HYDRAULIC RESISTANCE GENERATED BY FRAZIL ICE FORMATION

by

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(An Extended Abstract)

Frazil ice formation in rivers during freeze-up is characterized by distinct changes in stage, discharge and velocity profile. Field observations during frazil formation in Goldstream Creek near Fairbanks suggest that there are significant changes in hydraulic conditions in a river due to frazil, anchor, and edge ice formation at a time before the development of a congealed ice cover. We have devised several simple models in an effort to quantify the direct effect of frazil ice particles on flow resistance.

Modelling of the forces responsible for increased resistance on frazil particles requires distinguishing between frazil ice concentrations which are well mixed throughout the water column and those which may be considered two layer flow. The former state exists when turbulent diffusion is sufficiently high to keep particles uniformly mixed through the depth, while the latter case occurs when buoyant forces are sufficient to lift frazil particles to the surface. To distinguish these cases we establish two time scales, called the buoyant time scale, T_B and the diffusive time scale, T_D. These scales represent the time required for a frazil particle to rise by buoyancy from the river bottom to the water surface, and the time required for a frazil particle to be transported by turbulent diffusion through the depth. The ratio of these time scales T_B/T_D describes the nature of the layering; in particular, if $T_B/T_D << 1$, then buoyant forces can lift a particle much faster than turbulent diffusion can redistribute it, and the flow will be layered. Conversely, if $T_B/T_D >> 1$ the time required for turbulent diffusion through the depth is small relative to the time required for buoyancy to act thus the flow is well mixed.

The buoyant time scale is by definition equal to the river depth divided by the rise velocity of a frazil particle. This upward velocity is determined by a simple force balance on an individual frazil disc. When the assumption is made of a constant ratio between frazil diameter and thickness, it may be shown that the rise velocity of a frazil disc is directly proportional to its diameter. The diffusive time scale is determined from Taylor's (1953) model for dispersion in a homogeneous medium, namely, $T_D = h^2/2\ D_T$ where h represents the river depth, and D_T is the diffusion coefficient, ($D_T = .067\ h\ u^*$), as given by Fisher (1973) for river and open channel flow.

Applying the above analysis it may be shown that $T_B/T_D << 1$, the case of layered flow, corresponds to the criteria $u << 1$ m/sec for frazil particles of diameter 2 mm or less, where u is the mean river velocity. In the general case, the criteria $T_B/T_D << 1$ corresponds to the criteria $u/(v/0.009) << 1$ where v is the rise velocity of a frazil particle, a function of the particle diameter. This relationship may be considered a critical velocity formulation for the development of a layered flow from an initial well mixed condition. It also establishes a critical frazil particle size where larger particles tend to rise and smaller frazil particles will remain in suspension.

Two flow situations have been distinguished by this time scale analysis; the first case is well mixed flow, i.e. $T_B/T_D >> 1$, and the second is layered flow $T_B/T_D << 1$. For the well mixed flow case, two models of increased resistance due to frazil particles are suggested. The first is the Einstein-Stokes model, from sediment theory, which simulates

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a disturbance in the flow in the vicinity of spherical particles. This disturbance is due to the change in the pressure field in the wake of a particle, i.e. a pressure drag or form drag model. Einstein's effective viscosity is $\mu_{eff} = [1+5\alpha/2]$ where α is the volume fraction of ice and μ_w is the viscosity of water. The second model determines the surface drag on an individual disc shaped frazil particle. The governing equation is determined from a simple force balance in the streamwise direction on a frazil particle. This model predicts a net increase in shear force proportional to $\alpha(\rho_w - \rho_i)/\rho_w$ where ρ_w and ρ_i are densities of water and ice respectively. Thus, both the Einstein form drag model, and the shear drag model predict increased flow resistance of less than ten percent in contrast to measured velocity changes of up to 30 percent. These models are, of course, laminar flow models, and do not consider flocculation processes. They are, however, an initial investigation of flow resistance due to frazil particles without consideration of ice cover formation, and give some indication of a new direction for research in this area.

Finally, for the layered flow case, $T_B/T_D<<1$, we present a simple two layer viscous flow model, assuming constant but different viscosity and density in the top and bottom layers. Boundary conditions are given as no slip at the river bottom and continuous velocity at the interface, which is also assumed to be the location of the velocity maximum. The predicted velocity distribution is plotted in Figure 1, where velocity measurements in Goldstream Creek during a period of frazil ice formation are shown for comparison. The agreement is remarkably good considering the assumptions employed. One pertinent result is the ratio of ice drag at the surface to surface drag at the river bottom.

$$\frac{\tau\,(y = h_1 + h_2)}{\tau\,(y = 0)} = \frac{\rho_2\, h_2}{\rho_1\,(h_1 + h_2)}$$

Thus the model predicts a surface drag due to a stepwise change in the viscosity of the water. An extension of the model is planned to include more realistic river conditions, especially turbulent diffusion and momentum transfer at the interface.

All of the models described above are simple analyses of water - frazil ice interaction. Their further development depends upon the availability of data concerning the concentration and properties of frazil during its initial formation. Extension of these theoretical models must therefore be coupled with appropriate programs for field and laboratory measurements.

References

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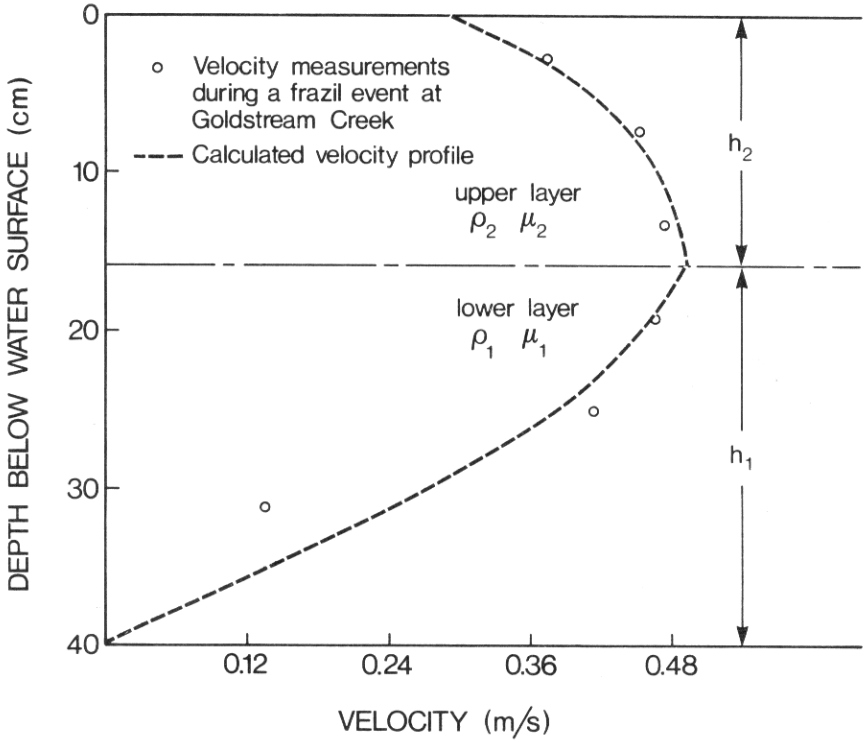

FIGURE 1. TWO LAYER VISCOUS FLOW MODEL – PARABOLIC SOLUTION.