

$$I(v_i,v_j) = \int_{v_{i-1/2}}^{v_{i+1/2}} \int_{v_{j-1/2}}^{v_{j+1/2}} Z C_E(v_i,v_j)\, dv_i\, dv_j \qquad \textbf{(9)}$$

where $I(v_i, v_j)$ is the number of nuclei produced per unit volume per unit time, v_i and v_j are volumetric sizes of the colliding particles, Z is the number of nuclei produced per unit collision energy and, $C_E(v_i, v_j)$ is the rate of collisional energy transfer to the crystals per unit volume. The parameter Z may be considered as a material constant for the small supercooling level in rivers, when, the impurity concentration is low. The function $C_E(v_i, v_j)$ can be expressed in terms of collision frequency and collision efficiency. The main contributors to these in open channel flows are turbulent shear and differential rising. The collision frequency function is the probability that two non-interfering particles of sizes v_i and v_j will collide in a unit time. For turbulent shear this is (Mercier 1984):

$$\beta_{sh}\,(v_i,v_j) = 0.39\,\frac{<\varepsilon>^{\frac{1}{2}}}{\nu^{\frac{1}{2}}}\,(v_i^{\frac{1}{3}} + v_j^{\frac{1}{3}})^3 \qquad \textbf{(10)}$$

The collision frequency function due to differential rising is (Findheisen 1939):

$$\beta_{dr}(\nu_i, \nu_j) = 0.1\frac{g}{\nu}\frac{|\rho_s - \rho_f|}{\rho_f}|\nu_j^{\frac{2}{3}} - \nu_i^{\frac{2}{3}}|(\nu_j^{\frac{1}{3}} + \nu_i^{\frac{1}{3}})^2 \quad \textbf{(11)}$$

Combining these equations, noting that the collisions are assumed inelastic, under conservation of momentum and energy yields:

$$C_E(\nu_i, \nu_j) = \frac{1}{2}\rho_i\frac{\nu_i\nu_j}{\nu_i + \nu_j}[b(\nu_i^{\frac{1}{3}} + \nu_j^{\frac{1}{3}})^5(\frac{<\varepsilon>}{\nu})^{\frac{3}{2}}E_{sh}$$
$$+ 0.00076(\frac{g}{\nu}\frac{(\rho_f - \rho_i)}{\rho_f}|\nu_j^{\frac{2}{3}} - \nu_i^{\frac{2}{3}}|)^3(\nu_j^{\frac{1}{3}} + \nu_i^{\frac{1}{3}})^2 E_{dr}]g(\nu_i)\,g(\nu_j) \quad \textbf{(12)}$$

where $b = 0.0066/K_u^{3/4}$, K_u = kurtosis of the velocity derivative (Mercier 1984), and $g(\nu_i)$ is the number density functions. The collision efficiency functions for differential rising and turbulent shear E_{dr} and E_{sh}, are introduced to account for particle interference effects. These functions are evaluated according to the procedure outlined by Pearson et al. (1984). The value of E_{dr} decreases when the difference in particle size increases.

Flocculation

The instantaneous expected number of collisions between all particles in the ith size class and jth size class per unit volume per unit time is

$$F_n = \beta(\nu_i,\nu_j)\,E(\nu_i,\nu_j)\,\phi_i\phi_j \quad \textbf{(13)}$$

in which, ϕ_i and ϕ_j = the number concentration of ith and jth-size particles. Each collision per unit volume will reduce the local number concentration of i- and j-particles each by one. The colliding particles will merge into a new particle of volume $\nu_{merge} = \nu_i + \nu_j - n\nu_1$, where $n\nu_1$ is the volume contributed to secondary nuclei production. The size ν_{merge} will not correspond exactly to the prescribed size groups, but will fall between ν_k and ν_{k+1}. In the simulation scheme, the merged particles will be distributed to the two neighboring size groups k and k+1. Based on the volume conservation method of Lawler, et al. (1980), a fraction

$$f = \frac{v_{k+1} - v_{merge}}{v_{k+1} - v_k} \tag{14}$$

of F_n is assigned to the kth size group, and a fraction (1-f) is assigned to the (k+1)th size class. When the merged particle size is larger than the size of the last size class m, then the fraction

$$f = \frac{v_{merge}}{v_m} > 1 \tag{15}$$

of F_n is assigned to the mth size class.

Source Terms

The source term, S_f, due to frazil growth, in Eq. 2, and the corresponding source term, S_C, in Eq. 3 can be determined from the heat transfer between the ice crystal and the ambient turbulent flow. The term S_f can be written as:

$$S_f = \sum_k C_k q_k (d_{fk} \rho_w C_p)^{-1} \tag{16}$$

where, d_{fk} is the mean face diameter of particles in the kth size group. Correspondingly, the rate of ice production per unit volume due to the thermal growth of ice mass in the kth size group is

$$S_k = 4 C_k q_k (d_{fk} \rho_f L)^{-1} \tag{17}$$

where, L is the latent heat of fusion of ice. The net increase in ice concentration of the kth size group due to thermal growth and flocculation then becomes

$$S_{Ck} = (\frac{S_{k-1}}{\Delta v_{k-1}} - \frac{S_k}{\Delta v_k})\, v_k \qquad (18)$$

in which $\Delta v_k = v_{k+1} - v_k$. The term on the right hand side of Eq. 18 represents the increase in C_k, contributed by the growth in the (k-1)th size group, and the decrease in C_k due to growth in the kth size group.

MODEL VERIFICATION

The present model is validated against experimental data of Carstens (1966). The experiments were conducted in a racetrack shape recirculating flume in a cold room. The flow, which was driven by a propeller, has a cross section of 20 cm by 20 cm. The tests were performed at a temperature of about -10°C in the cold room. Water temperatures were measured at a point located at 5 to 10 cm below the water surface. Due to the mixing effect of the propeller, the vertical temperature gradient was found to be negligible. Because of this, the well-mixed condition will be imposed in the numerical model for calibrations with the experimental data. The turbulence parameters are calculated from the measured flow conditions.

Two cases presented by Carstens are calibrated. The first case corresponds to Case A in Carstens' Figure 6. The second case corresponds to Figure 7 of Carsten's paper. Table 1 summarizes flow parameters and heat loss rates of these experiments.

Table 1. Parameters of calibration simulations

Case	U, m/s	$\overline{k}$, m^2/s^2	$\overline{\varepsilon}$, m^2/s^3	Q, J/m^3-s
1	0.50	0.00096	0.0012	1,400
2	0.33	0.00048	0.00038	600

In Table 1, the heat loss rate, Q, is estimated from the initial slope of the water temperature curve. The turbulence parameters are estimated from the observed flow data. Assuming smooth boundaries, the friction factor, f, can be determined from

in which $R_e = 4UR/\nu$, and R = hydraulic radius. The shear velocity $u^* = U(f/8)^{1/2}$.

$$\sqrt{f} = 2log(R_e f^{\frac{1}{2}}) - 0.8 \quad (19)$$

Assuming the vertical distribution of the energy dissipation rate is

$$\varepsilon(y) = \frac{u_*^3}{\kappa y}(1 - \frac{y}{h}) \quad (20)$$

The depth-averaged value $\overline{\varepsilon}$ can be determined from Eq. 17 by assuming the thickness of the viscous sublayer equals to ν/u_*. The turbulent kinetic energy can be approximated by

$$k(y) = \frac{u_*^2}{0.3}(1 - \frac{y}{h}) \quad (21)$$

The parameter Z and the initial seeding rate I_0 are calibrated to be 3 x 10^{15} nuclei/J and 10^4 nuclei/m^3-s.

Simulation Results

Figures 1 and 2 compares the simulated and measured water temperatures for both cases. These comparisons confirm the validity of the simulation model. Figure 3 shows the evolution of particle size distribution. A comparison of these figures shows initially the relative concentration of large particles increase continuously when the water temperature is steadily decreasing. The increase in large particles is due partly to the rapid rate of thermal growth of small particles, and partly to flocculation. As the particle concentration increases, especially for larger particles, a massive increase in secondary nuclei emanating from particle collisions. This leads to the rapid increase in small particles and the recovery of water temperature. Thereafter, the concentration of small size particles continue to increase mainly due to secondary nucleation, while concentration of large size particles continue to increase through thermal growth and flocculation of smaller particles. The water temperature remains essentially constant as a result of the balance of heat loss through the channel boundaries and the latent heat released from the growth of ice particles.

SENSITIVITY ANALYSIS

To examine the sensitivity of the process to various model parameters, additional simulations are performed for Case 1. Figure 4 shows the effect of the heat loss rate. As expected, a larger heat loss rate leads to a higher degree of supercooling and a faster temperature recovery. Figure 5 shows the effect of seeding rate. A larger seeding rate increases the number of small particles and hence faster water temperature recovery. Figure 6 shows the effect of dissipation rate. The dissipation rate reflects the level of turbulent intensity, which directly affects the collision frequency, as can be seen from Eq. 10. A higher dissipation rate leads to a higher rate of secondary nucleation and hence a faster rate of water temperature recovery. Figure 7 shows the effect of the parameter Z. A larger Z value corresponds to a higher rate of collisional breeding, and a faster recovery of water temperature.

CONCLUSION

In this paper, a mathematical model for formation and evolution of frazil ice in channels is developed. The accompanying variation of water temperature is also simulated. In the model the frazil formation is initiated by mass exchange of seeding crystals at the water surface. The model considered the distribution of frazil particle size in the turbulent channel flow. The size distribution is a result of the thermal growth and flocculation of frazil particles. Secondary nucleation and flocculation are modelled by considering binary collisions between frazil particles. The model is validated with laboratory data. The simulated results illustrate processes involved in the evolution of frazil ice in turbulent channel flows. Effects of the surface heat loss rate, the rate of seeding, the flow condition, and secondary nucleation parameters are examined. The model can be used together with well controlled laboratory experiments to give more insights to the frazil evolution process.

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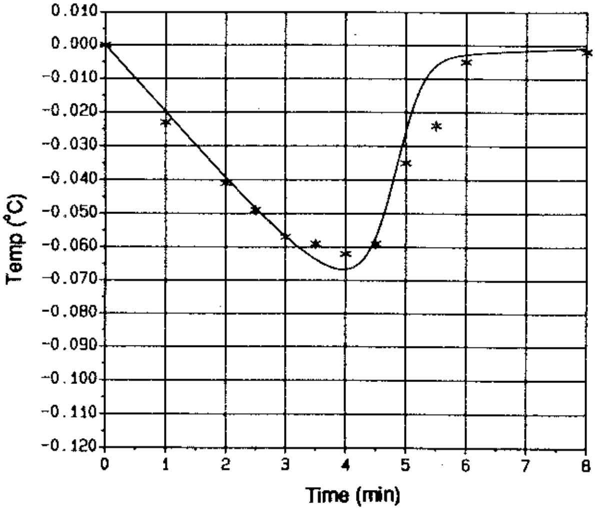

Fig. 1 Evolution of water temperature, Case 1.

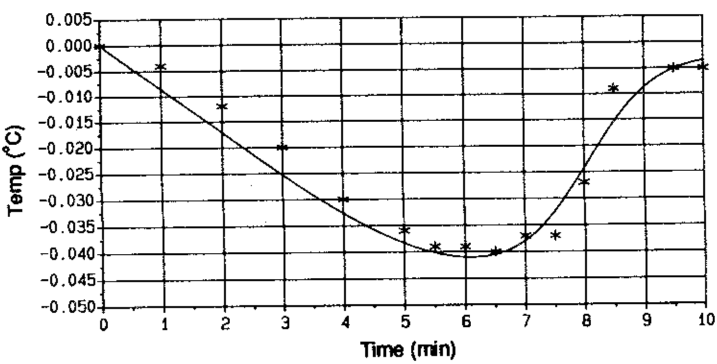

Fig. 2 Evolution of water temperature, Case 2.

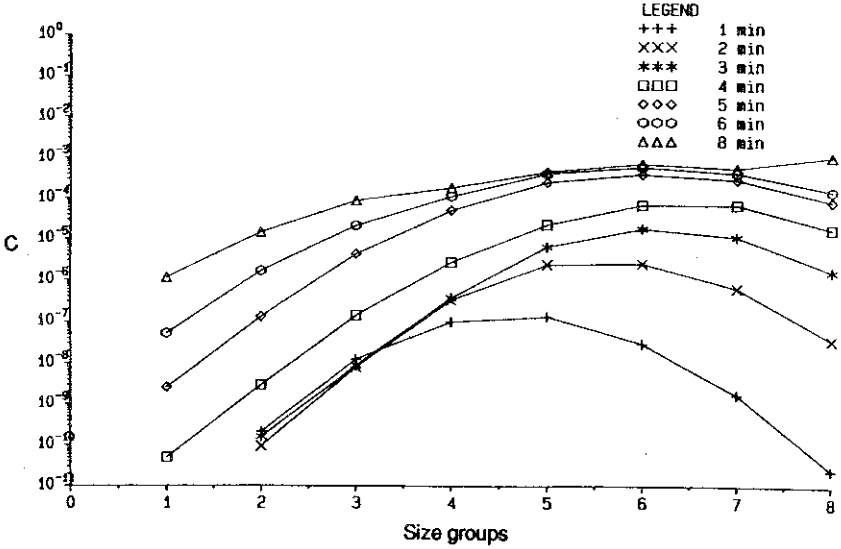

Fig. 3 Variations of concentration in different size groups, Case 1.

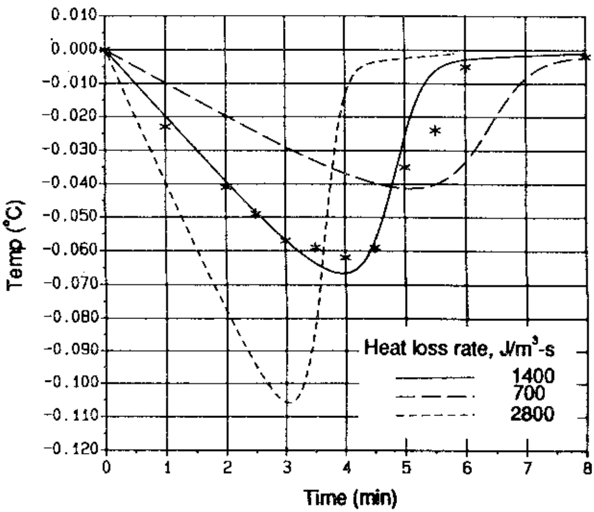

Fig. 4 Effect of heat loss rate.

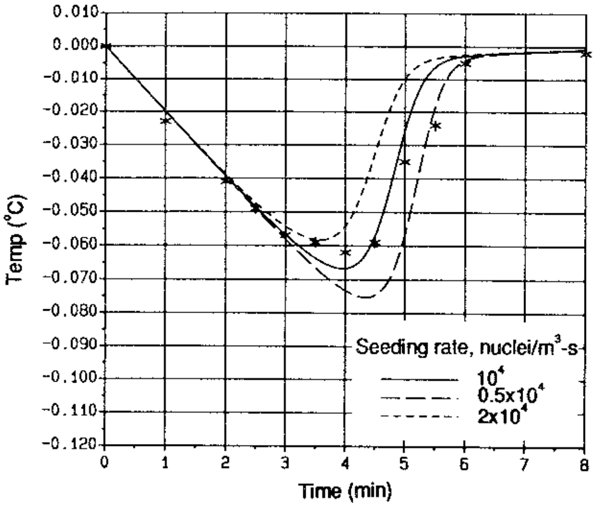

Fig. 5 Effect of Seeding rate.

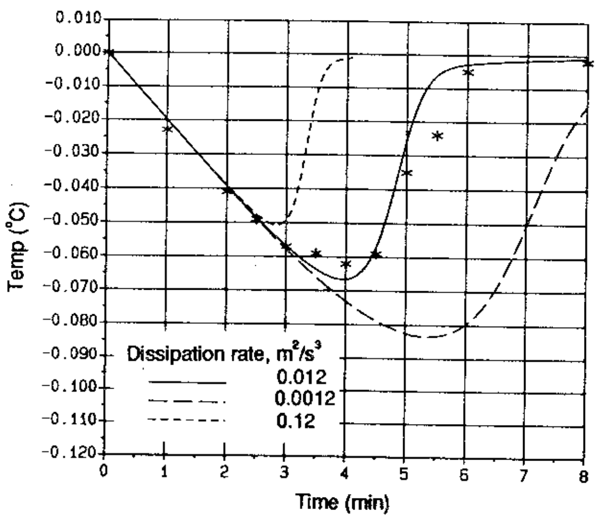

Fig. 6 Effect of dissipation rate.

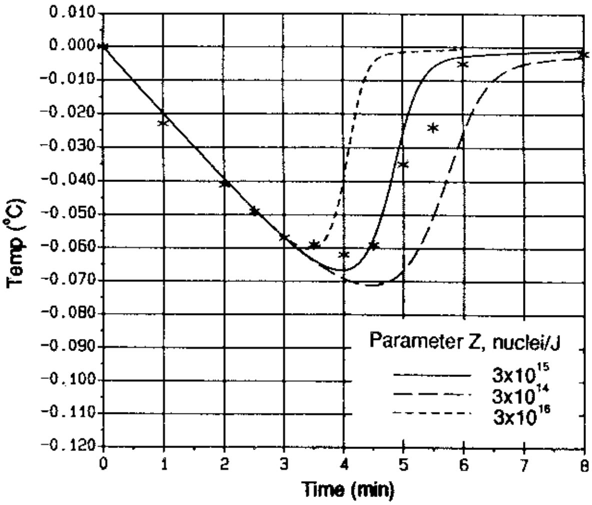

Fig. 7 Effect of the parameter Z.

DISCUSSION

Gee Tsang:

You have shown in your slide that the temperature/time curve is affected by flow turbulence. In Carstens' experiments, the flow was created with a propeller, which was not the same as in a river. How do you use your model in a river hydraulic situation?

Reply:

We assumed a well-mixed condition in simulating Carstens' experiments. However, the model is a two-dimensional model, which can simulate variations of flow and thermal/ice conditions along the channel as well as over the depth. Such an application can be found in our paper presented in the last workshop.

Darryl J. Calkins:

Field data on sub-cooling of streams seems to be in the range of -0.01°C to -0.03°C. What process(es) in the model would give temperature in this range?

Reply:

The water temperature variation is a result of the difference between the rate of heat loss through channel boundaries and the rate of latent heat released due to frazil ice growth. The water temperature is, therefore, a function of the rate of heat loss, seeding rate, secondary nucleation, and the turbulence intensity. The written paper has more detailed discussion on this.

Mike Ferrick:

I am not clear from the presentation what in the model provides downward mixing of particles. It must be the turbulence model. How are downward particle velocities obtained from calculations of k and ε?

Reply:

The mixing of frazil particles is due to the existence of the vertical concentration gradient and turbulence. More specifically, the vertical mixing is due to the term $\partial/\partial x_2\ (\nu_t/\sigma_c\ \partial C_k/\partial x_2)$ in Eq.3.