RIVER ICE JAMS: THEORY VERSUS CASE STUDIES

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INTRODUCTION

A major consequence of ice cover formation on northern rivers is the jamming that occurs during the spring breakup of the cover and, to a lesser degree, during the freeze up period. Due to their large aggregate thickness and hydraulic resistance relative to those of sheet ice, jams tend to cause unusually high water stages. This has repercussions in many operational and design problems such as overturning moment on river structures due to moving ice, forces on ice booms, spring flooding and associated stage-frequency relationships, river bed scour due to surges from released jams, to mention but a few.

At present, complete mathematical simulation of water stages during breakup is only a hope for the future. There are simply too many unknowns: It is not known whether, where and when a jam will form; even if it is assumed that a jam has been initiated at a specified location, it is not known exactly what occurs at the toe (downstream end) and thus it is not possible to formulate an appropriate boundary condition for the jam's subsequent evolution; and even if the configuration of an ice jam at a specified time is given or assumed, it is not known how, why and when the jam will release.

Faced with such difficulties, research has concentrated on the relatively simple problem of equilibrium jams, i.e. jams that no longer evolve. This approach has considerable practical merit since it can be argued that, under certain circumstances, the highest water stages occur when a jam has attained equilibrium. Theoretical work has resulted in a model for floating jams in equilibrium that has been tested with some success versus experimental results. Virtually nothing is known, however, about grounded (otherwise known as "dry") jams. Figure 1 gives a schematic illustration of a floating equilibrium jam. In Fig. 1, it has been assumed that the jam was initiated at the edge of an undisturbed ice sheet and attained a steady state condition. Typically, there are three regions within the length of such a jam:

(i) Upstream transition: For a certain distance below the head of the jam, the thickness increases and approaches an asymptotic value. The flow under the jam is of the gradually varied type. As will be discussed later, this should be the case for jams of the "wide channel" kind which represents a very common occurrence (see also pertinent measurements by Calkins 1978). "Narrow channel" jams may not exhibit an upstream transition; from theoretical considerations it could be shown that for a prismatic channel the thickness of a narrow jam should not change in the downstream direction.

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(ii) Equilibrium thickness reach: The thickness of the jam is relatively uniform and approximately equal to the asymptotic value mentioned earlier. This value has been termed the equilibrium thickness (Uzuner and J. F. Kennedy 1976). The flow under the jam is uniform and the water surface is equal to the channel bed slope S. This concept applies to prismatic channels but may be extrapolated to natural streams by replacing S with the open-water surface slope, provided (a) the reach under consideration is long enough to permit meaningful averaging of channel characteristics; and (b) the flow is free to assume a uniform condition, i.e. there are no significant control effects in the reach of interest.

(iii) Downstream transition: Below the equilibrium reach, the water surface profile steepens progressively to meet the relatively low water stage that prevails at the toe of the jam (e.g. see Doyle 1977; Doyle and Andres 1978,79). The jam and flow configurations in this region are difficult to assess and could vary depending on local conditions and mode of jam initiation.

It should be recognized that the type of jam depicted in Fig. 1 occurs often but is by no means universal. If the jam is too short, the equilibrium thickness region may not exist while in reaches that are strongly influenced by controls (e.g. river mouths) uniform flow may not occur under the jam.

In the following sections, it will be attempted to review and assess the available theory and data on ice jams.

THEORETICAL BACKGROUND

Hydraulics of Flow under an Ice Jam. - A river cross section located in a reach where a floating jam has formed is sketched in Fig. 2. In the following, flow through the voids of the jam will be neglected and the jam assumed to have an equilibrium thickness reach with uniform flow underneath. The longitudinal water surface slope is then equal to the river slope under open-water conditions. The velocity distribution in any one vertical will be as sketched in Fig. 2 where the dashed line represents the locus of the maximum velocity points; for a very wide channel, relative to its depth, the shear stress along this line is nearly zero. As a first approximation, it is assumed that the flows in the two subsections defined by the maximum velocity line are respectively controlled by the average shear stresses on the jam underside (τ_i) and on the river bed (τ_b).

Let Q_i, A_i, V_i and R_i be the discharge, area, average velocity and hydraulic radius for the ice-controlled subsection respectively. Then $V_i \equiv Q_i/A_i$ and $R_i \equiv A_i/W$ (see Fig. 2); the Manning roughness coefficient n_i, and friction factor, f_i for the jam underside are defined as $n_i \equiv V_i^{-1} R_i^{2/3} S^{1/2}$ (metric units) and $f_i \equiv 8\tau_i/\rho V_i^2$ with $\tau_i = \rho g R_i S$ and ρ=water density, g=acceleration of gravity. Similar relationships apply to quantities pertaining to the bed-controlled flow subsection; in the following, such quantities will be designated using the same symbols as above but with the suffix "b" in place of "i".

For the overall, composite roughness flow under the jam (designated with the suffix "o"), the well-known Sabaneev equations may be used, that is:

$$n_o = \left[(n_b^{3/2} + n_i^{3/2}) / 2 \right]^{2/3} \quad (1)$$

$$f_o = (f_i + f_b)/2 \quad (1a)$$

$$R_i/R_b = (n_i/n_b)^{3/2} \quad (2)$$

$$R_i/R_b = f_i/f_b \quad (2a)$$

where equivalent relationships in terms of the friction factors instead of the Manning coefficients are numbered as 1a, 2a, etc. Moreover, for wide channels:

$$R_i + R_b \simeq 2 R_o \quad (3)$$

It can be shown that Eqs. 1 and 2 are exact when $V_i = V_b$. For two-dimensional flow, this condition will be nearly satisfied if the shear velocities associated with the bed and jam respectively are either small compared to V_i and V_b or not very different from each other (Beltaos 1979). This is supported by laboratory and theoretical data (Lau 1982; Sayre and Song 1979). For wide channels of arbitrary cross-sectional shape, the condition $V_i = V_b$ can only be tested empirically; examination of field data by the writer (unpublished) showed V_i and V_b to be within 10 percent of each other. However, these data were associated with relatively smooth sheet ice covers that occur in mid-winter, hence this finding may not apply to the very rough flows under ice jams. To a large degree then, use of Eqs. 1 and 2 (or 1a and 2a) is justified by a lack of more reliable information.

Hydraulic resistance characteristics of the river bed (n_b and f_b) can be obtained from hydrometric surveys in the reach of interest during open-water conditions. Though jam stages are generally high, a large portion of the water depth is occupied by the jam itself and the flow part controlled by the jam underside (see also Fig. 2); thus, usual values of R_b may represent low open-water stages, at which n_b (or f_b) is stage-dependent. The dependence of n_b on open-water depth is assumed to apply to flow under an ice jam by using R_b in place of the open-water depth. For a given reach, one may thus write:

$$n_b \text{ (or } f_b) = \text{a function of } R_b \quad (4)$$

Resistance characteristics of the underside of an ice jam have not been documented widely to date. The few pertinent data known to the writer are discussed briefly below.

R. J. Kennedy (1958) investigated the characteristics of log jams; the bottom roughness of a jam was found to increase with increasing jam thickness, based on field and laboratory measurements. The roughness-thickness relationship suggested by R. J. Kennedy is shown in Fig. 3. Intuitively, one would expect that the curve of Fig. 3 should have a horizontal asymptote, i.e. the roughness should not increase indefinitely but attain a constant value beyond a limiting value of thickness. The limiting thickness and the maximum roughness would probably depend on the dimensions and geometry of the fragments that compose the jam.

Field data on ice jams in the Soviet Union have been reanalyzed by Nezhikhovskiy (1964) for the period of freeze up. Nezhikhovskiy found that the jam roughness, defined as the average deviation from the mean of numerous thickness measurements, increased with the average jam thickness t and thence proceeded to establish empirical relationships between n_i and t. Three types of accumulations were identified, respectively comprising loose slush, dense slush and ice floes. The latter type is thought to be the most relevant to spring jams. The writer (1979) noted that, for the very rough boundaries of ice jams, n_i should depend on R_i as well as t and attributed Nezhikhovskiy's findings to a restricted range of R_i values (approximately 1.0 m to 1.5 m). With this interpretation, it was possible to account for the effects of R_i and derive the following empirical equation:

$$n_i = 0.072\, R_i^{-0.23}\, t^{0.4} \qquad (5)$$

$$f_i = 0.4\, (t/R_i)^{0.8} \qquad (5a)$$

in which $0.5 \leq t/R_i \leq 3.0$ and $t \leq 3$ m. Equations 5 and 5a are based on Nezhikhovskiy's suggestion that the jam roughness increases linearly with t.

An alternative interpretation of Nezhikhovskiy's data can be obtained by assuming that the well-known logarithmic dependence of the friction factor on relative roughness (fully rough flow regime) applies to ice jams. Then, R_i may be taken as constant ($\simeq 1.25$ m) for Nezhikhovskiy's data and the jam roughness can be calculated as a function of thickness. For the variation of f_i, an equation proposed by Limerinos (1970) has been used, derived from natural stream data:

$$f_i = \left[1.16 + 2 \log (R_i/d_{i,84})\right]^{-2} \qquad (6)$$

in which $d_{i,84}$ is (by analogy with stream beds) the roughness value that exceeds 84 percent of the values in a representative set of individual roughness measurements. The writer (1979) has found Limerinos' equation to apply in the range $0.4 \leq R/d_{84} \leq 150$, using data of several other investigators. Using the identity $n/R^{1/6} = 0.113\sqrt{f}$, values of $d_{i,84}$ were computed to fit Nezhikhovskiy's n_i-t relationship for ice floes and are shown plotted versus t in Fig. 4. The graph in Fig. 4 is adequately represented by the empirical equation (metric units):

$$d_{i,84} = 1.43 \left[1 - \exp \left\{ -0.734 (t - 0.15) \right\} \right] \tag{7}$$

It is noteworthy that the curve of Fig. 4 is similar to that of Fig. 3 that applies to log jams.

Equations 5 (or 5a) and 6 (with 7) give two different interpretations of Nezhikhovskiy's results, both designed to account for the effects of R_i; at present, it is difficult to state which is more realistic, though the latter is more appealing to intuition. Finally, it is noted that laboratory data by Tatinclaux and Cheng (1978) and Calkins and Müller (1980) also showed f_i and n_i to increase with t.

Ice Jam Theory - A review of theoretical work by others on ice jams has been carried out by the writer (1981) and the main findings are described briefly below. Strictly speaking, the present discussion applies to spring ice jams; these may be considered granular accumulations of ice fragments with no cohesion. It is not clear at this time whether cohesionless jams can form during freeze up, due to complications arising from thermal effects. From the practical point of view, a cohesionless jam is of more interest as it has a greater flooding potential than a cohesive jam.

For most natural streams (width-to-depth ratio larger than about 10), the thickness of a spring jam is controlled by its ability to resist streamwise forces caused by its own weight and the shear stress exerted by the flow on its underside. This is the "wide channel jam" according to the terminology introduced by Pariset et al (1966). According to the theory, the thickness of the jam satisfies the following equation

$$\mu \rho_i (1 - s_i) g t^2 = W (\rho g R_i S + \rho_i g t S) \tag{8}$$

in which μ = dimensionless coefficient that depends solely on the internal friction of the jam; ρ_i = specific gravity of ice; $\rho g R_i S$ and $\rho_i g t S$ represent respectively the shear stress exerted by the flow on the underside of the jam and the downstream component of the jam's own weight per unit area. Using Eqs. 2a and 3 and solving Eq. 8 for t gives

$$t = \frac{WS}{2\mu (1-s_i)} \left[1 + \sqrt{1 + \frac{2\mu (1-s_i)}{s_i} \left(\frac{f_i}{f_o}\right) \left(\frac{h}{WS}\right)} \right] \tag{9}$$

in which h ($=2R_o \simeq$ average flow depth under the jam) can be computed from the following resistance equation

$$h = \left[q/(4gS/f_o)^{1/2} \right]^{2/3} \tag{10}$$

with q = Q/W = discharge intensity. Assuming that the porosity of the jam is the same both above and below the water surface, the overall water depth, H, is given by

$$H = h + s_i t \tag{11}$$

and can be expressed in terms of the discharge using Eqs. 9 and 10, i.e.

$$\eta \equiv H/WS = 0.63\, f_o^{1/3}\, \xi + \frac{5.75}{\mu} \left\{ 1 + \sqrt{1 + 0.11\, \mu\, f_o^{1/3} \left(\frac{f_i}{f_o}\right) \xi} \right\} \tag{12}$$

in which s_i has been fixed at 0.92 and

$$\xi \equiv (q^2/gS)^{1/3} / WS \tag{13}$$

Equations 12 and 13 show that the dimensionless total water depth η depends primarily on the dimensionless discharge ξ and the internal friction of the jam; and secondarily on the friction factors of the flow boundaries. It is felt that Eq. 12 is more convenient than Pariset et al's final expression which relates H^4W/Q^2 to t/H because: (a) in practice it is desired to estimate H given Q, W and S and this can be accomplished more directly using Eq. 12 than Pariset et al's expression; and (b) in analysing field data on spring jams, for which H, Q, W and S are measurable but t is not, it is simpler to work with Eq. 12 than with Pariset et al's expression. Further, it may be noted that in Eq. 12, η increases continuously with increasing ξ; there are no minima or maxima that might occasionally result in uncertainties of interpretation.

An interesting feature of Eq. 12 is that η does not vanish when Q (and thence ξ) are zero. This result is contrary to intuition and can be explained as follows. Firstly, it is noted that at $\xi = 0$, the flow depth vanishes but the (submerged) jam thickness becomes equal to $11.5\ WS/\mu$. Recalling Eq. 8 shows that the jam thickens to withstand two types of force: the hydraulic friction and the streamwise component of the jam's own weight. When $Q \to 0$, the former vanishes but the latter does not since S remains constant. Putting $R_i = 0$ (i.e., shear stress = 0) and $s_i = 0.92$ in Eq. 8 gives again $11.5\ WS/\mu$ for the submerged jam thickness. It is now obvious that this implausible result is due to the assumption that the flow through thejam is negligible. This assumption is realistic under normal circumstances; however, as Q approaches zero, an increasing fraction of Q will flow through the voids of the jam and even before Q becomes zero, the jam will ground. When this occurs, the jam need not be as thick as indicated by Eqs. 8 and 9 because additional frictional resistance becomes available by contact with the river bed.

COMPARISON WITH CASE STUDIES

Description of Data. - Most of the ice jam case studies utilized herein derive from field research programs that have been described by Gerard (1975) and the writer (1978, 82). A novelty of these programs consists of documenting "instantaneous" water level profiles along any observed jam as follows: from small aircraft or from ground access points, photos are taken of the jam stage against the river banks and used later for identification and survey. Elevations so determined may be plotted versus traverse distance to obtain longitudinal profiles of the jam stage. An example is shown in Fig. 5. When a jam profile has a section that is parallel to the open-water surface, the jam can be assumed to be in equilibrium. From cross-sectional measurements and slope surveys, reach-averaged values of H, W and S can be determined. Unfortunately, it is not possible to determine directly h and t because there is no capability at present for measuring the thickness of a spring jam. Estimates of the average thickness of the jam in the equilibrium reach are possible only through an indirect analysis, as will be discussed later. The discharge Q assigned to each jam is that which prevailed at the time the jam was observed. The actual value of Q responsible for the formation of the jam can be higher than the assumed Q but not lower. Thus, what is observed is a jam with possibly oversized thickness for the assigned Q. Letting H_{obs} and Q_{obs} be the "observed" values of H and Q, as outlined above, and H_a, Q_a be the corresponding values at the time when the jam was formed, we have $Q_a > Q_{obs}$ and $H_a > H_{obs}$. The pair H_a, Q_a satisfies the conditions of ice jam formation, that is, it satisfies Eq. 12, if the theory is assumed valid. As Q decreases from Q_a to Q_{obs}, it is reasonable to assume further that the thickness of the jam does not change but the flow depth under the jam decreases, as indicated by Eq. 10. Hence, the pair H_{obs}, Q_{obs} will not satisfy Eq. 12; if H' is the value of H obtained from Eq. 12 with $Q = Q_{obs}$, then $H' < H_{obs}$. With plausible values of μ, f_i, f_o (see later discussion) Eq. 12 can be used as a rough guide to evaluate the relative error, $(H_{obs} - H')/H'$, inherent in the pair H_{obs}, Q_{obs}. Fortunately, it is found that even if Q_a is as large as $2Q_{obs}$, this error does not exceed 7 percent. This is acceptable considering the many other errors associated with field data pertaining to ice jams.

Testing of Theory. - Equation 12 indicates that, according to the theory, the dimensionless jam stage η should depend on ξ with f_i, f_o and μ as parameters. Available data are plotted in the form of η verus ξ in Fig. 6. The data are summarized in Tables 1 and 2 which will be discussed later. In Fig. 6, different symbols have been used to describe ice jams deemed to have been respectively in equilibrium and in evolution. Despite considerable scatter, the data points in Fig. 6 show an unquestionable trend for η to increase with ξ which qualitatively supports Eq. 12. For a quantitative test of Eq. 12, μ and f_i/f_o were fixed at 1.2 and 1.25 respectively (average values found from a detailed analysis to be discussed later) and η was calculated for $f_o = 0.1$ and 0.5. Comparison of the resulting curves with the data points indicates that the theory is basically sound while there seems to be a general trend for f_o to decrease with increasing ξ. The latter is plausible because f_o should decrease when t/H decreases and this can be shown to occur when ξ increases. It is of interest to note in Fig. 6 that the data points for evolving jams plot at or below the line defined by the points corresponding to equilibrium jams; this gives a measure of support to the expectation that the peak stage is attained at equilibrium.

DETAILED ANALYSIS - INDIRECT METHODS

Detailed analysis of the data available to date is hampered by a lack of means to measure ice jam thicknesses during breakup. Typically, the measurable quantities are H, Q, W, S, a few representative river cross sections, and the relationship of Eq. 4. It is desired to determine h, t, R_i, R_b and n_i, n_b (or f_i, f_b), i.e. a total of six unknowns. The available equations are five, i.e. Eqs. 1 to 4 and the flotation relation, Eq. 11. Clearly, the problem cannot be solved unless an additional relationship is introduced. Pariset et al (1966) assumed $n_i = n_b = n_o$ ($f_i = f_b = f_o$). This assumption is arbitrary as there is no a priori reason why n_i should be equal to n_b for all jams in all rivers. The writer believes that use of Nezhikhovskiy's data (1964), as interpreted in Eq. 5 (or 5a) or Eq. 6 (with 7), is preferable because these relationships have a basis on measurement. It is recognized that Nezhikhovskiy's data are subject to the usual inaccuracies one may expect for field observations of ice jams; in addition, these data have not been duplicated by other investigators (though indirectly corroborated by Calkins and Müller 1980; R. J. Kennedy 1958; Tatinclaux and Cheng 1978) neither for freeze up nor for breakup. Thus use of this approach cannot be considered "satisfactory" but may be viewed as the "least objectionable" at present.

The procedure of analysis is as follows. For an assumed value of t, plot the lower jam boundary (0.92 t below the water stage) on each river cross section and determine reach-averaged values of A, W, V(= Q/A), R_o (= A/2W) and thence n_o (or f_o). Use Eqs. 1 to 4 to determine the remaining four unknowns R_b, R_i and n_b, n_i (or f_b, f_i). Repeat for a few additional values of t and plot n_i (or f_i) versus t. The intersection of this plot with Eq. 5 (5a) or with Eq. 6 (with 7) which can be evaluated from the data already generated, gives the value of t that satisfies all of the specified relationships and is thus the desired jam thickness. With this information, the coefficient μ may be computed from

$$\mu = 12.5\,(SW/t)\left[1 + (R_i/0.92\,t)\right] \tag{14}$$

which is a rearranged version of Eq. 8.

Thirteen case studies, analyzed according to the above-mentioned procedure, are summarized in Table 1 where it may be noted that fairly wide ranges of stream width, slope and discharge are represented. The coefficient μ has an average value of about 1.2 and, for most case studies, individual μ's are close to this average. The lowest μ (= 0.6) was obtained for the jam on the Athabasca R. near Pelican rapids. The data for this jam are, however, uncertain because it was documented using post-breakup evidence. The highest value of μ (= 3.5) was obtained for the Smoky R. near Hunting Creek and does not seem to fit the pattern of the other jams. The sheet ice thickness in that case was about 0.6 m, i.e. one-half of the estimated jam thickness. It is possible that, as a jam approaches the configuration of a single layer of ice floes, μ will more and more reflect the effective ice-bank friction rather than the true internal friction of a granular ice mass. Note that the Thames R. jams, though only 0.7 - 0.9 m thick, represent jam thickness to sheet ice thickness ratios of more than 3.5. Overall, Table 1 gives a measure of support to the method of analysis used herein because it shows that μ takes on consistent values and at the same time is close to the value of 1.3 that has been reported earlier by Pariset et al (1966).

The parameters f_o and f_i/f_o that appear in Eq. 12 are seen in Table 1 to range from 0.09 to 0.67 and from 0.63 to 1.64 respectively; corresponding average values are 0.37 and 1.25. There is no consistent variation of f_i/f_o with ξ hence the average value of 1.25 could conveniently be substituted in Eq. 12 considering that η is insensitive to f_i/f_o. At the same time, there is a trend for f_o to decrease with increasing ξ, as earlier discussion concerning Fig. 6 had indicated. This trend is illustrated in Fig. 7. It should be understood at this point that no unique relationship between f_o and ξ can be expected because f_o should also depend on channel bed characteristics. This aspect is probably responsible for the large scatter of the data points in Fig. 7. For estimates of H using Eq. 12, it would nevertheless seem permissible to substitute "average" values of f_o from Fig. 7, given that a relatively small power of f_o appears in Eq. 12.

Table 2 summarizes additional but less comprehensive data for which the detailed analysis has not been performed. These data have also been used in Fig. 6.

SUMMARY AND CONCLUSIONS

The existing theory of floating "wide channel" ice jams has been examined in the light of field data accumulated in recent years. The theory predicts the jam thickness so that the jam, considered a granular mass, can withstand the applied forces. These forces are caused by the flow shear stress and by the streamwise component of the jam's own weight. The theory was shown to give the dimensionless jam thickness, t/WS, and overall depth H/WS, as functions of the dimensionless discharge ξ ($=(q^2/gS)^{1/3}/WS$). The internal friction of the jam along with the river bed and jam friction factors appear as parameters of these functions. The theory was tested with satisfactory outcome by plotting H/WS versus ξ using the available field data.

Because the thickness of spring jams cannot be measured at present, the data do not permit direct estimates of the applicable roughness and internal friction characteristics. A method has been outlined for arriving at such estimates indirectly, based on existing hydraulic resistance data for winter jams. This method was applied to 13 case studies with the following results:

(a) The coefficient μ which depends on the internal friction of the jam was between 0.6 and 3.5. The lower limit of this range was obtained under conditions of considerable uncertainty with regard to both jam stage and applicable discharge. The upper limit was found for a relatively thin jam (jam thickness $\simeq$ 2x sheet ice thickness) and thus might have been influenced by ice-bank friction. In the remaining 11 cases, μ was between 0.8 and 1.3 which was considered encouraging in view of the crudeness of both the data and the analytical procedure. An average value of 1.2 was suggested for applications.

(b) The composite friction factor, f_o, varied between 0.09 and 0.67 and exhibited a tendency to decrease with increasing dimensionless discharge ξ. The same tendency was also implied when comparing the theory with data in terms of the relationship between H/WS and ξ.

(c) The ratio f_i/f_o ranged between 0.6 and 1.6 but showed no tendency to vary with ξ.

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NOTATION

The following symbols are used in this paper:

A = cross-sectional area;

b = suffix denoting parameters pertaining to the river bed;

$d_{i,84}$ = roughness height that exceeds 84 percent of all roughness height values;

f = friction factor;

g = acceleration of gravity;

H = overall water depth;

h = depth of flow under a jam;

i = suffix denoting parameters pertaining to an ice jam;

n = Manning roughness coefficient;

o = suffix denoting composite roughness flow parameters;

Q = flow discharge;

q = discharge intensity;

R = hydraulic radius;

S = channel slope;

s_i = specific gravity of ice;

t = jam thickness;

V	=	average flow velocity;
W	=	channel width;
η	=	(dimensionless) overall water depth;
μ	=	(dimensionless) coefficient depending on angle of internal friction;
ξ	=	dimensionless flow discharge;
ρ, ρ_i	=	densities of water and ice;
τ_i, τ_b	=	fluid shear stresses on ice jam and river bed;

TABLE 1. - Analysis of 13 Case Studies of Ice Jams

Location (1)	Date (2)	Source (3)	Q in meters cubed per second (4)	W in meters (5)	S in meters per kilometer (6)	H in meters (7)	t in meters (8)	f_o (9)	$\frac{f_i}{f_o}$ (10)	μ (11)	ξ (12)	η (13)	Remarks (14)
Smoky R. near mouth	Apr. 9, 76	Writer	210	227	0.72	5.5	3.1	0.65	1.12	1.0	30.5	33.8	Slope taken from topographic maps.
Smoky R. near mouth.	Apr.10, 76	Writer	283	228	0.72	6.2	3.3	0.55	1.17	1.0	36.9	37.9	Slope taken from topographic maps.
Smoky R. near Hunting Creek	Apr. 7, 77	Writer	456	163	0.83	5.9	1.2	0.46	0.63	3.5	73.0	43.6	Slope surveyed at site.
Wapiti R. near Grande Prairie	Apr.12, 76	Writer	185	153	1.02	5.8	3.1	0.67	0.97	1.0	33.8	37.2	Slope surveyed at site.
Thames R. near Thamesville	Jan.12, 80	Writer	108	41	0.77	3.9	0.9	0.13	0.81	1.0	308	125	Slope surveyed at site.
Thames R. above Sherman Brown Bridge	Mar.19, 80	Writer	196	74	0.26	4.7	0.9	0.09	1.44	1.2	728	244	Slope surveyed at site.
Thames R. at mouth	Mar.20, 80	Writer	195	106	0.13	4.6	0.7	0.09	1.44	1.3	1005	334	Slope surveyed at site.
Athabasca R. near Ft. McMurray	Apr.16,77	Doyle	930	560	0.36	8.2	4.4	0.38	1.43	1.0	45.7	40.7	Slope surveyed at site
	Apr.18, 77	Doyle	1100	560	0.36	7.4	3.2	0.33	1.42	1.3	51.1	36.7	Slope surveyed at site
	Apr.20, 78	Doyle & Andres	800	410	0.35	8.1	4.0	0.31	1.64	0.9	72.2	56.4	Slope surveyed at site
	Apr.21, 78	Doyle & Andres	600	405	0.35	7.4	4.0	0.32	1.62	0.8	60.8	52.2	Slope surveyed at site
	Apr.30, 79	Doyle & Andres	1150	680	0.30	8.1	3.8	0.40	1.30	1.2	48.6	39.7	Slope surveyed at site
Athabaska R. near Pelican Rapids	≈ 17th of Apr. 78	Doyle & Andres	450	193	0.75	8.7	4.7	0.56	1.27	0.6	62.5	60.1	Data based on high ice marks after jam release; slope taken from topographic maps

TABLE 2. - Additional Case Studies (Writer's Data)

Location (1)	Date (2)	Q in meters cubed per second (3)	S in meters per kilometers (4)	W in meters (5)	H in meters (6)	ξ (7)	η (8)	Probable Condition of Jam (9)
Smoky R. below Hunting Creek	Apr. 7, 1977	400	0.86	145	4.6	77.3	36.8	Evolving
Smoky R. at Watino	Apr. 7, 1977	456	0.52	250	4.1	66.7	31.5	Evolving
Smoky R. near mouth	Apr. 30, 1979	1360	0.72	280	8.0	74.6	39.8	Evolving
Smoky R. near mouth	Apr. 29, 1979	1270	0.72	286	9.2	68.8	44.9	Evolving
Peace R. below Peace River	May 1, 1979	3930	0.15	600	9.3	342	103	Equilibrium
Heart R. near mouth	Apr. 8-9, 1977	10.5-13.3	4.36	36	2.8	8.0-9.4	17.8	Equilibrium
Thames R. near Middlemiss	Jan. 14, 1980	100	0.05	45	4.8	1766	584	Equilibrium
Thames R. near Bothwell	Jan. 14, 1980	165	0.26	56	4.4	1002	296	Equilibrium
Thames R. near Fairfield Museum	Mar. 18, 1980	130	0.81	44	4.2	290	118	Evolving

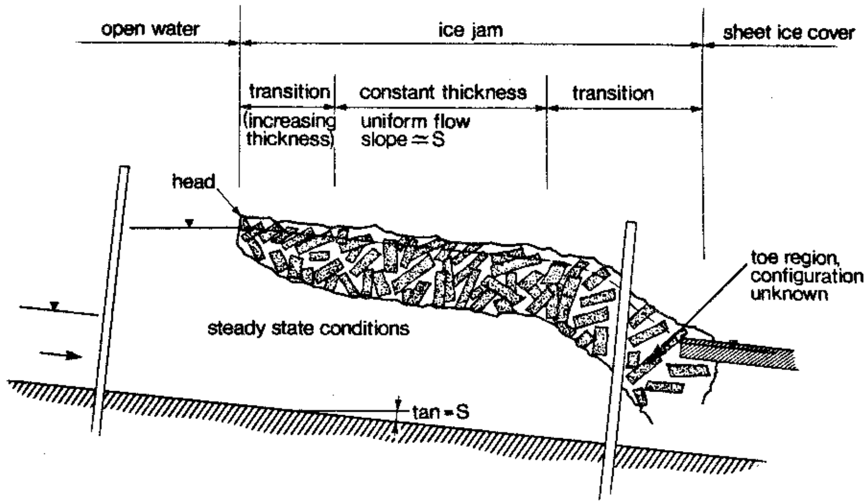

Fig. 1 Schematic illustration of an equilibrium jam

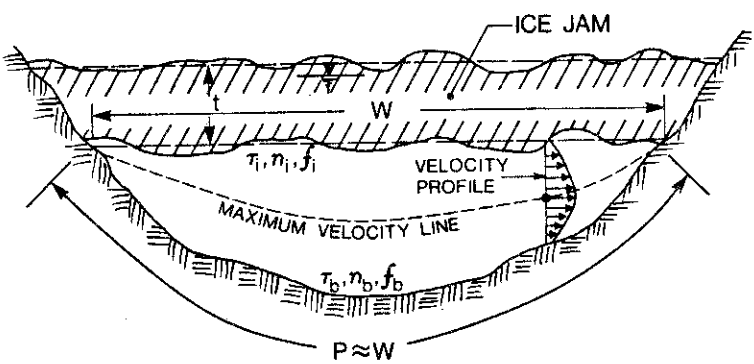

Fig. 2 River cross section within the equilibrium thickness region of a floating jam

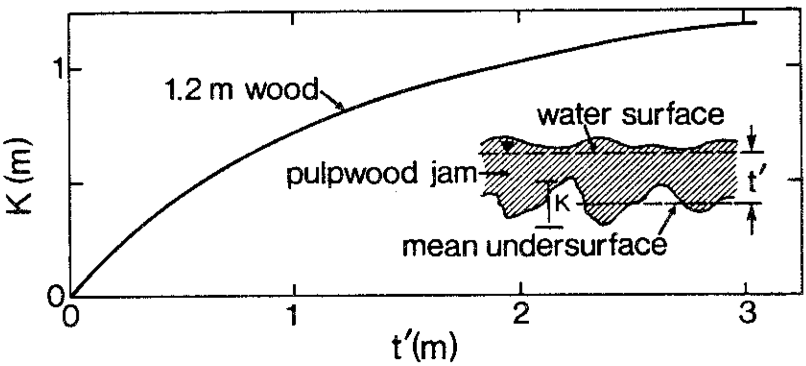

Fig. 3 R. J. Kennedy's roughness-thickness relationship for log jams (note $t'=s_w t$; s_w=specific gravity of wood)

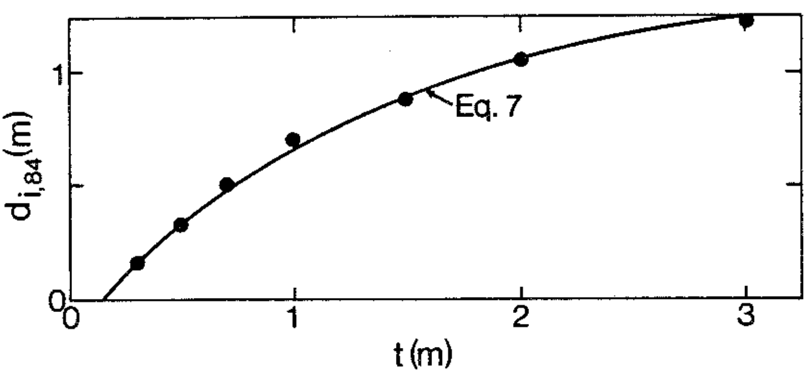

Fig. 4 Variation of $d_{i,84}$ with t as deduced from Nezhikhovskiy's results

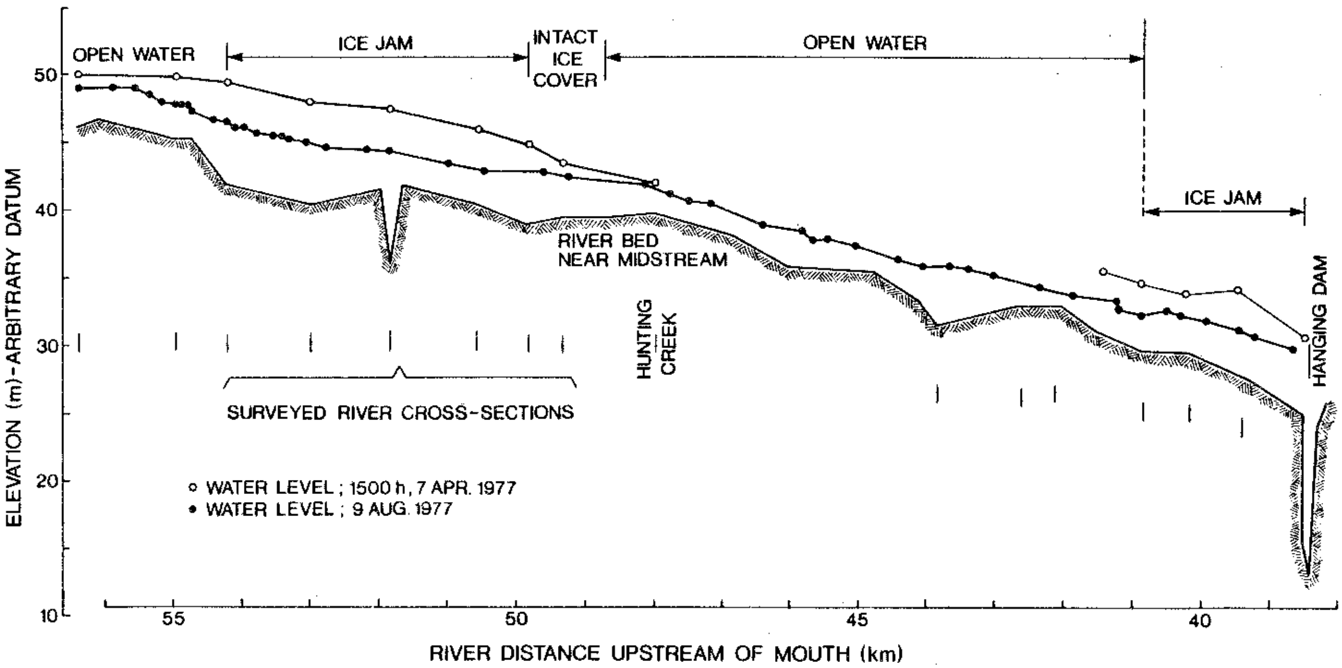

Fig. 5 Profiles of lower Smoky River under open-water and breakup (Apr.'77) conditions (38 to 56 km above the mouth)

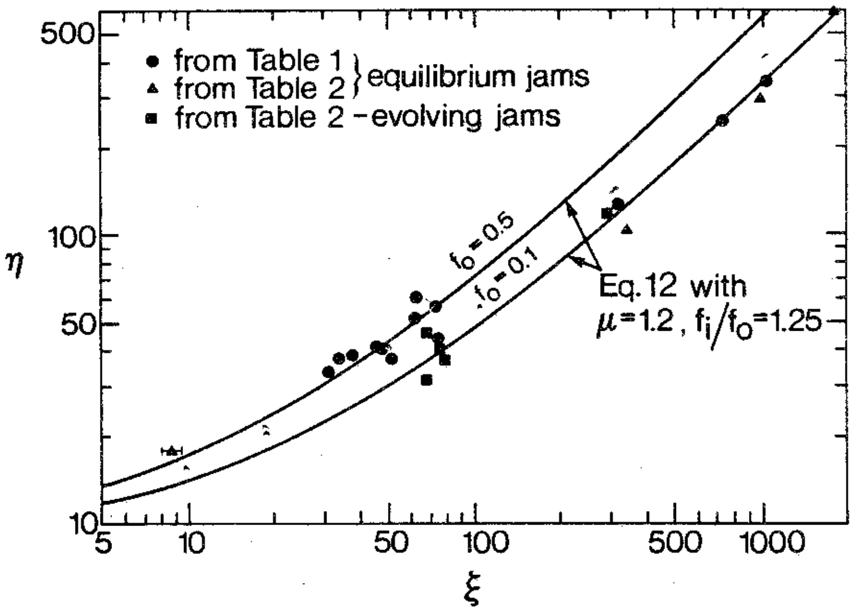

Fig. 6 Test of theory, Eq. 12

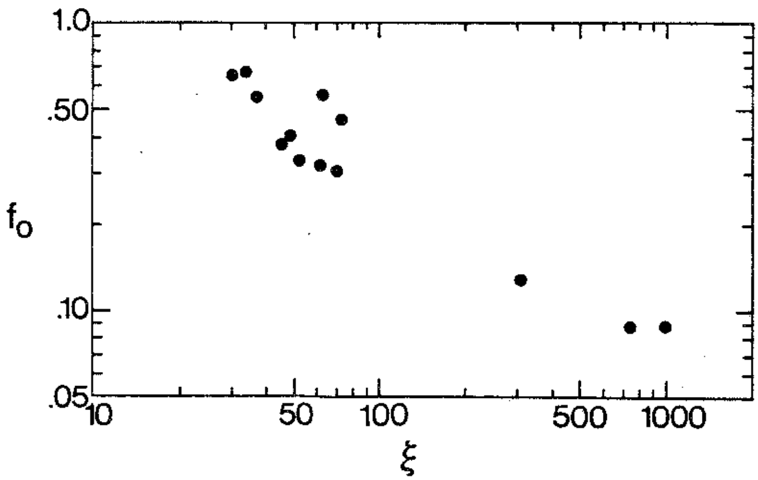

Fig. 7 Variation of f_o with ξ (Data from Table 1)