

**DESIGN OF HEAT BALANCE, ICE GENERATION
AND COVER MODULES**

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ABSTRACT

Frazil, skim and anchor may occur during the winter season under low temperature and weak solar radiation conditions. As a result an ice cover may form. Heat is either lost or gained at the water-air, ice-air and water-ice interfaces and this loss or gain is based on the components of the heat exchange at the interfaces. These components are in turn a function of the meteorological conditions.

The present paper presents the heat balance, open water ice generation and ice cover melting parts of the comprehensive numerical river ice model.

Key Words: Temperature, Frazil, Skim, Anchor, Ice, Melting, Cover, Radiation, Heat, Solar, Teriv.

INTRODUCTION

Relatively low temperatures and weak solar radiation occurring during the winter season result in the heat loss from the river to the atmosphere. As a result cooling of the river or lake takes place and ice generation in the form of frazil, skim and anchor may occur. Eventually an ice cover is formed.

The heat lost or gained at the water-air interface for open water reaches and at the ice-air and water-ice interfaces for ice covered cross sections is based on the components of the heat exchange at the interfaces evaluated in terms of shortwave solar radiation, longwave radiation, evapo-condensation heat flux, and convective heat transfer. These components are in turn a function of the meteorological conditions.

The present paper constitutes a part of the comprehensive numerical river ice model "RIVICE". This includes the heat balance module TERIV (module D) and the open water ice generation and anchor ice (module F). The change in the water temperature from one cross section to the other is accounted based on the hydraulic and meteorological conditions. The estimated flow conditions at a particular point in the river are a function of the history of previously encountered flow conditions. That is, the temperature of the flow at a particular cross section is a function of the cooling/warming that the flow has undergone in passing from the upstream end of the cross section. The heat loss would depend on the history of meteorological conditions encountered as well as the type of the ice cover encountered whether that may be in open water or ice covered reaches.

HEAT BALANCE MODULE (MODULE D)

The heat balance module evaluates the heat exchange fluxes between the open water surface and the atmosphere; the exchanges between the water and the ice cover; the effects of tributary and groundwater inflow, heat from frictional head losses and geothermal heat gain from the bed of the river. The combination of the heat exchange terms determine the quantities and types of ice generated or melted and the water temperature of the river as it progresses downstream from one cross section to another. The module therefore interacts closely with the open water ice generation routines (frazil, skim, and anchor) which determine how various types of ice evolve in each sub-section as each individual cross section is interrogated.

The heat flux components, shortwave radiation/longwave radiation, evapocondensation and conductive heat fluxes are evaluated based on the meteorological conditions and incoming solar radiation. Figure 1 shows the heat fluxes occurring at the ice-air, ice-water, and water-air interfaces.

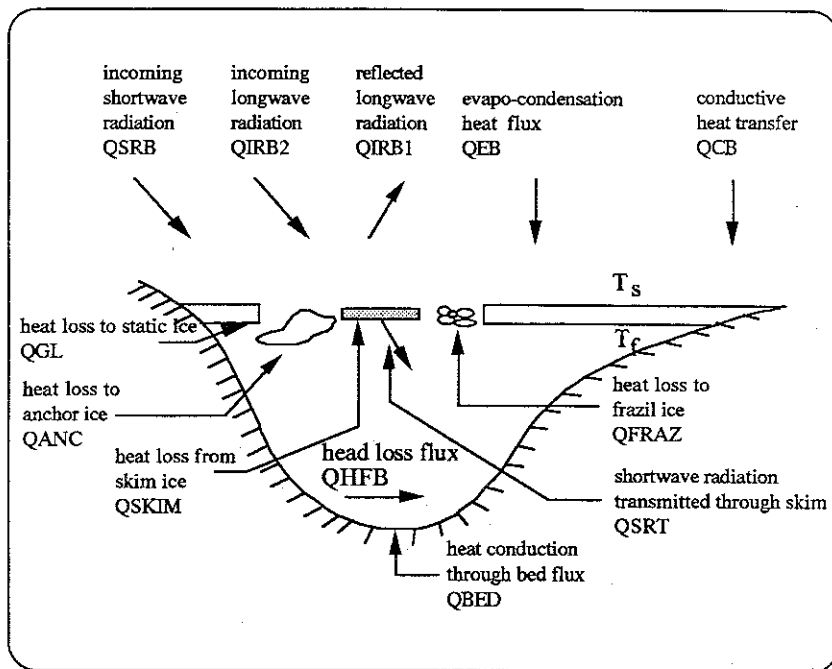


Figure 1. Heat Exchange Components.

The objective of the heat balance module is to calculate the heat fluxes occurring at the water-air, ice-air and water-ice interfaces, the water temperature, and the open water ice generation which is composed of:

- skim ice characteristics; and
- frazil ice characteristics;
- anchor ice characteristics.

The computations logic handled by the heat balance module, is shown in figure 2.

OPEN WATER ICE GENERATION (MODULE F)

The open water ice generation computations are treated as three routines to estimate the formation of skim ice, frazil ice, and anchor ice.

Skim Ice

The skim ice routine calculates the formation of skim ice at each cross section according to certain criteria which is based on the following parameters:

- flow velocity;
- air temperature and heat exchange fluxes;
- wind velocity;
- incoming volume of ice from upstream;
- local water temperature; and
- open water width.

Each time the skim ice routine is called, the above mentioned conditions are checked and if satisfied, a skim ice cover is formed, its discharge calculated, the resulting loss or gain of heat, and finally the corresponding modified local temperature. The same procedure is followed for frazil ice and anchor ice in the sequence mentioned herein, such that the final modification to the local water temperature would be the final temperature of the water.

The skim routine calculates at each time step the thickness of skim ice, the surface area dimensions and discharge of skim ice floating on the water surface. Two distinct, almost independent conditions of ice formation are handled. The first considers the section upstream to be ice free and the characteristics of new skim are determined. The second treats the case where the upstream section contains skim ice such that the thickness is calculated as it travels downstream, as well as the formation of new skim ice in the open water areas that remain.

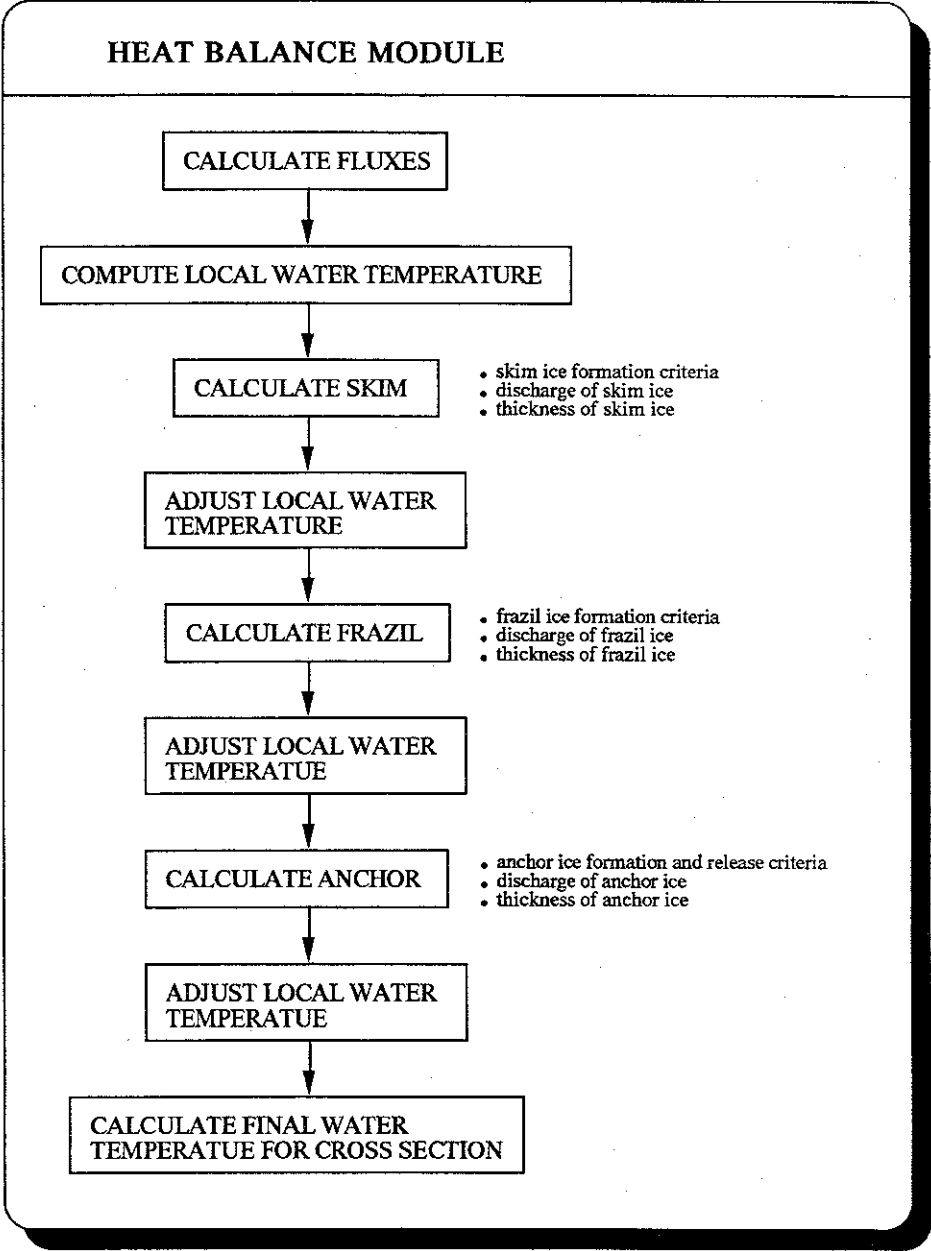


Figure 2. Heat Balance Module Logic Diagram.

First, the skim ice formation criteria are investigated. Three conditions have to be satisfied for skim ice to form. These are:

- a) the velocity has to be less than a velocity limit calculated as a function of the heat exchange and channel conveyance as determined by Matousek (7)

$$[1] \text{ VLIM} = 0.0182 * \sqrt{c} * (-\sqrt{Q_{\text{air}}})$$

- b) the ratio of the local water temperature to the air temperature from Hausser et al, (1)

$$[2] \frac{T_w}{-T_{\text{air}}} < 0.03$$

- c) the minimum open water width is greater than 100.0 m, Marcotte (2,3 & 4).

Skim ice forms in open water zones between border ice. Within the routine, the physical characteristics of the cross section are calculated, sub-section by sub-section. The total open water width is determined and the mean velocity. As the surface velocity is used in the calculations, it is calculated by increasing the mean velocity by 5%.

The formation of new skim is then calculated (case of no skim at the upstream section). The fraction of the width which freezes is based on the heat exchange and wind speed. The maximum width of skim that can form is assumed to be 98% of the open water width, based on observations (3).

For a reach which already contains skim ice, it is necessary to calculate its growth or melt as it floats downstream and is represented by the following equation, developed by Hausser et al., (1).

$$[3] \text{ TSK} = 0.1 \left(1 - 33.0 \left(\frac{T5}{-TAIR} \right) \right) \left(-1.0 + \sqrt{1 + 0.125 \text{ DD} > 0^\circ\text{C}} \right)$$

where TSK is the skim ice thickness (m),
T5 is the local water temperature (°C), and
TAIR is the air temperature (°C).

The speed of the skim ice sheets is taken as the surface velocity and can be corrected to account for the influence of wind (4).

Next, the mean velocity, distance and travel time between two adjacent cross sections are calculated. Four scenarios are investigated: a) open water at both adjacent cross sections; b) ice cover at both adjacent cross sections; c) ice cover at a cross section and open water at the one upstream; and d) open water at a cross section and ice cover at the one upstream. If the newly ice cover is insufficiently formed (too thin) or the velocity is too high, then it ruptures, else it thickens as a function of its exposure to the air temperature (freezing degree days). Thickening can also occur through rafting if the velocity at the section was higher at the previous time step. If the section is larger or narrower than the one upstream or is subject to high velocity, then adjustments to the width and thickness of the ice is made accordingly based on continuity.

If the temperature is very cold, new skim ice can form in the open areas between the skim ice if criteria of velocity, heat exchange rate and water temperature are satisfied. The equivalent diameter of the ice, the surface area, the discharge and the percentage of the open water surface covered are then calculated. Insufficiently formed skim is transformed into frazil and lastly, the change in water temperature associated with its generation (if the water is supercooled).

Frazil Ice

The routine FRAZIL calculates the quantity of frazil generated in the reach comprised between adjacent sections. As a consequence, the temperature of the water is adjusted, the frazil in suspension separated from the frazil emerging at the surface and finally the characteristic (thickness, diameter, and porosity) of the floating frazil determined. The routine is based on work by Marcotte (2 & 3).

Firstly, if the air temperature is above 3°C or if the section has an ice cover, frazil ice cannot form. If there is no frazil ice at the section upstream and the water temperature is greater than -0.01°C or the heat exchange flux greater than -100 kcal/m²/day or the open water width zero, no frazil can form either.

If the above criteria is satisfied, the individual sub-section characteristics are calculated at each cross section and the mean velocity is then calculated by multiplying the mean velocity by 1.05. The mass of new frazil formed is calculated by taking 20% of the average surface heat exchange flux between the current and proceeding section, taking into account any ice bridges or segments that may have formed and multiplying the value by the net open water area, exposure time and a coefficient. The total frazil discharge is then calculated. The mean velocity between the current and upstream sections is then calculated, the distance of open water and the travel time. The formation of frazil absorbs a known quantity of heat from the super-cooled water, and a new temperature is calculated.

Next, the quantity of frazil which rests in suspension is separated from that which floats to the surface. The total travel time from the start of frazil formation which may have originated many section upstream, is calculated. If the time is less than 12 minutes (0.008 hrs) (8), there is no surface frazil. In the other case, the travel time is long enough and frazil surfaces. The travel time between two adjacent sections is considered, then between the two adjacent sections immediately upstream. If each of these two values is less than 12 minutes, then 75% of the frazil submerged at the previous section is considered to have emerged. The quantity of frazil which is in suspension at the section under consideration therefore consists of the 25% which rests plus all the frazil formed in the present section. In the intermediate case, where one of the travel times exceed 12 minutes, all the frazil formed upstream has surfaced and the quantity which stays in suspension is a proportion of the quantity formed locally and therefore becomes less as the local travel times increase.

The physical characteristics of the emerging frazil are then determined. Two conditions are treated; one where the section under consideration is the first cross section at which surface frazil occurs and the general case where surface frazil already exists upstream.

In the first case, the thickness of the floating frazil is at least 40mm thick and grows as a function of distance. The diameter is equal to 5 times the thickness and the specific weight equal to 200 kg/m^3 . These values allow the porosity and surface area covered by frazil to be calculated.

In the second case, there is frazil on the surface at the previous section. The thickness is proportional to the distance travelled except the zones where velocities are higher than 1.5 m/s and thickness reduces as a function of the velocity. The ice thickness in all cases cannot exceed 250 mm or be less than 50mm. The specific weight increases by 30 kg/m³ per kilometer travelled to a maximum of 500 kg/m³. This value can however be exceeded due to another phenomena, the effect of freezing of water in the ice interstices in contact with cold air. The calculation is made by evaluating the number of degree-days since first emergence of the frazil which allows the frozen thickness to be determined. The specific weight is adjusted and then the final values of mean porosity and surface area occupied by the floating frazil calculated.

Anchor Ice

The anchor ice routine calculates the thickness of the generated anchor ice which is attached to the bed as well as the discharge of the detached anchor ice which floats to the surface. The anchor ice thicknesses are established by evaluating the rate of ice growth, which is a function of the super-cooled water temperature. Another calculation takes into account warming by solar radiation or contact with warm water, which results in the ice detaching from the bed. The rate at which the ice detaches is a function of the water temperature, ice density (which is a function of the water velocity) and the type of bed material. The routine is based on work done by Marcotte (5) and Marcotte and Robert (6).

The routine identifies first the case where the particular cross section being analyzed corresponds to the upstream limit of an open water reach. If the water temperature is positive, only ice detachment is considered because there is no ice formation.

A) Formation and growth of anchor ice

No anchor ice forms if the distance from the upstream limit or ice segment to the section that is being considered is less than 1500 meters or if the local depth is greater than 12 meters. It should be noted these empirical values were determined from ice observations made by HydroQuebec on lake St. Louis in Montreal. These values may be site specific and could be changed later depending on test results obtained at other locations. Also, no anchor ice forms if the local water temperature at the bed is positive. This bed temperature is equal to the local water temperature plus a term which depends on the solar radiation received, the depth and the local velocity. The solar radiation which reaches the bed varies in effect with the depth, by attributing the water a transparency $\text{ETA}=0.8 \text{ m}^{-1}$. The solar

radiation heats a layer of water not less than 30 cm thick which grows proportionally with the velocity when it exceeds 30 cm/s.

The quantity of heat available to form anchor ice is then calculated, the initial thickness of ice and local warming of water. The local density of the anchor ice is proportional to the local velocity and reaches that of ice without voids under 3 m/s. The thickness of the anchor ice can be calculated once the density is known. If the velocity is higher than 3.3 m/s, the thickness is only 1% of the value calculated previously.

The density calculated is that of new formed ice in contact with the flow. This is added to the ice already formed which results in a compaction of the ice mass and an increase in density. A factor of density is introduced such that the growth of ice on the bed can be modified due to compaction. The factor of density has a minimum value of 0.5 and may reach 1.0 when the density of the ice reaches 600kg/m^3

Finally, the variation in water temperature during the process is calculated for each sub-section as a function of the local conditions.

B) Detachment of anchor ice and characteristics of floating ice

A part of the incoming solar radiation reaches the bed and heats the water next to it. When the mean water temperature becomes positive, there is thermal erosion of the ice which melts on contact with this relatively warm water and causes a reduction in thickness.

This is not however always the principal process of anchor ice loss. The program evaluates the rate of detachment by calculating for each sub-section the percentage of the anchor ice thickness which detaches during 1 hour. The time constant calculated which reduces exponentially the anchor ice thickness according to this constant if the hydro-meteorological conditions stay constant.

The time constant is at first given a value equal to 500 hrs which is later corrected by taking into account the following four factors:

1. the type of bed: the time constant is 2 hours if the bed is sand on gravel.
2. the flow velocity: the time constant is 20 hours if the velocity is in excess of 2m/s; 10 hours if it exceeds 3m/s.

3. the present and past values of the water temperature at the bed. The following is successively evaluated:

$$f_1 = 2 Q_0; \quad f_1 \leq 10 \%$$

$$f_2 = 6 Q_3; \quad f_2 \leq 20 \%$$

$$f_3 = 12 Q_6; \quad f_3 \leq 25 \%$$

$$f_4 = 24 Q_{12}; \quad f_4 \leq 25 \%$$

$$f_5 = 48 Q_{24}; \quad f_5 \leq 30 \%$$

where Q_0 is the temperature at the bed and Q_3, Q_6, \dots, Q_{24} are the mean water temperatures during the 3, 6, ...24 hours preceeding. The percentage of the anchor ice which detaches each hour is equal to the biggest factor f_1, \dots, f_5 (disregarding negative water temperatures). The time constant is therefore inversely proportional to the biggest mean temperature during the preceding 3, 6, 12 or 24 hours.

4. maximum and minimum periods: the calculated time constant is corrected to be never longer than 300 hours or less than 1 hour.

Once the time constant is evaluated, the detached thickness and the volume of anchor ice which floats to the surface are calculated as a function of the time step. The other characteristics (thickness, density, ... etc) are adjusted accordingly. The detached volume allows the new discharge of anchor ice and the river surface area covered to be determined. In sandy zones, the detached anchor ice is reproduced as frazil which is then treated by the FRAZIL routine.

Finally, the new water temperature is calculated which has been modified by the melting of the anchor ice in contact with warm water.

THERMAL ICE COVER THICKNESS CHANGES (COVER ROUTINE)

The thermal growth and decay of an ice cover is a result of the heat exchanges at the top and bottom surfaces. The heat exchanges at the top surface are a function of the meteorological conditions, the solar radiation, the surface temperature of the ice cover, and the ice cover thickness,

while the heat exchanges at the bottom surface are a function of, the hydraulic conditions, hydrothermal conditions, heat transfer coefficient at the underside of the ice cover, and the ice cover thickness.

The growth and decay of an ice cover under two different conditions is considered. Therefore two scenarios, cross sections under Border Ice conditions, and cross sections under full ice cover conditions are treated following the method proposed by Shen (10) and Wasantha (11). The COVER routine uses the cross section characteristics such as the bathymetry and the subsection characteristics to evaluate the melting or thickening of the ice cover at every subsection (panel). Therefore, a transversal variation in the ice cover thickness is calculated at every time step.

Scenario 1: Cross Sections Under Border Ice Conditions

The initial Border ice thickness is taken as 0.2 m in by default, from the BORDICE module. The cover routine computes the melting and thickening of the Border Ice cover under the assumption that it has grown as solid ice. Therefore, the porosity of the Border Ice is taken as 1.

In the presence of Border ice, the heat fluxes occur at the ice-air and water-ice interfaces. The heat flux at the ice-air interface is obtained from TERIV and by the following components:

- the incoming solar shortwave radiation;
- the longwave radiation emitted from the river's surface;
- the net absorbed atmospheric radiation;
- the evapo-condensation heat flux; and
- the conductive heat transfer.

Figure 3 shows how the growth and decay of a solid ice cover takes place. This logic is used for the melting and thickening of Border Ice in the COVER routine. Assuming linear temperature variation across the solid ice cover thickness, the melting from the top surface of the ice cover is given by equation 4, as follows:

$$[4] \quad \phi_s \left(1 - \beta_1 e^{-\tau_1 \theta} \right) - \phi_b - \phi_e - \phi_c + K_i \frac{T_f - T_s}{\theta} = - \rho_i L_i \frac{\Delta \theta}{\Delta T}$$

where: ϕ_s is the net incoming shortwave radiation, ϕ_b is the net back longwave radiation, ϕ_e is the evapo-condensation heat flux, ϕ_c is the conductive heat transfer, K_i is the conductivity of the ice cover, L_i is the latent heat of fusion of ice, T_f is the freezing point temperature of fresh water, T_s is the surface temperature, ρ_i is the density of ice, ΔT is the time step, and θ is the solid ice cover thickness.

Since the components of the heat exchanges (shortwave radiation, longwave radiation, conductive heat flux and evapocondensation heat flux) at the ice-air, water-frazil ice, solid ice - frazil ice, and solid ice -water interfaces, are a function of the temperature at the ice surface, it is important to have the surface temperature computed at each time step. Using the boundary condition at the ice-air interface and by linearizing it, the following approximate solution can be obtained, in which it can be solved explicitly:

$$[5] \quad T_s^{(k)} = \frac{\phi}{K_i} \left\{ \left(1 - \beta e^{-\tau_i \theta^{(k-1)}} \right) \phi_s^{(k)} + \phi_{ba}^{(k)} - \phi_{br}^{(k)} - \phi_{bs} [T_s^{(k-1)}] - \phi_c [T_s^{(k-1)}] - \phi_c [T_s^{(k-1)}] \right\}$$

The melting from the bottom surface of the ice cover given by equation 6 is a function of the following variables: the temperature gradient between the water and the underside of the ice cover, the ice cover thickness, and the heat flux at the water-ice interface.

$$[6] \quad K_i \frac{T_f - T_s}{\theta} \cdot \phi_{wi} = -\rho_i L_i \frac{\Delta \theta}{\Delta T}$$

where: ϕ_{wi} is the heat flux at the water-ice interface.

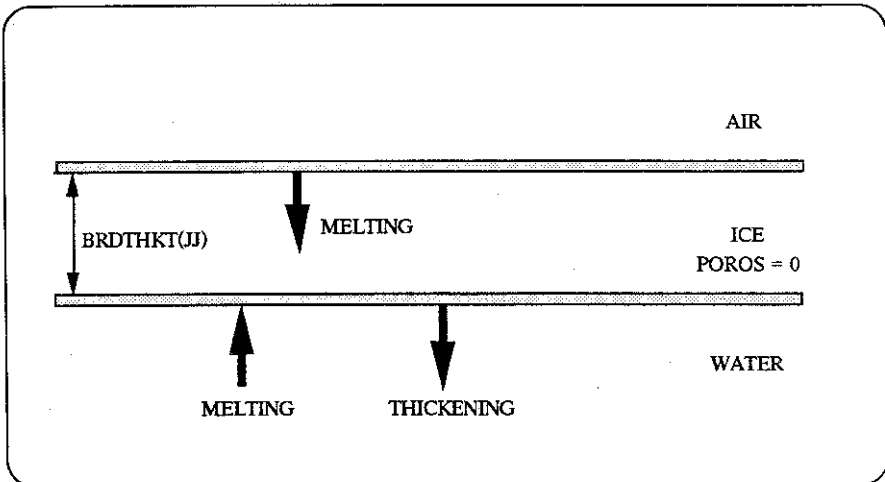


Figure 3. Growth and decay of Border Ice.

Under Border ice conditions, the surface temperature as well as the local water temperature play a major role in determining the change in thickness of the ice cover. If the surface temperature of the ice cover is greater than zero, then it is set to zero and thinning occurs at the top surface of the ice cover, at the ice-air interface. The melting at the bottom surface of the ice cover depends on the local temperature of the water.

If the surface temperature is less than zero, then the change of the ice cover thickness at the top surface is not computed, and thinning/thickening at the bottom surface occurs depending on the hydrothermal characteristics of the flow underneath of the ice cover.

Scenario 2: Cross Sections Under Complete Ice Cover Conditions

Surface ice particles swept underneath the ice cover at the leading edge, and the suspended ice in the flow can form accumulations on the underside of the ice cover. Undercover accumulation can occur when ice particles move into a slow-moving region. The discharge of ice coming into an ice cover section at the leading edge is computed by the open water ice generation module which is controlled by TERIV. The ICE COVER EVOLUTION module determines the progression of the ice cover in the river system based on the incoming ice in suspension, which may be freshly broken ice from an ice cover upstream and from open water sections where ice has been generated.

In concordance with the above logic, the COVER routine separates between a frazil ice layer and a solid ice layer as shown in figure 4. Therefore, the melting or growth of the ice cover is computed for the frazil and solid ice layers. When frazil ice deposit exists on the underside of the cover, equation 7 is used to evaluate the melting or thickening in the solid ice and equation 8 is used to calculate the melting or thickening in the frazil ice layer. Figure 4 presents the treatment of growth and/or decay of an ice cover under such a case.

$$[7] \quad \phi_{ia} = e_f \rho_i L_i \frac{\Delta\theta_i}{\Delta T}$$

$$[8] \quad -\phi_{wi} = (1 - e_f) \rho_i L_i \frac{\Delta\theta_i}{\Delta T}$$

where e_f is the porosity of the frazil ice layer, ϕ_{ia} is the heat flux at the ice-air interface, and ϕ_{wi} is the heat flux at the water-ice interface.

Equation 7 describes the growth of the solid ice into the frazil ice layer while equation 8 represents the melting of the frazil layer. Melting at the top surface of the solid ice cover occurs when the surface temperature computed in equation 5 is greater than the melting temperature of ice. If the surface temperature is inferior to the melting temperature of ice, then the growth of the solid ice cover can occur, into the frazil ice layer. If the surface temperature is equal to the melting temperature, then the ice cover remains isothermal and the growth of the ice cover is not possible since no heat is gained or lost across the ice cover interfaces.

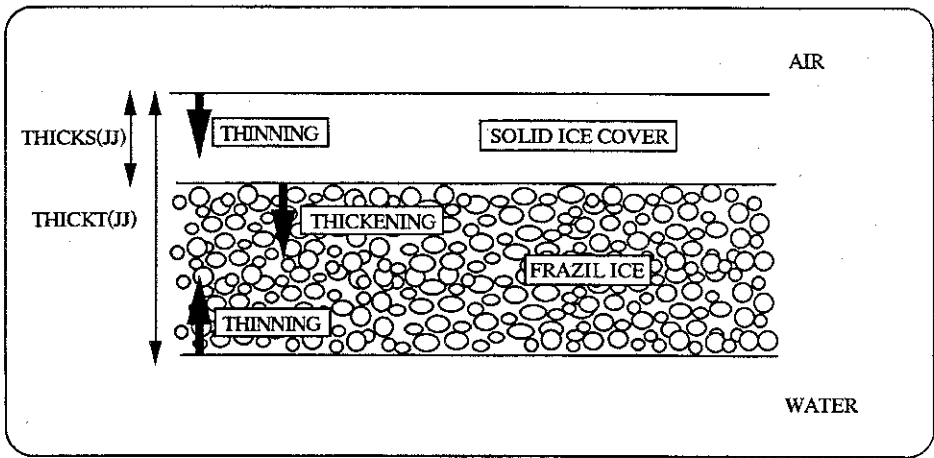


Figure 4. Growth and decay of a full Ice.cover.

Under complete ice covered conditions, the temperature at the top surface of the ice cover as well as the local water temperature underneath play a major role in determining the change in thickness of the ice cover. Similar to the Border ice condition, if the surface temperature of the ice cover is greater than zero, then it is set to zero and thinning occurs at the top surface of the ice cover, at the ice-air interface. The melting at the bottom surface of the ice cover depends on the local temperature of the water.

If the surface temperature is less than zero, then the change of the ice cover thickness at the top surface is not computed, and thinning at the frazil ice - water interface may occur depending on the local water temperature. Thickening takes place at the solid ice - frazil ice interface such that the growth occurs into the frazil ice layer.

CONCLUSION

The Heat Balance Module and Ice Generation routines have been linked successfully with the Rivice Driver and follow the reach/node connectivity system.

To date, tests are being run on a single channel four reach theoretical system with a constant discharge (Test Case 1). The results from these runs are discussed in "Examples of Simulated Ice Conditions with RIVICE".

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LIST OF SYMBOLS

α	=	surface albedo
ϵ_s	=	emissivity of water or ice surface
θ	=	ice cover thickness, in m
τ_i	=	bulk extinction coefficient, in cm^{-1}
ϕ_b	=	effective back radiation, in $\text{cal cm}^{-2} \text{day}^{-1}$
ϕ_{ba}	=	atmospheric radiation, in $\text{cal cm}^{-2} \text{day}^{-1}$
ϕ_{bn}	=	net atmospheric radiation, in $\text{cal cm}^{-2} \text{day}^{-1}$
ϕ_{bs}	=	longwave radiation emitted by the river surface, in $\text{cal cm}^{-2} \text{day}^{-1}$
ϕ_c	=	conductive heat transfer, in $\text{cal cm}^{-2} \text{day}^{-1}$
ϕ_e	=	evapo-condensation flux, in $\text{cal cm}^{-2} \text{day}^{-1}$
ϕ_{lat}	=	latitude on earth's surface, in degrees
ϕ_{ri}	=	incoming shortwave radiation, in $\text{cal cm}^{-2} \text{day}^{-1}$
ϕ_s	=	net shortwave radiation, in $\text{cal cm}^{-2} \text{day}^{-1}$
ϕ_{sp}	=	shortwave penetration into the waterbody, in $\text{cal cm}^{-2} \text{day}^{-1}$